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Total dominator color class total dominating sets in Dutch windmill graph and coconut tree graph

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Abstract
Let G be a finite, undirected and connected graph with minimum degree at least one. A proper coloring $\mathscr{C}$ of G is said to be a total dominator color class total dominating set of G if each vertex properly dominates a color class in $\mathscr{C}$ and each color class in $\mathscr{C}$ is properly dominated by a vertex in $\mathrm{V(G)}$. A total dominator color class total dominating set D of G is a minimal total dominator color class total dominating set if no proper subset of D is a total dominator color class total dominating set of G. The total dominator color class total domination number is the minimum cardinality taken over all minimal total dominator color class total dominating sets in G and is denoted by $\gamma_\chi^{td}(G)$. Here we obtain $\gamma_\chi^{td}(G)$ for dutch windmill graph and coconut tree graph.

Keywords
Chromatic number, Domination number, Total domination, Dominator color class dominating set, Total dominator color class total domination number.

AMS Subject Classification
05C15, 05C69.

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1. Introduction

All graphs considered in this paper are finite, undirected graphs with minimum degree at least one and we follow standard definitions of graph theory as found in [8]. Let $\mathrm{G} = (\mathrm{V},\mathrm{E})$ be a connected graph with no isolated vertices. The open neighborhood $\mathrm{N(v)}$ of a vertex $\mathrm{v} \in \mathrm{V(G)}$ consists of the set of all vertices adjacent to v . The closed neighborhood of v is $\mathrm{N[v]} = \mathrm{N(v)U\{v\}}$. For a set $\mathrm{S} \subseteq \mathrm{V}$, the open neighborhood $\mathrm{N(S)}$ is defined to be $\mathrm{U}_{\mathrm{v}\in\mathrm{S}}N(v)$, and the closed neighborhood of S is $\mathrm{N[S]} = \mathrm{N(S)} \cup \mathrm{S}$. For any set H of vertices of G, the induced sub graph $\langle \mathrm{H} \rangle$ is the maximal sub graph of G with vertex set H. A subset S of V is called a dominating set if every vertex in $\mathrm{V} - \mathrm{S}$ is adjacent to some vertex in S.

A dominating set S is called a minimal dominating set if no proper subset of S is a dominating set of G. The domination number $\gamma(\mathrm{G})$ is the minimum cardinality taken over all minimal dominating sets of G. A γ-set of G is any minimal dominating set with cardinality γ. A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The smallest number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by $\chi(\mathrm{G})$. A total dominator coloring of G is a proper coloring of G with the extra property that every vertex in G properly dominates a color class. The total dominator chromatic number is denoted by $\chi_{td}(\mathrm{G})$. This notion was introduced by [9]. A color class dominating set of G is a proper coloring $\mathscr{C}$ of G with the extra property that every color classes in $\mathscr{C}$ is dominated by a vertex in G. A color class dominating set is said to be a minimal color class dominating set if no proper subset of $\mathscr{C}$ is a color class dominating set of G. The color class domination number of G is the minimum cardinality taken over all minimal color class dominating sets of G and is denoted by $\gamma_\chi(\mathrm{G})$. This notion was introduced by [4].

A dominator color class dominating set of G is a proper coloring $\mathscr{C}$ of G with the extra property that each vertex v in

G is dominated by a color class $\mathscr{C}_i \in \mathscr{C}$ and each color class $\mathscr{C}_i \in \mathscr{C}$ is dominated by a vertex in G. The dominator color class domination number of G is the minimum cardinality taken over all dominator color class dominating sets in G and is denoted by $\gamma_\chi^d(\mathrm{G})$. This notion was introduced by [5]. The join $\mathrm{G}_1 + \mathrm{G}_2$ of graphs G_1 and G_2 with disjoint vertex set V_1 and V_2 and edge sets E_1 and E_2, respectively, is the graph union $G_1 \cup G_2$ together with each vertex in V_1 is adjacent to every vertices in V_2. The dutch windmill graph D_n^m is the graph obtained by taking m copies of the cycle with a vertex in common. The dutch windmill graph is also called as friendship graph. A coconut tree $\mathrm{CT(m,n)}$ is the graph, for all positive integer n and $m \geq 2$ is obtained from the path P_m by appending $'n'$ new pendant edges at an end vertex of P_m.

We use the following observation from [1].

Theorem 1.1. *([1]): Let G be P_n or C_n. Then for* $\mathrm{n} \geq 5$,

$$\gamma_\chi^{td}(P_n) = \gamma_\chi^{td}(C_n) = \begin{cases} \left\lfloor \frac{2n}{3} \right\rfloor & \text{if } n \equiv 0(\bmod 3) \\ 2\left\lfloor \frac{n+2}{3} \right\rfloor & \text{if } n \not\equiv 0(\bmod 3) \end{cases}$$

2. Main Results

Definition 2.1. *Let G be a connected graph with minimum degree at least one. A proper coloring $\mathscr{C}$ of G is said to be a total dominator color class total dominating set of G if each vertex properly dominates a color class in $\mathscr{C}$ and each color class in $\mathscr{C}$ is properly dominated by a vertex in* $\mathrm{V(G)}$. *A total dominator color class total dominating set D is a minimal total dominator color class total dominating set if no proper subset of D of G is a total dominator color class total dominating set of G. The total dominator color class total domination number is the minimum cardinality taken over all minimal total dominator color class total dominating sets in G and is denoted by $\gamma_\chi^{td}(G)$.*

Theorem 2.2. *For the Dutch windmill graph D_n^m,*

$$\gamma_\chi^{td}(D_n^m) = \begin{cases} \left(\frac{2mn}{3}\right) - 2(m-1) & \text{if } n \equiv 0(\bmod 3) \\ 2m\left(\frac{n+2}{3}\right) - 2(m-1) & \text{if } n \equiv 1(\bmod 3) \\ 2m\left(\frac{n+1}{3}\right) - 2(m-1) & \text{if } n \equiv 2(\bmod 3) \end{cases}$$

Proof. Let D_n^m be a Dutch windmill graph with $\mathrm{V}(D_n^m) = \{v_{ij} / i = 1,2,\ldots,m \,\&\, j = 1,2,\ldots n\}$. For each i $\{1 \leq i \leq m\}$, let $\{v_{i1}, v_{i2}, \ldots, v_{in}\}$ be the vertices of i^{th} copy of the cycle C_n and $v_{11} = v_{21} = v_{31} = \cdots = v_{n1} = v$ (say), a common vertex. We assign colors 1 and 2 to the vertices $\{v\}$ and $\{v_{i2}, v_{in} / i = 1,2,\ldots,m\}$ respectively. For each $i = 1,2,\ldots.m$, let $H_i = \langle v_{i3}, v_{i4}, \ldots, v_{i(m-1)} \rangle$ then $H_i \cong P_{n-3}, \forall i = 1,2,\ldots,m-1$. We consider 3 cases.

Case (**i**) : When $n \equiv 0(\bmod 3)$

Since $n-3 \equiv 0(\bmod 3)$ and by theorem (A), assign $\mathrm{m}\gamma_\chi^{td}(P_{n-3})$ distinct colors, we assign $\frac{n-3}{3}$ distinct colors say $2\mathrm{l}+1 \& 2\mathrm{l}+2, \left(1 \leq l \leq \frac{n-3}{3}\right)$ to the vertices $\{v_{i(3k)}\}, \{v_{i(3k+2)}\}$ and $\{v_{i(3k+1)}\}, \forall k = 1,2,\ldots,\frac{n-3}{3}$ respectively, we attain a γ_χ^{td} coloring of D_n^m. So

$$\begin{aligned} \gamma_\chi^{td}(D_n^m) &= 2 + \mathrm{m}\gamma_\chi^{td}(P_{n-3}) \\ &= \left(\frac{2mn}{3}\right) - 2(m-1) \end{aligned}$$

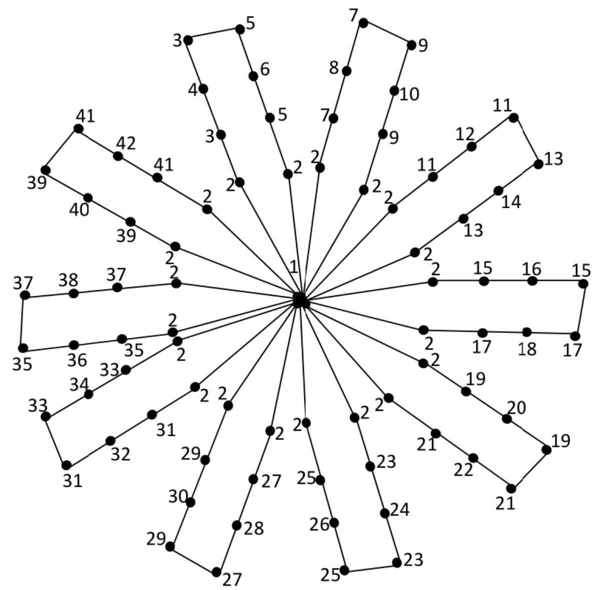

Figure 1. $\gamma_\chi^{td}\left(D_9^{10}\right) = 42$

Case (ii): When $n \equiv 1(\bmod 3)$

We assign $4i (1 \leq i \leq m)$ distinct colors, say $12i-1, 12i, 12i+1, 12i+2$ to the vertices $\{v_{i(n-4)}\}, \{v_{i(n-3)}\}, \{v_{i(n-2)}\}$ & $\{v_{i(n-1)}\}$ respectively. Since $n-7 \equiv 0(\bmod 3)$, by theorem (A) & case(i), we assign $\mathrm{m}\gamma_X^{td}(P_{n-7})$ distinct colors to the remaining vertices, we attain a γ_χ^{td} coloring of D_n^m. So

$$\begin{aligned} \gamma_\chi^{td}(D_n^m) &= 2 + 4m + \mathrm{m}\gamma_\chi^{td}(P_{n-7}) \\ &= 2m\left(\frac{n+2}{3}\right) - 2(m-1) \end{aligned}$$

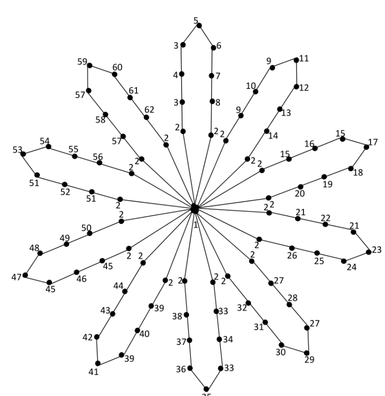

Figure 2. $\gamma_X^{td}\left(D_{10}^{10}\right) = 62$

Case (iii): When $n \equiv 2(\bmod 3)$
Assign $2i(1 \leq i \leq m)$ distinct colors to the vertices $\{v_{i(n-2)}\}$ & $\{v_{i(n-1)}\}$ respectively. Since $n-5 \equiv 0(\bmod 3)$ and by theorem (A)& case(i) we assign m$\gamma_{\chi}^{td}(P_{n-5})$ distinct colors to the remaining vertices, we get a γ_{x}^{td} coloring of D_n^m. So

$$\begin{aligned}\gamma_{X}^{td}(D_n^m) &= 2+2m+\mathrm{m}\gamma_{X}^{td}(P_{n-5}) \\ &= 2m\left(\frac{n+1}{3}\right)-2(m-1)\end{aligned}$$

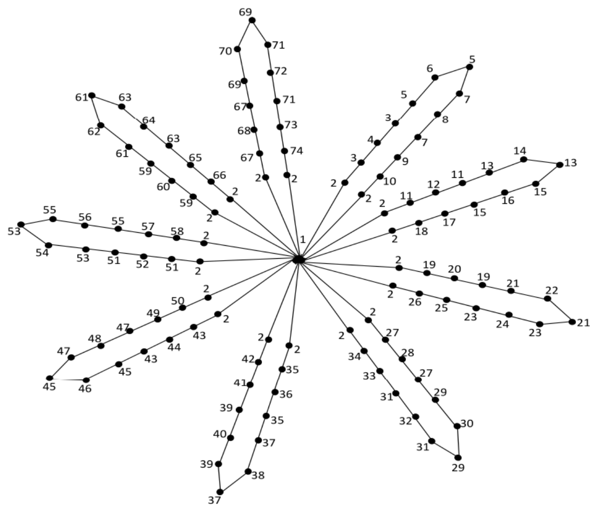

Figure 3. $\gamma_{X}^{td}(D_{14}^{9}) = 74$

□

Theorem 2.3. *For a coconut tree graph, $CT_{m,n}$*

$$\gamma_{X}^{td}(CT_{m,n}) = \begin{cases} \frac{2m}{3}+2, & \text{if } m \equiv 0(\bmod 3) \\ \left\lceil \frac{2m}{3} \right\rceil + 1, & \text{if } m \equiv 1(\bmod 3) \\ \left\lceil \frac{2m}{3} \right\rceil, & \text{if } m \equiv 2(\bmod 3) \end{cases}$$

Proof. Let $\mathrm{V}(CT_{m,n}) = \{u_i, v_i / 1 \leq i \leq m, 1 \leq j \leq n\}$, where $\{u_i / 1 \leq i \leq m\}$ be a vertices of the path and $\{v_1, v_2, v_3, \ldots. v_n\}$ be the pendant vertices attached with the end vertex $\mathrm{u_m}$ of the path $\mathrm{P_m}$ with degree $u_m = n+1$. We consider 3 cases.
Case (i): When $m \equiv 0(\bmod 3)$. Since $m-1 \equiv 2(\bmod 3)$, by theorem 2.5, $\gamma_{\chi}^{td}(P_{m-1}) = 2\left\lfloor \frac{m+1}{3} \right\rfloor$. We assign 2 new colors say $\frac{2m}{3}+1$ and $\frac{2m}{3}+2$ to the vertices $\{u_m\}$ and $\{v_1, v_2, v_3, \ldots. v_n\}$ respectively. We attain a $\gamma_{\chi}^{td}-$ coloring of $CT_{m,n}$. Thus $\gamma_{\chi}^{td}(CT_{m,n}) = \frac{2m}{3}+2$.
Case (ii): When m $\equiv 1(\bmod 3)$. Since $m-4 \equiv 0(\bmod 3)$, by theorem 2.5, $\gamma_{X}^{td}(P_{m-4}) = \frac{2m}{3}-2$. Assign 4 new colors say $\frac{2(m-1)}{3}-1, \frac{2(m-1)}{3}, \frac{2(m-1)}{3}+1, \frac{2(m-1)}{3}+2$ to the vertices say $\{u_{m-3}\}, \{u_{m-2}\}, \{u_m\}$ and $\{u_{m-1}, v_1, v_2, v_3, \ldots. v_n\}$ respectively. We attain a $\gamma_{\chi}^{td}-$ coloring of $CT_{m,n}$. Thus $\gamma_{\chi}^{td}(CT_{m,n}) = \left\lceil \frac{2m}{3} \right\rceil + 1$.
Case (iii): When $m \equiv 2(\bmod 3)$. Since $m-2 \equiv 0(\bmod 3)$, by theorem 2.5, $\gamma_{\chi}^{td}(P_{m-2}) = \left\lfloor \frac{2(m-2)}{3} \right\rfloor$. We assign 2 new colors

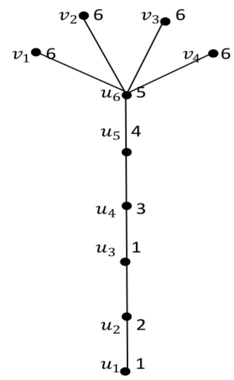

Figure 4. $\gamma_{\chi}^{td}(CT_{6,4}) = 6$

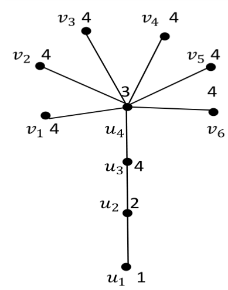

Figure 5. $\gamma_{\chi}^{td}(CT_{4,6}) = 4$

say $\frac{2m}{3}$ and $\frac{2m}{3}+1$ to the vertices $\{u_m\}$ and $\{u_{m-1}, v_1, v_2, v_3, \ldots. v_n\}$ respectively. We attain a γ_{χ}^{td} -coloring of $CT_{m,n}$. Thus $\gamma_{\chi}^{td}(CT_{m,n}) = \left\lceil \frac{2m}{3} \right\rceil$.

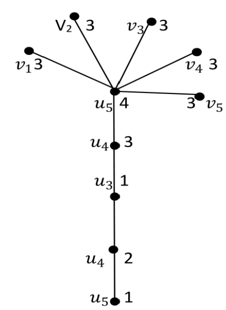

Figure 6. $\gamma_{\chi}^{td}(CT_{5,5}) = 4$

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