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Total dominator color class total dominating sets in ladder and mobius ladder graph

A. Vijayalekshmi [1]* and S. Abisha [2]

Abstract

Let G be a finite, undirected and connected graph with minimum degree at least one. A proper coloring $\mathscr{C}$ of G is said to be a total dominator color class total dominating set of G if each vertex properly dominates a color class in $\mathscr{C}$ and each color class in $\mathscr{C}$ is properly dominated by a vertex in $\mathrm{V(G)}$. A total dominator color class total dominating set D of G is a minimal total dominator color class total dominating set if no proper subset of D is a total dominator color class total dominating set of G. The total dominator color class total domination number is the minimum cardinality taken over all minimal total dominator color class total dominating sets in G and is denoted by $\gamma_\chi^{td}(G)$. Here we obtain $\gamma_\chi^{td}(G)$ for ladder graph and mobius ladder graph.

Keywords

Chromatic number, Domination number, Total domination, Dominator color class dominating set, Total dominator color class total domination number.

AMS Subject Classification

05C15, 05C69.

[1] *Department of Mathematics, S.T.Hindu College, Nagercoil- 629002, Tamil Nadu, India.*

[2] *Research Scholar, Reg.No: 20213152092019, Department of Mathematics, S.T.Hindu College, Nagercoil- 629002, Tamil Nadu, India.*

***Corresponding author**: [1] vijimath.a@gmail.com

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1. Introduction

All graphs considered in this paper are finite, undirected graphs with minimum degree at least one and we follow standard definitions of graph theory as found in [8]. Let $\mathrm{G=(V,E)}$ be a connected graph with no isolated vertices. The open neighborhood $\mathrm{N(v)}$ of a vertex $\mathrm{v \in V(G)}$ consists of the set of all vertices adjacent to v. The closed neighborhood of v is $\mathrm{N[v] = N(v)U\{v\}}$. For a set $\mathrm{S \subseteq V}$, the open neighborhood $\mathrm{N(S)}$ is defined to be $\mathrm{U_{v\in S}}N(v)$, and the closed neighborhood of S is $\mathrm{N[S] = N(S) \cup S}$. For any set H of vertices of G, the induced subgraph $\langle \mathrm{H} \rangle$ is the maximal sub graph of G with vertex set H. A subset S of V is called a dominating set if every vertex in $\mathrm{V-S}$ is adjacent to some vertex in S.

A dominating set S is called a minimal dominating set if no proper subset of S is a dominating set of G. The domination number $\gamma(\mathrm{G})$ is the minimum cardinality taken over all minimal dominating sets of G. A γ-set of G is any minimal dominating set with cardinality γ. A proper coloring of G is an assignment of colors to the vertices of G such that adjacent vertices have different colors. The smallest number of colors for which there exists a proper coloring of G is called chromatic number of G and is denoted by $\chi(\mathrm{G})$. A total dominator coloring of G is a proper coloring of G with the extra property that every vertex in G properly dominates a color class. The total dominator chromatic number is denoted by $\chi_{td}(\mathrm{G})$. This notion was introduced by [9]. A color class dominating set of G is a proper coloring $\mathscr{C}$ of G with the extra property that every color classes in $\mathscr{C}$ is dominated by a vertex in G. A color class dominating set is said to be a minimal color class dominating set if no proper subset of $\mathscr{C}$ is a color class dominating set of G. The color class domination number of G is the minimum cardinality taken over all minimal color class dominating sets of G and is denoted by $\gamma_\chi(\mathrm{G})$. This notion was introduced by [4].

A dominator color class dominating set of G is a proper coloring $\mathscr{C}$ of G with the extra property that each vertex v in G is dominated by a color class $\mathscr{C}_i \in \mathscr{C}$ and each color class $\mathscr{C}_i \in \mathscr{C}$ is dominated by a vertex in G. The dominator color class domination number of G is the minimum cardinality taken over all dominator color class dominating sets in G and is denoted by $\gamma_\chi^d(\mathrm{G})$. This notion was introduced by [5]. For any two graphs G and H, we define the cartesian product, denoted by $\mathrm{G} \times \mathrm{H}$, to be the graph with vertex set $V(G) \times V(H)$ and edges between two vertices (u_1, v_1) and (u_2, v_2) are adjacent iff either $u_1 = u_2$ and $v_1 v_2 \in E(H)$ or $u_1 u_2 \in E(G)$ and $v_1 = v_2$. A ladder graph can be defined as $P_2 \times P_n$, where $n \geq 2$ and is denoted by L_n. A mobius ladder graph M_n is a graph obtained from the ladder graph $P_2 \times P_n$ by joining the opposite end points of the two copies of P_n.

2. Main Results

Definition 2.1. *Let G be a connected graph with minimum degree at least one. A proper coloring $\mathscr{C}$ of G is said to be a total dominator color class total dominating set of G if each vertex properly dominates a color class in $\mathscr{C}$ and each color class in $\mathscr{C}$ is properly dominated by a vertex in* $\mathrm{V(G)}$*. A total dominator color class total dominating set D is a minimal total dominator color class total dominating set if no proper subset of D of G is a total dominator color class total dominating set of G . The total dominator color class total domination number is the minimum cardinality taken over all minimal total dominator color class total dominating sets in G and is denoted by $\gamma_\chi^{td}(G)$.*

Theorem 2.2. *Let L_p be a ladder graph of order* $2n$*. Then*
$$\gamma_\chi^{td}(L_p) = \begin{cases} n+2 & \text{if } n \text{ is even} \\ n+1 & \text{if } n \text{ is odd} \end{cases}$$

Proof. Let $L_p = L_{2n} = P_2 \times P_n$ and let $V(L_p) = \{v_1, v_2, \dots v_n, v_{n+1}, \dots v_{2n}\}$ with $\deg(v_i) = 2$ for $i = 1, n, (n+1), 2n$ and $\deg(v_j) = 3$ for $j \neq i$. We take $N(v_i) = \{v_{i-1}, v_{i+1}, v_{i+n}\}$ for $i = 2, 3, \dots (n-1)$ and $N(v_j) = \{v_{j-1}, v_{j+1}, v_{j-n}\}$ for $j = (n+2), (n+3), \dots, (2n-1)$. We consider two cases.
Case (i): When $n \equiv 1(\bmod 2)$
The vertices $\{v_{4i+1}\}$ $\left(1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor\right)$ receive distinct colors, say $4i+2$ and the vertices $\{v_{n+4i-1}\}$ $\left(1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor\right)$ receive distinct colors, say $4i$ respectively. Assign colors, say n and $(n+1)$ to the vertices $\{v_{n-1}, v_{2n}\}$ & $\{v_n\}$ if $n \equiv 1(\bmod 4)$ and the vertices $\{v_n, v_{2n-1}\}$ & $\{v_{2n}\}$ if $n \equiv 3(\bmod 4)$ respectively. Assign distinct colors, say $(4i-1)$&$(4i+1)$ $\left(1 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor\right)$ to the vertices $\{v_{4i-1}, v_{n+4i-2}, v_{n+4i}\}$ and $\{v_{4i}, v_{4i+2}, v_{n+4i+1}\}$ respectively, we attain a $\gamma_\chi^{td}-$ coloring of L_p,so $\gamma_\chi^{td}(L_p) = n+1$. Case (ii): When $n \equiv 0(\bmod 2)$

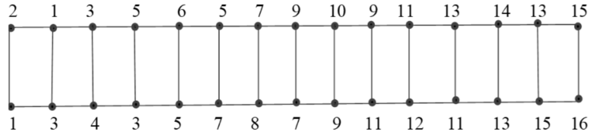

Figure 1. $\cdot\gamma_\chi^{td}(L_{15}) = 16$

When $n-1 \equiv 1(\bmod 2)$& by case (i), $\gamma_\chi^{td}(L_{n-1}) = n$. Assign two new colors, say $n+1$&$n+2$ to the vertices $\{v_n\}$ & $\{v_{2n}\}$ respectively, we obtain a γ_χ^{td} - coloring of L_p. Thus $\gamma_\chi^{td}(L_p) = n+2$.

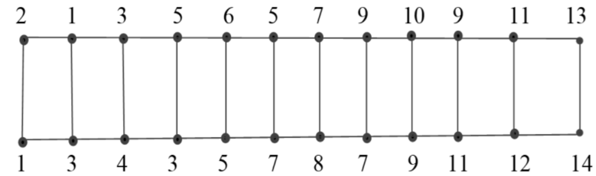

Figure 2. $\gamma_x^{td}(L_{12}) = 14$

□

Theorem 2.3. *Let M_n be a mobius ladder graph with* $\mathrm{n} \geq 5$*. Then*
$$\gamma_\chi^{td}(M_n) = \begin{cases} n+2 & \text{if } n \equiv 0(\bmod 4) \\ n+1 & \text{if } n \equiv 1,3(\bmod 4) \\ n & \text{if } n \equiv 2(\bmod 4) \end{cases}$$

Proof. Let M_n be a mobius ladder graph with $V(M_n) = \{u_i, v_i / i = 1, 2, \dots n\}$. We take
$\mathrm{N}(u_i) = \{u_{i-1}, u_{i+1}, v_i / i = 2, 3 \dots (n-1)\}$
$\mathrm{N}(v_i) = \{v_{i-1}, v_{i+1}, u_i / i = 2, 3 \dots (n-1)\}$
$\mathrm{N}(u_1) = \{u_2, v_1, v_n\}$
$\mathrm{N}(u_n) = \{u_{n-1}, v_1, v_n\}$
$\mathrm{N}(v_1) = \{v_2, u_1, u_n\}$
$\mathrm{N}(v_n) = \{u_n, u_1, v_{n-1}\}$
We consider 3 cases.
Case (i): When $n \equiv 0(\bmod 4)$
Assign colors, say 1&2 to the vertices $\{u_n\}$ & $\{v_1, u_{n-1}, v_n\}$ respectively. Also assign colors say $(n-1), n, (n+1)$&$(n+2)$ to the vertices $\{u_{n-3}\}, \{u_{n-2}\}, \{v_{n-2}\}$ & $\{v_{n-1}\}$ respectively. Assign colors, say $2i-1$&$2i$ $\left(i = 2, 4, 6 \dots \frac{n}{2} - 2\right)$ to the vertices $\{u_{2i-3}, v_{2i-2}, u_{2i-1}\}$ & $\{u_{2i-2}\}$ respectively. Again receive colors, say $2i-1$&$2i$ $\left(i = 3, 5 \dots \frac{n}{2} - 1\right)$ to the vertices $\{v_{2i-3}, u_{2i-2}, v_{2i-1}\}$ & $\{v_{2i-2}\}$ respectively, we obtain a γ_χ^{td} - coloring of M_n. So $\gamma_\chi^{td}(M_n) = n+2$.
Case (ii): When $n \equiv 1, 3(\bmod 4)$
Assign colors 1&2 to the same vertices as in case (i). We consider 2 subcases.
Subcase 2.1. When $n \equiv 1(\bmod 4)$:
Assign colors, say $2i-1$&$2i$ $\left(i = 2, 4, 6 \dots \left\lfloor \frac{n}{2} \right\rfloor\right)$ to the vertices $\{u_{2i-3}, v_{2i-2}, u_{2i-1}\}$ & $\{u_{2i-2}\}$ respectively. Also assign colors, say $2i-1$&$2i$ $\left(i = 3, 5 \dots \frac{n}{2} + 1\right)$ to the vertices $\{v_{2i-3}, u_{2i-2}, v_{2i-1}\}$ & $\{v_{2i-2}\}$ respectively. In addition, that assign two distinct colors say n and $\mathrm{n}+1$ to the vertices $\{v_{n-2}\}$ & $\{v_{n-1}\}$, we get a γ_χ^{td} - coloring of M_n. Thus $\gamma_\chi^{td}(M_n) = n+1$.
Subcase 2.2. When $n \equiv 3(\bmod 4)$:
Assign colors, say $2i-1$&$2i$ $\left(i = 2, 4, 6 \dots \left\lfloor \frac{n}{2} \right\rfloor - 1\right)$ to the vertices $\{u_{2i-3}, v_{2i-2}, u_{2i-1}\}$ & $\{u_{2i-2}\}$ respectively. Also assign colors, say $2i-1$&$2i$ $\left(i = 3, 5 \dots \left\lfloor \frac{n}{2} \right\rfloor - 2\right)$ to the vertices $\{v_{2i-3}, u_{2i-2}, v_{2i-1}\}$ & $\{v_{2i-2}\}$ respectively. In addition that assign 4 distinct colors say $n-2, n-1, n$ and $n+1$ to the

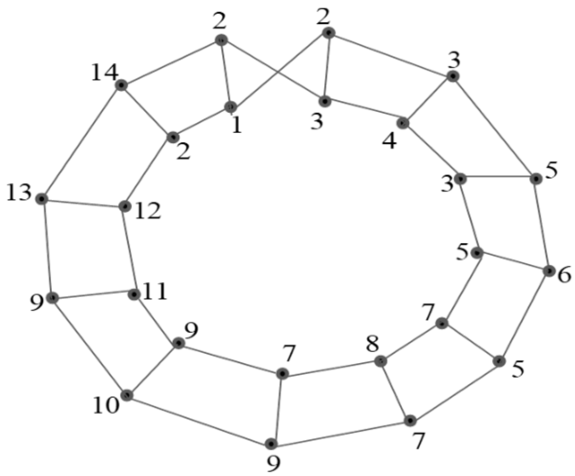

Figure 3. $\gamma_x^{td}(L_{12}) = 14$

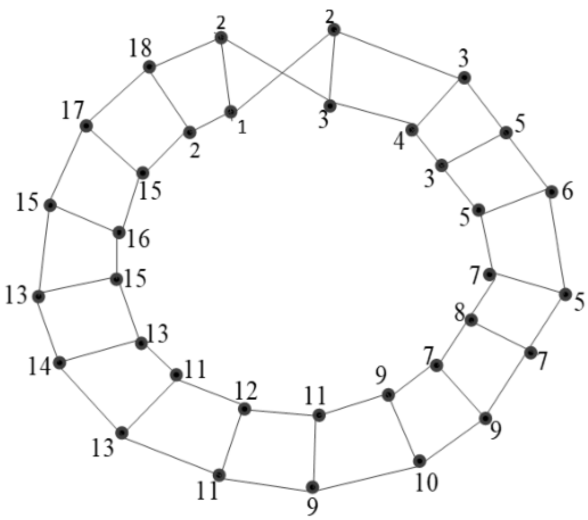

Figure 4. $\gamma_\chi^{td}(L_{17}) = 18$

vertices $\{v_{n-4}, u_{n-3}\}, \{v_{n-3}\}, \{u_{n-2}, v_{n-1}\}$ & $\{v_{n-2}\}$ respectively, we attain a γ_χ^{td} - coloring of M_n. Hence $\gamma_\chi^{td}(M_n) = n+1$

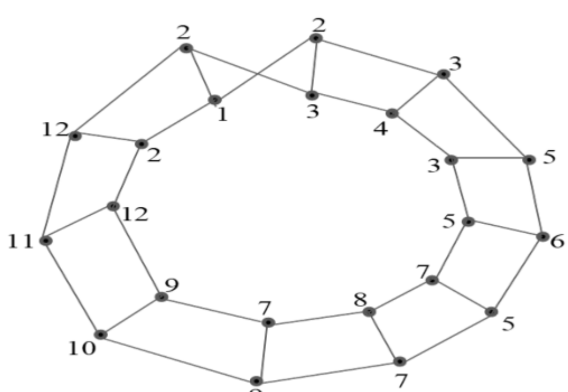

Figure 5. $\gamma_\chi^{td}(L_{11}) = 12$

Case (iii): When $n \equiv 2(\bmod 4)$:
Assign colors 1&2 to the same vertices as in case (i). We assign colors, say $2i-1$&$2i$ $\left(i = 2,4,6\ldots\frac{n}{2}-1\right)$ to the same vertices as in case (i) & assign colors, say $2i-1$&$2i$ $\left(i = 3,5\ldots\frac{n}{2}\right)$ to the same vertices as in case (i). We get the required result hence $\gamma_\chi^{td}(M_n) = n$

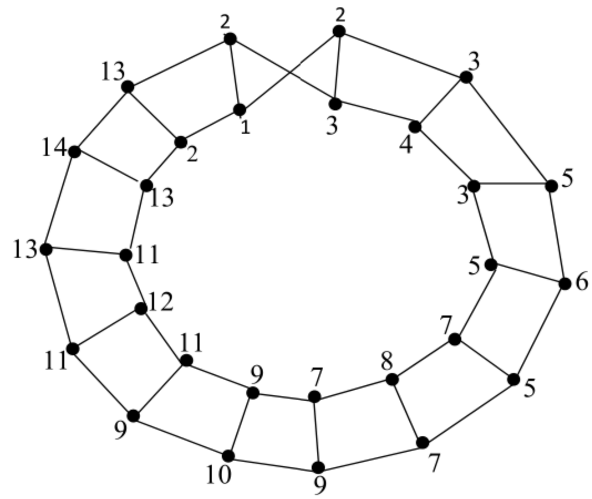

Figure 6. $\gamma_X^{td}(L_{14}) = 14$

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