MALAYA JOURNAL OF MATEMATIK
Malaya J. Mat. **13(01)**(2025), 46–53.
http://doi.org/10.26637/mjm1301/006

Soft β^*-open sets in soft topological spaces

Abeda S Dodamani *[1], **Leonard Winston Aiman**‡[2]

[1] *Department of Mathematics, University of Eswatini, Kwaluseni Campus, Eswatini.*
[2] *Department of Mathematics, Dayananda Sagar University, Bengaluru, Karnataka, India.*

***R**eceived 15 April 2024; **A**ccepted 10 October 2024*

Abstract. In this present paper we define the concepts of soft β^*-open sets, soft β^*-closed sets in soft topological space and study some of their properties. Further, the notion of soft β^*-interior and soft β^*-closure of a set are introduced and their basic properties are explored.
Keywords: soft set, soft toplogical space, soft β^*-open set, soft β^*-closed set, soft β^*-interior, soft β^*-closure.

Contents

1. Introduction and Background

In tradition set theory, elements of a set either belongs to it or it doesn't. However in practical applications, there are often situations where the memberships of elements in a set is not clear leading to the need of a more flexible approach. It was to deal such uncertainties and vagueness in a data, Molodstov [16] in 1999 introduced soft sets as a general mathematical framework.

In soft sets, instead of a crisp membership function, we have a characteristic function which assigns a degree of belongingness to each element with respect to the set. This allows for a more nuanced representation of uncertainty where elements can have partial or full membership in a set. In areas like Perron integration, Reimann integration etc, soft set theory was applied with great success.

Continuing the work, Maji et al. [12] in 2002 studied soft sets and their properties. It was in 2011 that Shabir and Naz [25]proposed soft topological space that are determined over an initial universe having parameters that are fixed. They also defined soft open sets, soft closed sets, soft closure and soft interior along with soft separation axioms for soft topological spaces.

***Corresponding author.** Email addresses: `abeda.dodamani@gmail.com` (Abeda S Dodamani), `leonard.aiman@sju.edu.in` (Leonard Winston Aiman)
‡**Corresponding author.** Email addresses: `abeda.dodamani@gmail.com` (Abeda S Dodamani), `leonard.aiman@sju.edu.in` (Leonard Winston Aiman)

Many researchers like Zorlutuna et al.[27], Aygunoglu et al.[7] and Hussain et al.[10] further contributed to the study of soft topology.

Chen [8]was the first to investigate and study the weaker forms of soft open sets along with it's properties. His work on soft semi-open sets is highly significant.

Researchers like Akdag and Ozkan [13] who introduced soft α-open sets then continued the study of soft β-open sets[14]. Arockiarani and Arokialancy [6]proposed β-open sets and studied its properties. Alzahrani Samirah et al.[4] in 2022 worked on soft γ-open sets. They also presented its properties. The study on weak forms of soft open sets is on the rise now as great amount of work is being done [1–3, 5, 9, 11, 15, 21–24, 26].

The study on soft β-open sets where continued by Benchalli et al. who investigated and came up with soft β-separation axioms[17], soft β-connected space[18], soft β-compactness [19, 20] and soft β-first countable space and soft β-second countable space.

Here in our work, we define soft β^*-open sets and study some of its properties. We further proceed to define soft β^*-interior and soft β^*-closure and explore its characteristics. The basic definitions and findings that we will utilize throughout the paper are provided. Throughout the entire paper, we will consider (X, τ, E) to be soft topological space.

Definition 1.1. *[18] Let the initial universe be X and E be the set of parameters. We shall denote $P(X)$ to be the power set of X. If we have a non empty subset A of E, then the pair (F, A) is called a soft set over X where F is a mapping $F : A \to P(X)$.*
In other words, a soft set over X is a parameterized family of subsets of X. For $\epsilon \in A, F(\epsilon)$ can be considered as a set of ϵ-approximate elements of soft set (F, A). Clearly a soft set is not a set.

Definition 1.2. *[7] If τ is the collection of soft sets over X, then τ is a soft topology on X if the following conditions hold:*

1. *X, ϕ belong to τ.*
2. *Union of any number of soft sets in τ belongs to τ.*
3. *Intersection of any two sets in τ belongs to τ.*

Definition 1.3. *[14] Let (X, τ, E) be a soft topological space over X and (F, E) be the soft set over X. Then*

1. *Soft interior of (F, E) is the soft set int $(F, E) = \cup\{(O, E) : (O, E)$ which is soft open and $(O, E) \subset (F, E)\}$.*
2. *Soft closure of (F, E) is the soft set cl$(F, E) = \cap\{(G, E) : (G, E)$ which is soft open and $(F, E) \subset (G, E)\}$.*

   Clearly cl(F, E) is the smallest soft closed set over X which contains (F, E) and int(F, E) is the largest soft open set over X which is contained in (F, E).

Definition 1.4. *[13] A soft set (A, E) of a soft topological space (X, τ, E) is a soft α- open set if*

$$(A, E) \subset int(cl(int(A, E))).$$

The compliment of soft α-open set is soft α-closed set.

Definition 1.5. *[13] A soft set (A, E) of a soft topological space (X, τ, E) is a soft β- open set if*

$$(A, E) \subset cl(int(cl(A, E))).$$

The compliment of soft β-open set is soft β-closed set.

Definition 1.6. *[18] Let (X, τ, E) be a soft topological space and (F_1, E) and (F_2, E) be two soft β-open sets over X. Then these soft β-open sets are said to be soft β-separated if*

$$(F_1, E) \cap s\beta cl(F_2, E) = \phi \text{ and } s\beta cl(F_1, E) \cap (F_2, E) = \phi$$

Definition 1.7. *[18] Let (X, τ, E) be a soft topological space over X. Then (X, τ, E) is soft β- connected if there does not exist a pair (F_1, E) and (F_2, E) of non-empty disjoint open subsets of (X, τ, E) such that $(F_1, E) \cup (F_2, E) = X$. Otherwise (X, τ, E) is said to be soft β-disconnected. In this case, the pair (F_1, E) and (F_1, E) are called soft β-disconnection of X.*

2. Main Results

Definition 2.1. *A soft set (D, E) in a soft topological space (X, τ, E) is said to be*

1. *Soft β^*-open if $(D, E)\tilde{\subseteq}cl(int(cl((D, E))))\tilde{\cup}int(cl_\delta((D, E)))$.*

2. *Soft β^*-closed if $int(cl(int((D, E))))\tilde{\cap}cl(int_\delta((D, E)))\tilde{\subseteq}(D, E)$.*

The soft complement of soft β^*-open set is soft β^*-closed set.

2.1. Remark

The chart below holds for every soft set (D, E) of X.

soft open set $\rightarrow$ soft α-open set $\rightarrow$ soft semi-open set $\rightarrow$ soft γ-open set $\rightarrow$ soft β-open set $\rightarrow$ soft β^*-open set.

These implications need not be reversible.

2.2. Example

Let $X = \{h_1, h_2, h_3\}, E = \{e_1, e_2\}$ and $\tau = \{\phi, \tilde{X}, (D_1, E), (D_2, E), (D_3, E), (D_4, E), (D_5, E), (D_6, E)\}$ where $(D_1, E), (D_2, E), (D_3, E), (D_4, E), (D_5, E), (D_6, E)$ are soft open sets over X, which are defined as follows:
$(D_1, E) = \{(e_1, \{h_2\})\}$, $(D_2, E) = \{(e_1, \{h_2\}), (e_2, \{h_1\})\}$, $(D_3, E) = \{(e_1, \{h_2, h_3\}), (e_2, \{h_1, h_2\})\}$,
$(D_4, E) = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$, $(D_5, E) = \{(e_1, \{h_1, h_2\}), (e_2, \{h_1\})\}$,
$(D_6, E) = \{(e_1, \{h_2\}), (e_2, \{h_2, h_3\})\}$.
Thus τ is a soft topology on X and hence (X, τ, E) forms a soft topology over the space X.
Understandably the soft closed sets are $\tilde{X}, \phi, (D_1, E)^c, (D_2, E)^c, (D_3, E)^c, (D_4, E)^c, (D_5, E)^c, (D_6, E)^c$.

The soft set (D, E) given by $(D, E) = \{(e_1, \{h_2\}), (e_2, \{h_2, h_3\})\}$ is soft β^*-open set but it is not soft β-open set.

Theorem 2.2. *In a soft topological space (X, τ, E), the arbitrary union of soft β^*-open sets is soft β^*-open.*

Proof. Let $\{(D_i, E) : i \in I\}$ is a collection of soft β^*-open sets. Then $\forall i \in I$, $(D_i, E) \subseteq cl(int(cl((D_i, E))))$ $\tilde{\cup}int(cl_\delta((D_i, E)))$. Hence it implies that $\tilde{\bigcup}_{i\in I}(D_i, E)\tilde{\subseteq}\tilde{\bigcup}_{i\in I}cl(int(cl((D_i, E))))\tilde{\cup}int(cl_\delta((D_i, E)))\tilde{\subseteq}cl(int(cl(\tilde{\bigcup}_{i\in I}(D_i, E))))\tilde{\cup}int(cl_\delta(\tilde{\bigcup}_{i\in I}(D_i, E)))$.
Therefore arbitrary union of soft β^*-open sets is soft β^*-open. i.e. $\tilde{\bigcup}_{i\in I}(D_i, E)$ is soft β^* open. ■

Intersecting any two soft β^*-open sets may not be soft β^*-open, as demonstrated in the example below.

2.3. Example:

For the example mentioned in Example 2.2, consider two soft β^*-open sets $(A,E)=\{(e_1,\{h_1\}),(e_2,\{h_1\})\}$ and $(B,E)=\{(e_1,\{h_2\}),(e_2,\{h_2,h_3\})\}$. Let their intersection be, $(A,E)\tilde{\cap}(B,E)=(C,E)$ and hence $(C,E)=\{(e_1,\phi\}),(e_2,\phi\})\}$. So we get $cl(int(cl((C,E))))\tilde{\cup}int(cl_\delta(C,E))=\phi$. Thus (C,E) is not soft β^*-open.

Theorem 2.3. *Let (X,τ,E) be a soft topological space. Then the arbitrary intersection of soft β^*-closed sets is soft β^*-closed.*

Proof. Let $\{(D_i,E) : i \in I\}$ be a collection of soft β^*-closed sets. Then $\forall i \in I, int(cl(int((D_i,E))))\tilde{\cup}cl(int_\delta((D_i,E)))\tilde{\subseteq}(D_i,E)$. This implies that $\tilde{\bigcap}_{i\in I}[int(cl(int((D_i,E))))\tilde{\cup}cl(int_\delta((D_i,E)))]\tilde{\subseteq}\tilde{\bigcap}_{i\in I}(D_i,E)$ and hence $int(cl(int(\tilde{\bigcap}_{i\in I}(D_i,E))))\tilde{\cup}cl(int_\delta(\tilde{\bigcap}_{i\in I}(D_i,E)))\tilde{\subseteq}\tilde{\bigcap}_{i\in I}(D_i,E)$. Therefore arbitrary intersection of soft β^*-closed sets is a soft β^*-closed set. ■

The example to follow illustrates that union of two soft β^*-closed sets is not necessarily a soft β^*-closed set.

2.4. Example:

For the example mentioned in Example 2.2, consider two soft β^*-closed sets $(A,E)=\{(e_1,\{h_1\}),(e_2,\{h_1\})\}$ and $(B,E)=\{(e_1,\{h_2\}),(e_2,\{h_2,h_3\})\}$. Let their union be, $(A,E)\tilde{\cup}(B,E)=(C,E)$ and hence $(C,E)=\{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2,h_3\})\}$. So we get $int(cl(int((C,E))))\tilde{\cap}cl(int_\delta((C,E)))=\{(e_1,\{h_1,h_2,h_3\}),(e_2,\{h_1,h_2,h_3\})\}\not\subseteq\{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2,h_3\})\}$. Therefore the union of two soft β^*-closed sets is not β^*-closed set.

Proposition 2.4. *For every proper soft subset of soft indiscrete structure (X,τ,E), the subsequent results hold true.*

1. *Each proper soft subset of X is soft β^*-open set of X but not soft semi-open set of X.*
2. *Soft pre-open set of X = soft γ-open set of X = soft β-open set of X = soft β^*-open set of X.*

Proof. 1. Consider (D,E) to be a proper soft subset of a soft indiscrete structure. Then $cl(int(cl((D,E))))\tilde{\cup}int(cl_\delta((D,E)))=X$ and $cl(int((D,E))=\phi$. Thus every proper soft subset on X is a soft β^*-open set of X but not soft semi-open set of X.

2. As for each proper soft subset (D,E) of a soft indiscrete structure (X,τ,E) we have $int(cl((D,E)))=X$, $int(cl((D,E)))\tilde{\cup}cl(int((D,E)))=X$, $cl(int(cl((D,E))))=X$ and $cl(int(cl((D,E))))\tilde{\cup}int(cl_\delta((D,E)))=X$. Hence soft pre-open set of X = soft γ-open set of X = soft β-open set of X = soft β^*-open set of X.

■

Theorem 2.5. *For any soft β^*-open set (K,E), if (G,E) belongs to soft open set of X, then $(K,E)\tilde{\cap}(G,E)$ belongs to soft β^*-open set of X.*

Proof. Consider a soft β^*-open set. Let (G,E) be a soft open set. Then $(K,E)\cap(G,E)\tilde{\subseteq}[cl(int(cl((K,E))))\tilde{\cup}int(cl_\delta((K,E)))]\tilde{\cap}(G,E)\tilde{\subseteq}$
$[cl(int(cl((K,E))))\tilde{\cap}(G,E)]\tilde{\cup}[int(cl_\delta((K,E))\tilde{\cap}(G,E)]\tilde{\subseteq}cl[int(cl((K,E)))\tilde{\cap}(G,E)]$
$\tilde{\cup}int[cl_\delta((K,E))\tilde{\cap}(G,E)]\tilde{\subseteq}cl(int[cl((K,E))\tilde{\cap}(G,E)]\tilde{\cup}int(cl_\delta[(K,E)\tilde{\cap}(G,E)]\tilde{\subseteq}$
$cl(int(cl[(K,E)\tilde{\cap}(G,E)]\tilde{\cup}int(cl_\delta[(K,E)\tilde{\cap}(G,E)]$. Therefore $(K,E)\tilde{\cap}(G,E)\in$ soft β^*-open set. ■

For any soft β^*-closed set (N,E), if $(M,E)\in$ soft closed set then $(N,E)\tilde{\cap}(M,E)\in$ soft β^*-closed set.

Definition 2.6. *Let (X,τ,E) be a soft topological space and let (M,E) be a soft set over X.*

1. ***Soft*** β^****-interior*** *of a soft set (M,E) is represented as $s\beta^*int((M,E))$ and is defined as $s\beta^*int((M,E)) = \tilde{\bigcup}\{(D,E) : (D,E)$ is a soft β^*-open set and $(D,E)\tilde{\subset}(M,E)\}$.*
   i.e. $s\beta^* - int((M,E))$ is the largest soft β^*-open set over X which is contained in (M,E).

2. ***Soft*** β^****-closure*** *of a soft set (M,E) is represented as $s\beta^*cl((M,E))$ and is defined as $s\beta^*cl((M,E)) = \tilde{\bigcap}\{(P,E) : (P,E)$ is a soft β^*-closed set and $(M,E)\tilde{\subset}(P,E)\}$.*
   i.e. $s\beta^*cl((M,E))$ is the smallest soft β^*-closed set over X containing (M,E).

Theorem 2.7. *Consider a soft topological space (X,τ,E) with a soft set (M,E) over X. Then (M,E) is soft β^*-open set if and only if $(M,E) = s\beta^*int((M,E))$.*

Proof. Let $(M,E) = s\beta^*int((M,E)) = \tilde{\bigcup}\{(G,E) : (G,E)$ is a soft β^* open set with $(G,E)\tilde{\subset}(M,E)\}$. It shows that $(M,E) \in \{(G,E) : (G,E)$ is a soft β^*-open set and $(G,E)\tilde{\subset}(M,E)\}$. Hence (M,E) is soft β^*-open set. Conversely, let (M,E) is soft β^*-open set. Since $(M,E)\tilde{\subseteq}(M,E)$ and (M,E) is soft β^*-open set, $(M,E) \in \{(G,E) : (G,E)$ is a soft β^*-open set and $(G,E)\tilde{\subset}(M,E)\}$. Further, $(M,E)\tilde{\subset}(G,E)$ for all such $(M,E)s$. This means that $(M,E) = \tilde{\bigcup}\{(G,E) : (G,E)$ is a soft β^*-open set and $(G,E)\tilde{\subset}(M,E)\}$. ■

Theorem 2.8. *Consider a soft topological space (X,τ,E) with a soft set (M,E) over X. Then (M,E) is soft β^*-closed set if and only if $(M,E) = s\beta^*cl((M,E))$.*

Proof. Let $(M,E) = s\beta^*cl((M,E)) = \tilde{\bigcap}\{(F,E) : (F,E)$ is a soft β^*-closed set and $(M,E)\tilde{\subset}(F,E)\}$. This shows that $(M,E) \in \{(F,E) : (F,E)$ is a soft β^*-closed set and $(M,E)\tilde{\subset}(F,E)\}$. Hence (M,E) is soft β^*-closed set. Conversely, let (M,E) is soft β^*-closed set. Since $(M,E)\tilde{\subseteq}(M,E)$ and (M,E) is soft β^*-closed set, $(M,E) \in \{(F,E) : (F,E)$ is a soft β^*-closed set and $(M,E)\tilde{\subset}(F,E)\}$. Further, $(M,E)\tilde{\subset}(F,E)$ for all such $(F,E)s$. This means that $(M,E) = \tilde{\bigcap}\{(F,E) : (F,E)$ is a soft β^*-closed set and $(M,E)\tilde{\subset}(F,E)\}$. ■

Theorem 2.9. *For a soft topological space (X,τ,E), let (P,E) and (Q,E) be two soft sets of the space X. Then,*

1. *$s\beta^*cl(\phi) = \phi$ and $s\beta^*cl(X) = X$.*
2. *$s\beta^*cl(s\beta^*cl((P,E))) = s\beta^*cl((P,E))$.*
3. *$(P,E)\tilde{\subseteq}(Q,E) \Longrightarrow s\beta^*cl((P,E))\tilde{\subseteq}s\beta^*cl((Q,E))$.*
4. *$s\beta^*cl((P,E))^c = s\beta^*int((Q,E)^c)$.*
5. *$s\beta^*cl((P,E)\tilde{\cup}(Q,E)) = s\beta^*cl((P,E))\tilde{\cup}s\beta^*cl((Q,E))$.*
6. *$s\beta^*cl((P,E)\tilde{\cap}(Q,E))\tilde{\subset}s\beta^*cl((P,E))\tilde{\cap}s\beta^*cl((Q,E))$.*

Proof.
1. As ϕ and X are soft β^* closed sets hence $s\beta^*cl(\phi) = \phi$ and $s\beta^*cl(X) = X$.
2. Since $s\beta^*cl((P,E))$ belongs to soft β^*-closed set in (X,τ,E), by Theorem 2.8 $s\beta^*cl(s\beta^*cl((P,E))) = s\beta^*cl((P,E))$.
3. By definition, $s\beta^*cl((P,E)) = \tilde{\bigcap}\{(R,E) : (R,E)$ is a soft β^*-closed set and $(P,E)\tilde{\subset}(R,E)\}$ and $s\beta^*cl((Q,E)) = \tilde{\bigcap}\{(S,E) : (S,E)$ is a soft β^*-closed set and $(Q,E)\tilde{\subset}(S,E)\}$. Since $(P,E)\tilde{\subseteq}s\beta^*cl((P,E))$ and $(Q,E)\tilde{\subseteq}s\beta^*cl((Q,E))$. Now, $(P,E)\tilde{\subseteq}(Q,E)\tilde{\subseteq}s\beta^*cl((Q,E))$ which implies $(P,E)\tilde{\subseteq}s\beta^*cl((Q,E))$. But $s\beta^*cl((P,E))$ is the smallest soft $s\beta^*$ closed set containing (P,E). Therefore $s\beta^*cl((P,E))\tilde{\subseteq}s\beta^*cl((Q,E))$.
4. $[s\beta^*cl((P,E))]^c = [\tilde{\bigcap}\{(R,E) : (R,E)$ is a soft β^*-closed and $(P,E)\tilde{\subset}(R,E)\}]^c = \tilde{\bigcup}\{(R,E)^c : (R,E)^c$ is a soft β^*-open set and $(R,E)^c\tilde{\subset}(P,E)^c\} = s\beta^*int((P,E)^c)$.

5. We have $(P,E)\tilde{\subset}(P,E)\tilde{\cup}(Q,E)$ and $(Q,E)\tilde{\subset}(P,E)\tilde{\cup}(Q,E)$. Therefore from third result of this theorem we have $s\beta^*cl((P,E))\tilde{\subset}s\beta^*cl((P,E)\tilde{\cup}(Q,E))$ and $s\beta^*cl((Q,E))\tilde{\subset}s\beta^*cl(P,E)\tilde{\cup}(Q,E))$. This implies $s\beta^*cl((P,E))\tilde{\cup}s\beta^*cl((Q,E))\tilde{\subset}s\beta^*cl((P,E)\tilde{\cup}(Q,E)) \rightarrow (1)$. Since $s\beta^*cl((P,E))$ and $s\beta^*cl((Q,E))$ are soft β^*-closed sets in (X,τ,E), we have $s\beta^*cl((P,E))\tilde{\cup}s\beta^*cl((Q,E))$ is also soft β^*-closed set in (X,τ,E). Then $(P,E))\tilde{\subset}s\beta^*cl((P,E)$ and $(Q,E))\tilde{\subset}s\beta^*cl((Q,E)$ which implies $(P,E)\tilde{\cup}(Q,E)\tilde{\subset}s\beta^*cl((P,E))\tilde{\cup}s\beta^*cl((Q,E))$. That is $s\beta^*cl((P,E))\tilde{\cup}s\beta^*cl((Q,E))$ is a soft β^*-closed set containing $(P,E)\tilde{\cup}(Q,E)$. Hence $s\beta^*cl((P,E)\tilde{\cup}(Q,E))\tilde{\subset}s\beta^*cl((P,E))\tilde{\cup}s\beta^*cl((Q,E)) \rightarrow (2)$ From (1) and (2) $s\beta^*cl((P,E)\tilde{\cup}(Q,E))=s\beta^*cl((P,E))\tilde{\cup}s\beta^*cl((Q,E))$.

6. We have $(P,E)\tilde{\cap}(Q,E)\tilde{\subset}(P,E)$ and $(P,E)\cap(\tilde{Q},E)\tilde{\subset}(Q,E)$. This implies $s\beta^*cl((P,E)\tilde{\cap}(Q,E))\tilde{\subset}s\beta^*cl((P,E))$ and $s\beta^*cl((P,E)\tilde{\cap}(Q,E))\tilde{\subset}s\beta^*cl((Q,E))$. Therefore, $s\beta^*cl((P,E)\tilde{\cap}(Q,E))\tilde{\subset}s\beta^*cl((P,E))\tilde{\cap}s\beta^*cl((Q,E))$.

■

Theorem 2.10. *For a soft topological space (X,τ,E), let (P,E) and (Q,E) be two soft sets of the space X.Then,*

1. $s\beta^*int(\phi)=\phi$ *and* $s\beta^*int(X)=X$.
2. $s\beta^*int(s\beta^*int((P,E)))=s\beta^*int((P,E))$.
3. $(P,E)\tilde{\subseteq}(Q,E)\Longrightarrow s\beta^*int((P,E))\tilde{\subseteq}s\beta^*int((Q,E))$.
4. $(s\beta^*int((P,E)))^c=s\beta^*int((P,E)^c)$.
5. $s\beta^*int((P,E))\tilde{\cup}s\beta^*int((Q,E))\tilde{\subset}s\beta^*int((P,E)\tilde{\cup}(Q,E))$.
6. $s\beta^*int((P,E)\tilde{\cap}(Q,E))=s\beta^*int((P,E))\tilde{\cap}s\beta^*int((Q,E))$.

Proof. 1. As ϕ and X are soft β^* open sets hence $s\beta^*int(\phi)=\phi$ and $s\beta^*int(X)=X$.

2. Since $s\beta^*int((P,E))$ belongs to soft β^*-open set in (X,τ,E), by Theorem 2.7, $s\beta^*int(s\beta^*int((P,E)))=s\beta^*int((P,E))$.

3. By definition, $s\beta^*int((P,E)) = \tilde{\bigcup}\{(R,E) : (R,E)$ is a soft β^*-open set and $(R,E)\tilde{\subset}(P,E)\}$ and $s\beta^*int((Q,E)) = \tilde{\bigcup}\{(S,E) : (S,E)$ is a soft β^*-open set and $(S,E)\tilde{\subset}(Q,E)\}$. Since $s\beta^*int((P,E))\tilde{\subseteq}(P,E)$ and $s\beta^*int((Q,E))\tilde{\subseteq}(Q,E)$. Now, $s\beta^*int((P,E))\tilde{\subseteq}(P,E)\tilde{\subseteq}(Q,E)$ which implies $s\beta^*int((P,E))\tilde{\subseteq}(Q,E)$. Since $s\beta^*int((Q,E))$ is the largest soft β^*-open set contained in (Q,E). Therefore $s\beta^*int((P,E))\tilde{\subseteq}s\beta^*int((Q,E))$.

4. $[s\beta^*int((P,E))]^c = [\tilde{\bigcup}\{(R,E) : (R,E)$ is a soft β^*-open set and $(R,E)\tilde{\subset}(P,E)\}]^c = \tilde{\bigcap}\{(R,E)^c : (R,E)^c$ is a soft β^*-closed set and $(P,E)^c\tilde{\subset}(R,E)^c\} = s\beta^*int((P,E)^c)$.

5. We have $(P,E)\tilde{\subset}(P,E)\tilde{\cup}(Q,E)$ and $(Q,E)\tilde{\subset}(P,E)\tilde{\cup}(Q,E)$. This implies that $s\beta^*int((P,E))\tilde{\subset}s\beta^*int((P,E)\tilde{\cup}(Q,E))$ and $s\beta^*int((Q,E))\tilde{\subset}s\beta^*int((P,E)\tilde{\cup}(Q,E))$. Therefore, $s\beta^*int((P,E))\tilde{\cup}s\beta^*int((Q,E))\tilde{\subset}s\beta^*int((P,E)\tilde{\cup}(Q,E))$.

6. Since $(P,E)\tilde{\cap}(Q,E)\tilde{\subset}(P,E)$ and $(P,E)\cap(\tilde{Q},E)\tilde{\subset}(Q,E)$. This implies $s\beta^*int((P,E)\tilde{\cap}(Q,E))\tilde{\subset}s\beta^*int((P,E))$ and $s\beta^*int((P,E)\tilde{\cap}(Q,E))\tilde{\subset}s\beta^*((Q,E))$. Hence, $s\beta^*int((P,E)\tilde{\cap}(Q,E))\tilde{\subset}s\beta^*int((P,E))\tilde{\cap}s\beta^*int((Q,E)) \rightarrow (1)$. Since $s\beta^*int((P,E))$ and $s\beta^*int((Q,E))$ are soft β^*-open sets in $(X,\tau,E), s\beta^*int((P,E))\tilde{\cap}s\beta^*int((Q,E))$ is also a soft β^*-open set in (X,τ,E). Then $s\beta^*int((P,E))\tilde{\subset}(P,E)$ and $s\beta^*int((Q,E))\tilde{\subset}(Q,E)$. So $s\beta^*int((P,E))\tilde{\cap}s\beta^*int((Q,E))\tilde{\subset}(P,E)\tilde{\cap}(Q,E)$. That is $s\beta^*int((P,E))\tilde{\cap}s\beta^*int((Q,E))$ is a soft

β^*-open set contained in $(P,E)\tilde{\cap}(Q,E)$ $\rightarrow$ (2). From (1) and (2) $s\beta^*int((P,E)\tilde{\cap}(Q,E)) = s\beta^*int((P,E))\tilde{\cap}s\beta^*int((Q,E))$.

■

3. Conclusion

The theory of soft sets is an immensely useful tool to deal uncertainty. In this paper we introduce a weaker form of soft set which is soft β^*open set in soft topological spaces. Soft β^*closed sets are also established and their properties have been investigated. We then define the closure and interior of soft β^* sets and finally examine a few of its properties. These findings will further help in the study of soft topology in the future.

4. Acknowledgement

The author is thankful to the referee for his valuable suggestions which improved the presentation of the paper.

References

[1] ABEDA S. DODAMANI, LEONARD W. AIMAN, On Half Soft β Open Sets, *J. Int. Acad. Phys. Sc.*, **28**(2024), 215-221.

[2] ABD EL-LATIF, ALAA, Soft extra strongly semi star generalized closed sets, *Gazi Univ. J. Sci.*, **30**(2017), 421–430.

[3] ALKAN OZKAN,METIN AKDAG,FETHULLAH EROL., Soft b- compact spaces, *New trends. Math. Sci.*, **4**(2016), 211–219.

[4] ALZAHRANI, SAMIRAH AND NASEF, AA AND YOUNS, N AND EL-MAGHRABI, AI AND BADR, MS, Soft topological approaches via soft γ-open sets, *AIMS Math*, **7**(2022), 12144–12153.

[5] ANJAN MUKHERJEE, BISHNUPADA DEBNATH,, On soft e-Open sets and soft e-Continuity in soft Topological space, *J. New. Theory*, (2017), 1–18.

[6] AROCKIARANI, I AND LANCY, A AROKIA, Generalized soft gβ-closed sets and soft gsβ-closed sets in soft topological spaces, *Intl. J. Math. Arch*, **4**(2013), 1–7.

[7] AYGÜNOĞLU, ABDÜLKADIR AND AYGÜN, HALIS, Some notes on soft topological spaces, *Neural Comput. Applic.*, **21**(2012), 113–119.

[8] CHEN, BIN, Soft semi-open sets and related properties in soft topological spaces, *Appl. Math. Inf. Sci,* **7**(2013), 287–294.

[9] HARIWAN Z. IBRAHIM, , ON SOFT GENERALIZED CLOSED SETS., *TWS J. Appl. Engi. Math.*, **12**(2022).

[10] HUSSAIN, SABIR AND AHMAD, BASHIR, Some properties of soft topological spaces, *Comput. Math. Appl.,* **62**(2011), 4058–4067.

[11] KALAVATHI, A., AND G. SAI SUNDARA KRISHNAN., Soft g* closed and soft g* open sets in soft topological spaces, *J. Interdiscip. Math.,* **19**(2016), 65–82.

[12] MAJI, PK AND ROY, AKHIL RANJAN AND BISWAS, RANJIT, An application of soft sets in a decision making problem, *Comput. Math. Appl.,* **44**(2002), 1077–1083.

[13] METIN AKDAG, ALKAN OZKAN. Soft -Open Sets and Soft -Continuous Functions, *Abst. Appl. Anal.,* **2014**(2014).

[14] METIN AKDAG, ALKAN OZKAN. Soft β-Open Sets and Soft β-Continuous Functions, *The sci. world J.,* **2014**(2014).

[15] METIN AKDAG, ALKAN OZKAN. Soft b-open sets and soft b-continuous functions, *Math. Sci.,* **8**(2014), 1–9.

[16] MOLODTSOV, DMITRIY, Soft set theory—first results, *Comput. Math. Appl.*, **37**(1999), 19–31.

[17] S.S.BENCHALLI, PRAKASH GOUDA PATIL, ABEDA S. DODAMANI, Some Properties of Soft β-Separation Axioms in Soft Topological Spaces, *Int. J. Scient. Innov. Math Research*, **3**(2015), 254–259.

[18] S.S.BENCHALLI, PRAKASH GOUDA PATIL, ABEDA S. DODAMANI, Some Properties of Soft β-Connected Spaces in Soft Topological Spaces, *Int. J. Pure. Math. Sci.*, (2016).

[19] S.S.BENCHALLI, PRAKASH GOUDA PATIL, ABEDA S. DODAMANI, Soft β-Compactness in Soft Topological Spaces, *Math. Sci. Int. Research J.*, **4**(2016), 214–218.

[20] S.S.BENCHALLI, PRAKASH GOUDA PATIL, ABEDA S. DODAMANI, Some Properties of Soft β-Compact and Related Soft Topological Spaces, *Int. J. Pure. Math. Sci.*, **38**(2017), 487–496.

[21] SABIHA I MAHMOOD, On Weak Soft N-Open Sets and Weak Soft DN˜-Sets in Soft Topological Spaces, *Al-Nahrain J. Sci.*, **20**(2017), 131–141.

[22] SANDEEP KAUR, TAREQ M. AL-SHAMI, ALKAN ÖZKAN, AND M. HOSNY , A new approach to soft continuity, *Mathematics*, **11**(2023), 3164.

[23] SAVITA RATHEE, RIDAM GIRDHAR, , On Soft ω-Connectedness in Soft Topological Spaces, *Comput. Math. Appl.*, **12**(2021), 457–474.

[24] SAZIYE YÜKSEL, NAIME TOZLU, ZEHRA ERGÜL Z GÜZEL ERGÜL , Soft regular generalized closed sets in soft topological spaces, (2014).

[25] SHABIR, MUHAMMAD AND NAZ, MUNAZZA, On soft topological spaces, *Comput. Math. Appl.*, **61**(1999), 1786–1799.

[26] YUMAK, YUNUS AND KAYMAKCI, AYNUR KESKIN, Soft β-open sets and their applications, *J. New Theory*, (2015), 80–89.

[27] ZORLUTUNA, IDRIS AND AKDAG, M AND MIN, WK AND ATMACA, S, Remarks on soft topological spaces, *Ann. Fuzzy Math. Inform.*, **2**(2012), 171–185.