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Existence results for impulsive neutral stochastic functional integrodifferential systems with infinite delay

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Abstract

This paper is devoted to build the existence of mild solutions of impulsive neutral stochastic functional integrodifferential equations (INSFIDEs) with infinite delay at abstract phase space in Hilbert spaces. Under the uniform Lipschitz condition, we obtain the solution for INSFIDEs. Sufficient conditions for the existence results are derived with the help of Krasnoselski-Schaefer type fixed point theorem. An example is provided to illustrate the theory.

Keywords: Impulsive neutral stochastic integrodifferential equations, infinite delay, Krasnoselski-Schaefer type fixed point theorem, semigroup theory.

2010 MSC: 34A37, 37H10, 60H20, 34K50, 34K05.

1 Introduction

Stochastic differential equations are well known to model problems from many areas of science and engineering, wherein, quite often the future state of such systems depends not only on the present state but also on its past history (delay) leading to stochastic functional differential equations and it has played an important role in many ways such as option pricing, forecast of the growth of population, etc., [32, 36, 37]. Random differential and integral equations play an important role in characterizing numerous social, physical, biological and engineering problems and for more details reader may refer [16, 25] and reference therein.

From time in memory, the theory of nonlinear functional differential or integrodifferential equations has become an active area of investigation due to their application in many physical phenomena. Several authors [3, 7, 8, 22] have investigated the integrodifferential equations with or without impulsive conditions in Banach spaces. Recently impulsive neutral differential and integrodifferential equations have generated considerable interest among the researchers [20].

Impulsive dynamical systems exhibit the various evolutionary process, including those in engineering, biology and population dynamics, undergo abrupt changes in their state at certain moments between intervals of continuous evolution. Since many evolution process, optimal control models in economics, stimulated neutral networks, frequency- modulated systems and some motions of missiles or aircrafts are characterized by the impulsive dynamical behavior. Nowadays, there has been increasing interest in the analysis and synthesis of impulsive systems due to their significance both in theory and applications. Thus the theory of impulsive differential equations has seen considerable development. For instance, see the monograph of Lakshmikantham et al. [35], Bainov and Simeonov [6] and Somoilenko and Perestuk [44] for the ordinary impulsive differential system and [26, 27, 28, 29, 41, 42] for the partial differential and partial functional differential equations with impulses and for more details reader may refer [2, 3, 4, 10, 11, 18, 19, 39, 45] and reference therein. The stochastic differential equations combined with impulsive conditions with unbounded delay have been studied

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by few authors, [1, 5, 12, 15, 24, 40] and the papers of [8, 13, 14, 31, 33, 43], where the numerous properties of their solutions are studied.

In [9] Balachandran et al. studied the existence for impulsive neutral evolution integrodifferential equations with infinite delay and Krasnoselski-Schaefer type fixed point theorem, whereas A. Lin et al. [34] proved on neutral impulsive stochastic integrodifferential equations with infinite delay via fractional operators and Sadovskii fixed point theorem, and Yong Ren et al. [40] established the controllability of impulsive neutral stochastic functional differential inclusions with infinite delay and Dhage's fixed point theorem. Recently, Jing Cui et al. [23] derived nonlocal Cauchy problem for some stochastic integrodifferential equations in Hilbert spaces and Leray-Schauder nonlinear alternative fixed point theorem.

Inspired by the above mentioned works [9, 23, 34, 40], in this paper, we are interested in studying the existence of solutions of the following impulsive neutral stochastic differential equations with infinite delay;

$$d[x(t)-g(t,x_t)] = A\Big[x(t)+e\Big(t,x_t,\int_0^t h_1(t,s,x_s)ds\Big)\Big]dt + f(t,x_t)dt + \sigma\Big(t,x_t,\int_0^t h_2(t,s,x_s)ds\Big)dw(t),$$

$$t\in J:=[0,b],\quad t\neq t_k,\quad k=1,2,\dots,m, \tag{1.1}$$

$$\Delta x(t_k) = x(t_k^+)-x(t_k^-) = I_k(x(t_k^-)),\quad k=1,2,\dots,m, \tag{1.2}$$

$$x_0=\phi\in\mathcal{B}_h,\quad t\in J_0=(-\infty,0], \tag{1.3}$$

where A is the infinitesimal generator of an analytic semigroup of bounded linear operator $\{T(t)\}_{t\geq 0}$ in the Hilbert space H. The history $x_t:(-\infty,0]\to H, x_t(s)=x(t+s), s\leq 0$, belong to an abstract phase space $\mathcal{B}_h$, which will be described axiomatically in Section 2. Let K be the another separable Hilbert space with inner product $(\cdot,\cdot)_K$ and the norm $\|\|_K$. Suppose $\{w(t): t\geq 0\}$ is a given K- valued Brownian motion or Wiener process with a finite trace nuclear covariance operator $Q\geq 0$ defined on a complete probability space $(\Omega,\mathcal{F},P)$ equipped with a normal filtration $\{\mathcal{F}_t\}_{t\geq 0}$, which generated by the Wiener process w. We now employing the same notation $\|\cdot\|$ for the norm $L(K;H)$, where $L(K;H)$ denotes the space of all bounded linear operator from K into H. Here $g,f: J\times\mathcal{B}_h\to H$, $e: J\times\mathcal{B}_h\times H\to H$, $h_1,h_2: J\times J\times\mathcal{B}_h\to H$ and $\sigma: J\times\mathcal{B}_h\times H\to L_Q(K,H)$ are given functions, where $L_Q(K,H)$ denotes the space of all Q-Hilbert-Schmidt operator from K into H which will be defined in Section 2. The initial data $\phi=\{\phi(t): -\infty<t\leq 0\}$ is an $\mathcal{F}_0$-adapted, $\mathcal{B}_h$- valued random variable independent of the Wiener process w with finite second moment. Furthermore, the fixed times t_k satisfies $0=t_0<t_1<t_2<\cdots<t_m<b$, $x(t_k^+)$ and $x(t_k^-)$ denote the right and left limits of $x(t)$ at $t=t_k$. And $\Delta x(t_k)=x(t_k^+)-x(t_k^-)$ represents the jump in the state x at time t_k, where I_k determines the size of the jump.

The outline of the paper is as follows. We review some basic facts about semigroups, the theory of SDEs, as preliminaries in Section 2. Then, Section 3 is devoted to the development of our main existence results and our basic tool include Krasnoselski-Schaefer fixed point theorem. Finally, the paper is conclude with an example to illustrate the obtained results.

2 Preliminaries

Let $(K,\|\cdot\|_K)$ and $(H,\|\cdot\|_H)$ be the two separable Hilbert space with inner product $\langle\cdot,\cdot\rangle_K$ and $\langle\cdot,\cdot\rangle_H$, respectively. We denote by $\mathcal{L}(K,H)$ be the set of all linear bounded operator from K into H, equipped with the usual operator norm $\|\cdot\|$. In this article, we use the symbol $\|\cdot\|$ to denote norms of operator regardless of the space involved when no confusion possibly arises.

Let $(\Omega,\mathcal{F},P,H)$ be the complete probability space furnished with a complete family of right continuous increasing σ- algebra $\{\mathcal{F}_t, t\in J\}$ satisfying $\mathcal{F}_t\subset\mathcal{F}$. An H- valued random variable is an $\mathcal{F}$- measurable function $x(t):\Omega\to H$ and a collection of random variables $S=\{x(t,\omega):\Omega\to H\setminus t\in J\}$ is called stochastic process. Usually we write $x(t)$ instead of $x(t,\omega)$ and $x(t): J\to H$ in the space of S. Let $\{e_i\}_{i=1}^{\infty}$ be a complete orthonormal basis of K. Suppose that $\{w(t): t\geq 0\}$ is a cylindrical K-valued wiener process with a finite trace nuclear covariance operator $Q\geq 0$, denote $\mathrm{Tr}(Q)=\sum_{i=1}^{\infty}\lambda_i=\lambda<\infty$, which satisfies that $Qe_i=\lambda_i e_i$. So, actually $\omega(t)=\sum_{i=1}^{\infty}\sqrt{\lambda_i}\omega_i(t)e_i$, where $\{\omega_i(t)\}_{i=1}^{\infty}$ are mutually independent one-dimensional standard Wiener processes. We assume that $\mathcal{F}_t=\sigma\{\omega(s): 0\leq s\leq t\}$ is the σ-algebra generated by ω and $\mathcal{F}_t=\mathcal{F}$. Let $\Psi\in\mathcal{L}(K,H)$ and define

$$\|\Psi\|_Q^2 = \mathrm{Tr}(\Psi Q\Psi^*) = \sum_{n=1}^{\infty}\|\sqrt{\lambda_n}\Psi e_n\|^2.$$

If $\|\Psi\|_Q < \infty$, then Ψ is called a Q-Hilbert-Schmidt operator. Let $\mathcal{L}_Q(K, H)$ denote the space of all Q-Hilbert-Schmidt operators $\Psi : K \to H$. The completion $\mathcal{L}_Q(K, H)$ of $\mathcal{L}(K, H)$ with respect to the topology induced by the norm $\|\cdot\|_Q$ where $\|\Psi\|_Q^2 = \langle \Psi, \Psi \rangle$ is a Hilbert space with the above norm topology.

The collections of all strongly measurable, square integrable, H-valued random variable, denoted by $L_2(\Omega, \mathcal{F}, P, H) \equiv L_2(\Omega, H)$, is a Banach space equppied with norm $\|x(\cdot)\|_{L_2} = (E\|x(\cdot, \omega)\|^2)^{\frac{1}{2}}$, where the expectation, E is defined by $Ex = \int_\Omega x(\omega)dP$. Let $C(J, L_2(\Omega, H))$ be the Banach space of all continuous map from J into $L_2(\Omega, H)$ satisfying the condition $\sup_{t\in J} E\|x(t)\|^2 < \infty$. An important subspace is given by $L_2^0(\Omega, H) = \{f \in L_2(\Omega, H) : f \quad \text{is} \quad \mathcal{F}_0 - \text{measurable}\}$.

Let A be the infinitesimal generator of an analytic semigroup $T(t)$ in H. Suppose that $0 \in \rho(A)$ where $\rho(A)$ denotes the resolvent set of A and that semigroup $T(\cdot)$ is uniformly bounded that is to say, $\|T(t)\| \leq M_1$ for some constant $M_1 \geq 1$ and for every $t \geq 0$. Then for $\alpha \in (0, 1]$, it is possible to define the fractional power operator $((-A)^\alpha)$ as a closed linear invertible operator on its domain $D((-A)^\alpha)$. Furthermore, the subspace $D((-A)^\alpha)$ is dense in H and the expression

$$\|x\|_\alpha = \|(-A)^\alpha x\|, \; x \in D((-A)^\alpha),$$

defines the norm on $H_\alpha = D((-A)^\alpha)$.

It should be pointed out that, to study of abstract impulsive functional differential systems with infinite delay, the abstract phase space $\mathcal{B}_h$(which is similar to that used in [46]) is very appropriate. Now we present we present the abstract phase space $\mathcal{B}_h$ as given in [21].

Assume that $h : (-\infty, 0] \to (0, +\infty)$ is a continuous function with $l = \int_{-\infty}^0 h(s)ds < +\infty$. For any $a > 0$, we define,

$$\mathcal{B} = \{\psi : [-a, 0] \to X \text{ such that } \psi(t) \text{ is bounded and measurable}\},$$

and equip the space $\mathcal{B}$ with the norm,

$$\|\psi\|_{[-a,0]} = \sup_{s\in[-a,0]} \|\psi(s)\|, \quad \forall \psi \in \mathcal{B}.$$

Let us define,

$$\mathcal{B}_h = \Big\{\psi : (-\infty, 0] \to H : \; (E\|\psi(\theta)\|^2)^{\frac{1}{2}} \text{is a bounded and measurable function on } [-a, 0]$$
$$\text{and} \int_{-\infty}^0 h(s) \sup_{s\leq\theta\leq 0} (E\|\psi(\theta)\|^2)^{\frac{1}{2}} ds < +\infty\Big\}.$$

If $\mathcal{B}_h$ is endowed with the norm,

$$\|\psi\|_{\mathcal{B}_h} = \int_{-\infty}^0 h(s) \sup_{s\leq\theta\leq 0} (E\|\psi(\theta)\|^2)^{\frac{1}{2}} ds, \quad \text{for all} \quad \psi \in \mathcal{B}_h,$$

then, it is easy to see that $(\mathcal{B}_h, \|\cdot\|_{\mathcal{B}_h})$ is a Banach space [30].

Now, we consider the space,

$$\mathcal{B}_h' = \Big\{x : (-\infty, b] \to H \;\text{ such that } x_k \in C(J_k, H) \quad \text{and there exist} \quad x(t_k^+)$$
$$\text{and} \quad x(t_k^-) \text{with} \quad x(t_k^+) = x(t_k^-), \quad x_0 = \phi \in \mathcal{B}_h, k = 1, 2, \cdots, m\Big\},$$

where, x_k is the restrictions of x to $J_k = (t_k, t_{k+1}], \; k = 1, 2, \cdots, m$. Set $\|\cdot\|_b$ be a seminorm in $\mathcal{B}_h'$ defined by,

$$\|x\|_b = \|\phi\|_{\mathcal{B}_h} + \sup\{(E\|x(s)\|^2)^{\frac{1}{2}} : s \in [0, b]\}, \quad x \in \mathcal{B}_h'.$$

Next, we recall some basic definitions and lemmas which are used throughout this paper.

Lemma 2.1. *([21]) Assume that $x \in \mathcal{B}_h'$, then for $t \in J$, $x_t \in \mathcal{B}_h$. Moreover,*

$$l\Big(E\|x(t)\|^2\Big)^{\frac{1}{2}} \leq \|x_t\|_{\mathcal{B}_h} \leq \|x_0\|_{\mathcal{B}_h} + l \sup_{s\in[0,t]} \Big(E\|x(s)\|^2\Big)^{\frac{1}{2}},$$

where $l = \int_{-\infty}^0 h(t)dt < +\infty$.

Lemma 2.2. *([17]) Let H be a Hilbert space and Φ_1, Φ_2 be the two operator on H such that*

(a) Φ_1 is a contraction and

(b) Φ_2 is completely continuous.

Then either

(i) the operator equation $\Phi_1 x + \Phi_2 x = x$ has a solution or

(ii) the set $G = \{x \in H : \lambda\Phi_1(\frac{x}{\lambda}) + \lambda\Phi_2 x = x\}$ is unbounded for $\lambda \in (0, 1)$.

Lemma 2.3. *([27]) Let $v(\cdot)$, $w(\cdot) : [0, b] \to [0, \infty)$ be continuous function. If $w(\cdot)$ is nondecreasing and there exist two constants $\theta \geq 0$ and $0 < \alpha < 1$ such that*

$$v(t) \leq w(t) + \theta \int_0^t \frac{v(s)}{(t-s)^{1-\alpha}} ds, \quad t \in J,$$

then

$$v(t) \leq e^{\theta^n (\Gamma(\alpha))^n t^{n\alpha}/\Gamma(n\alpha)} \sum_{j=0}^{n-1} \Big(\frac{\theta b^\alpha}{\alpha}\Big)^j w(t),$$

for every $t \in [0, b]$ and every $n \in N$ such that $n\alpha > 1$ and $\Gamma(\cdot)$ is the Gamma function.

Lemma 2.4. *([38]) Suppose the following properties are satisfied.*

(i) Let $0 \leq \alpha \leq 1$. Then H_α is a banach space.

(ii) If $0 < \beta < \alpha \leq 1$, then $H_\alpha \subset H_\beta$ and the imbedding is compact whenever the resolvent operator of A is compact.

(iii) For every $0 < \alpha \leq 1$, there exists a positive constant $M_\alpha > 0$ such that;

$$\|(-A)^\alpha T(t)\| \leq \frac{M_\alpha}{t^\alpha}, \ for\ all\ 0 < t \leq b. \tag{2.4}$$

Definition 2.1. *A map $F : J \times \mathcal{B}_h \to H$ is said to be L^2- Caratheodory if*

(i) $t \to F(t, v)$ is a measurable for each $v \in \mathcal{B}_h$;

(ii) $v \to F(t, v)$ is continuous for almost all $t \in J$;

(iii) for each $q > 0$, there exist $h_q \in L^1(J, R_+)$ such that

$$\|F(t, v)\|^2 = \sup_{f \in F(t,v)} E\|f\|^2 \leq h_q(t), \quad forall \quad \|v\|^2_{\mathcal{B}_h} \leq q \quad and\ for\ a.e.\ t \in J.$$

Definition 2.2. *An $\mathcal{F}_t$-adapted stochastic process $x : (-\infty, b] \to H$ is called mild solution of the system (1.1)-(1.3) if $x_0 = \phi \in \mathcal{B}_h$ satisfying $x_0 \in L_2^0(\Omega, H)$, for each $s \in [0, b)$ the function $AT(t-s)e\Big(s, x_s, \int_0^s h_1(s, \tau, x_\tau)d\tau\Big)$ is integrable and the following conditions hold:*

(i) $\{x_t : t \in J\}$ is $\mathcal{B}_h$ valued and the restrictions of $x(\cdot)$ to the interval $(t_k, t_{k+1}]$, $k = 1, 2, \ldots, m$ is continuous;

(ii) $\Delta x(t_k) = I_k(x_{t_k})$, $\quad k = 1, 2, \ldots, m$;

(iii) for each $t \in J$, $x(t)$ satisfies the following integral equation

$$\begin{aligned} x(t) &= T(t)[\phi(0) - g(0, \phi)] + g(t, x_t) + \int_0^t T(t-s)f(s, x_s)ds + \int_0^t AT(t-s)g(s, x_s)ds \\ &+ \int_0^t AT(t-s)e\Big(s, x_s, \int_0^s h_1(s, \tau, x_\tau)d\tau\Big)ds + \int_0^t T(t-s)\sigma\Big(s, x_s, \int_0^s h_2(s, \tau, x_\tau)d\tau\Big)dw(s) \\ &+ \sum_{0<t_k<t} T(t-t_k)I_k(x(t_k^-)), \quad t \in J. \end{aligned} \tag{2.5}$$

3 Existence Results

In this section, we present and prove the existence results for the problem (1.1) – (1.3). In order to prove the main theorem of this section, we list the following hypotheses:

($\mathbf{H_1}$) The function $f : J \times \mathcal{B}_h \to X$ satisfies the following coditions:

(i) For $x : (-\infty, b] \to H$ such that $x_0 \in \mathcal{B}_h$ and $x|_J \in \mathcal{B}'_h$, the function $t \to f(t, x_t)$ is strongly measurable. i.e., $f(., x_t) : J \to H$ is a strongly measurable.

(ii) For each $t \in J$, the function $f(t, .) : \mathcal{B}_h \to H$ is continuous.

(iii) There exists integrable function $m(t) : J \to [0, \infty)$ and a continuous nondecreasing function $\Omega : [0, \infty) \to (0, \infty)$ such that,

$$E\|f(t, \psi)\|^2 \leq m(t)\Omega_1(E\|\psi\|^2_{\mathcal{B}_h}); \quad (t, \psi) \in J \times \mathcal{B}_h.$$

($\mathbf{H_2}$) A is the infinitesimal generator of a compact analytic semigroup and $0 \in \rho(A)$ such that

$$\|T(t)\|^2 \leq M_1, \quad \text{for all} \quad t \geq 0 \quad \text{and} \quad \|(-A)^{1-\beta}T(t-s)\|^2 \leq \frac{M^2_{1-\beta}}{(t-s)^{2(1-\beta)}} \ 0 \leq t \leq b.$$

($\mathbf{H_3}$) There exists a constant $M_{h_1} \geq 0$, such that

$$\| \int_0^t [h_1(t, s, x) - h_1(t, s, y)]\|^2 \leq M_{h_1}\|x - y\|^2_{\mathcal{B}_h}.$$

($\mathbf{H_4}$)) There exists constants $0 < \beta < 1$, such that e is H_β-valued, $(-A)^\beta e : J \times \mathcal{B}_h \to H$ is completely continuous,

(i) The function $e : J \times \mathcal{B}_h \times H \to H$ for $t \in J$, $x_1, x_2 \in \mathcal{B}_h$ and $y_1, y_2 \in H$ such that the function M_e satisfies the Lipschitz condition:

$$E\|(-A)^\beta e(t, x_1, y_1) - (-A)^\beta e(t, x_2, y_2)\|^2 \leq M_e[\|x_1 - x_2\|^2_{\mathcal{B}_h} + \|y_1 - y_2\|^2].$$

Let $\tilde{c}_1 = b \ \sup_{t\in J} \|h_1(t, s, 0)\|^2$, $\tilde{c}_2 = \|(-A)^\beta\|^2 \sup_{t\in J} \|e(t, 0, 0)\|^2$, $\|(-A)^{-\beta}\|^2 = M_0$.

(ii) There exist constants $0 < \beta < 1$, C_0, c_1, c_2, M_g such that g is H_β-valued, $(-A)^\beta g$ is continuous, and

$$E\|(-A)^\beta g(t, x)\|^2 \leq c_1\|x\|^2_{\mathcal{B}_h} + c_2, \quad t \in J, \quad x \in \mathcal{B}_h,$$
$$E\|(-A)^\beta g(t, x_1) - (-A)^\beta g(t, x_2)\|^2 \leq M_g\|x - y\|^2_{\mathcal{B}_h}, \quad t \in J, \quad x_1, x_2 \in \mathcal{B}_h, \text{with}$$
$$C_0 \equiv l^2 \left\{ M_g M_0 + \Big[M_g + M_e(1 + M_{h_1})\Big] \frac{(M_{1-\beta}b^\beta)^2}{2\beta - 1} \right\} < 1.$$

($\mathbf{H_5}$) There exist constants d_k such that $\|I_k(x)\|^2 \leq d_k$, k= 1, 2, ..., m, for each $x \in H$.

($\mathbf{H_6}$) Foe each $(t, s) \in J \times J$, the function $h_2(t, s, \cdot) : \mathcal{B}_h \to H$ is continuous for each $x \in \mathcal{B}_h$, the function $h_2(\cdot, \cdot, x) : J \times J \to H$ is strongly measurable. There exists an integrable function $m : J \to [0, \infty)$ and a constant $\gamma \geq 0$, such that

$$\|h_2(t, s, x)\|^2 \leq \gamma m(s)\Omega_3(\|x\|^2_{\mathcal{B}_h}),$$

where $\Omega_3 : [0, \infty) \to (0, \infty)$ is a continuous nondecreasing functions. Let us assume that the finite bound of $\int_0^t \gamma m(s)ds$ is L_0.

($\mathbf{H_7}$) The function $\sigma : J \times \mathcal{B}_h \times H \to H$ satisfies the following Caratheodory conditions:

(i) $t \to \sigma(t, x, y)$ is measurable for each $(x, y) \in \mathcal{B}_h \times H$,

(ii) $(x, y) \to \sigma(t, x, y)$ is continuous for almost all $t \in J$.

($\mathbf{H_8}$) $E\|\sigma(t,x,y)\|^2 \leq p(t)\Omega_2(\|x\|^2_{\mathcal{B}_h} + \|y\|^2)$ for almost all $t \in J$ and all $x \in \mathcal{B}_h$, $y \in H$, where $p \in L^2(J, R_+)$ and $\Omega_2 : R_+ \to (0,\infty)$ is continuous and increasing with

$$\widehat{m}(s) \leq \int_{B_0 K_1}^{\infty} \frac{ds}{\Omega_1(s) + \Omega_2(s) + \Omega_3(s)}, \quad \text{where}$$

$$N_0 = 2l^2\Big\{64\|(-A)^{-\beta}\|^2 c_1\Big\}, \tag{3.6}$$

$$N_1 = 2\|\phi\|^2_{\mathcal{B}_h} + 2l^2\overline{F}, \tag{3.7}$$

$$N_2 = 128 l^2 b M^2_{1-\beta}(c_1 + M_e(1 + M_{h_1})),$$

$$\widehat{m}(t) = max[B_0 K_3 m(t), B_0 K_4 p(t), \gamma m(t)],$$

$$B_0 = e^{K_2^n(\Gamma(2\beta-1))^n b^{n2\beta-1}/\Gamma(n(2\beta-1))} \sum_{j=0}^{n-1} \Big(\frac{K_2 b^{2\beta-1}}{2\beta-1}\Big)^j,$$

$$N_3 = 128 l^2 M_1, \qquad N_4 = 128 l^2 M_1 Tr(Q), \tag{3.8}$$

$$K_1 = \frac{N_1}{(1-N_0)}, \quad K_2 = \frac{N_2}{(1-N_0)}, \quad K_3 = \frac{N_3}{(1-N_0)}, \quad K_4 = \frac{N_4}{(1-N_0)}, \tag{3.9}$$

$$\begin{aligned}\overline{F} &= 64 M_1\|\phi\|^2_{\mathcal{B}_h} + 64\|(-A)^{-\beta}\|^2 M_1(c_1\|\phi\|^2_{\mathcal{B}_h} + c_2) + 64\|(-A)^{-\beta}\|^2 M_g c_2 \\ &+ 64\frac{M^2_{1-\beta} c_2 b^{2\beta}}{2\beta-1} + 64(M_e\tilde{c}_1 + \tilde{c}_2)\frac{M^2_{1-\beta} b^{2\beta}}{2\beta-1} + 64 M_1 \sum_{k=1}^{m} d_k.\end{aligned} \tag{3.10}$$

We consider the operator $\Phi : \mathcal{B}'_h \to \mathcal{B}'_h$ defined by

$$\Phi x(t) = \begin{cases} \phi(t), \ t \in (-\infty, 0], \\ T(t)[\phi(0) - g(0,\phi)] + g(t, x_t) + \int_0^t T(t-s) f(s, x_s) ds \\ \quad + \int_0^t AT(t-s) g(s, x_s) ds \\ \quad + \int_0^t AT(t-s) e\Big(s, x_s, \int_0^s h_1(s,\tau,x_\tau)d\tau\Big) ds \\ \quad + \int_0^t T(t-s)\sigma\Big(s, x_s, \int_0^s h_2(s,\tau,x_\tau)d\tau\Big) dw(s) \\ \quad + \sum_{0<t_k<t} T(t-t_k) I_k(x(t_k^-)), \quad t \in J. \end{cases} \tag{3.11}$$

From, hypothesis ($\mathbf{H_3}$) – ($\mathbf{H4}$) and Lemma 2.4, the following inequality holds:

$$\begin{aligned}\|AT(t-s)e(s, x_s, \int_0^t h_1(s,\tau,x_\tau)d\tau)\|^2 &\leq \|(-A)^{1-\beta} T(t-s)(-A)^\beta e(s, x_s, \int_0^t h_1(s,\tau,x_\tau)d\tau)\|^2 \\ &\leq \frac{M^2_{1-\beta}}{(t-s)^{2(1-\beta)}}\big[M_e(1+M_{h_1})\|x_s\|^2_{\mathcal{B}_h} + M_e\tilde{c}_1 + \tilde{c}_2\big].\end{aligned}$$

Then, from the Bochner theorem, it follows that $AT(t-s)e(s, x_s, \int_0^t h_1(s,\tau,x_\tau)d\tau)$ is integrable on $[0,t)$. For $\phi \in \mathcal{B}_h$, we defined $\tilde{\phi}$ by

$$\tilde{\phi}(t) = \begin{cases} \phi(t), & t \in (-\infty, 0], \\ T(t)\phi(0), & t \in J, \end{cases}$$

and then, $\tilde{\phi} \in \mathcal{B}'_h$. Let $x(t) = y(t) + \tilde{\phi}(t)$, $-\infty < t \leq b$. It is easy to see that x satisfies (2.5) if and only if y satisfies $y_0 = 0$ and

$$\begin{aligned} y(t) &= -T(t)g(0,\phi) + g(t, y_t + \tilde{\phi}_t) + \int_0^t T(t-s) f(s, y_s + \tilde{\phi}_s) ds \\ &+ \int_0^t AT(t-s) g(s, y_s + \tilde{\phi}_s) ds \\ &+ \int_0^t AT(t-s) e\Big(s, y_s + \tilde{\phi}_t, \int_0^s h_1(s,\tau, y_\tau + \tilde{\phi}_\tau)d\tau\Big) ds \end{aligned}$$

$$+\int_0^t T(t-s)\sigma\Big(s, y_s+\tilde{\phi}_s, \int_0^s h_2(s,\tau,y_\tau+\tilde{\phi})d\tau\Big)dw(s)$$
$$+\sum_{0<t_k<t} T(t-t_k)I_k(y(t_k^-)+\tilde{\phi}(t_k^-)).$$

Let $\mathcal{B}_h'' = \{y \in \mathcal{B}_h' : y_0 = 0 \in \mathcal{B}_h\}$. For any $y \in \mathcal{B}_h''$, we have

$$\|y\|_b = \|y_0\|_{\mathcal{B}_h} + \sup_{0\le s\le b}(E\|y(s)\|^2)^{\frac{1}{2}} = \sup_{0\le s\le b}(E\|y(s)\|^2)^{\frac{1}{2}}.$$

Thus, $(\mathcal{B}_h'', \|\cdot\|_b)$ is a Banach space. Set

$$\mathcal{B}_q = \{y \in \mathcal{B}_h'' : \|y\|_b \le q\} \quad \text{for some } q \ge 0,$$

then $\mathcal{B}_q \subseteq \mathcal{B}_h''$ is uniformly bounded. Moreover, for $y \in \mathcal{B}_q$, from Lemma 2.1, we have

$$\begin{aligned} E(\|y_t+\tilde{\phi}_t\|^2_{\mathcal{B}_h}) &\le 2(\|y_t\|^2_{\mathcal{B}_h}+\tilde{\phi}_t\|^2_{\mathcal{B}_h}) \\ &\le 2l^2\sup_{0\le s\le t}E\|y(s)\|^2+2\|y_0\|^2_{\mathcal{B}_h}+2l^2\sup_{0\le s\le t}E\|\tilde{\phi}(s)\|^2+2\|\tilde{\phi}_0\|^2_{\mathcal{B}_h} \\ &\le 2l^2(q^2+M_1E\|\phi(0)\|^2)+2\|\phi\|^2_{\mathcal{B}_h} \\ &= q'. \end{aligned} \tag{3.12}$$

Define the operator $\tilde{\Phi} : \mathcal{B}_h'' \to \mathcal{B}_h''$ by

$$\tilde{\Phi}y(t) = \begin{cases} 0, \quad t\in(-\infty,0], \\ -T(t)g(0,\phi)+g(t,y_t+\tilde{\phi}_t)+\int_0^t T(t-s)f(s,y_s+\tilde{\phi}_s)ds \\ +\int_0^t AT(t-s)g(s,y_s+\tilde{\phi}_s)ds \\ +\int_0^t AT(t-s)e\Big(s,y_s+\tilde{\phi}_t,\int_0^s h_1(s,\tau,y_\tau+\tilde{\phi}_\tau)d\tau\Big)ds \\ +\int_0^t T(t-s)\sigma\Big(s,y_s+\tilde{\phi}_s,\int_0^s h_2(s,\tau,y_\tau+\tilde{\phi})d\tau\Big)dw(s) \\ +\sum_{0<t_k<t}T(t-t_k)I_k(y(t_k^-)+\tilde{\phi}(t_k^-)), \quad t\in J. \end{cases}$$

Now, we decompose $\tilde{\Phi}$ as $\tilde{\Phi}_1+\tilde{\Phi}_2$ where

$$\begin{aligned} \tilde{\Phi}_1y(t) &= -T(t)g(0,\phi)+g(t,y_t+\tilde{\phi}_t)+\int_0^t AT(t-s)g(s,y_s+\tilde{\phi}_s)ds \\ &\quad+\int_0^t AT(t-s)e\Big(s,y_s+\tilde{\phi}_s,\int_0^s h_1(s,\tau,y_\tau+\tilde{\phi}_\tau)d\tau\Big)ds, \quad t\in J, \\ \tilde{\Phi}_2y(t) &= \int_0^t T(t-s)\sigma\Big(s,y_s+\tilde{\phi}_s,\int_0^s h_2(s,\tau,x_\tau+\tilde{\phi})d\tau\Big)dw(s)+\int_0^t T(t-s)f(s,y_s+\tilde{\phi}_s)ds \\ &\quad+\sum_{0<t_k<t}T(t-t_k)I_k(y(t_k^-)+\tilde{\phi}(t_k^-)), \quad t\in J. \end{aligned}$$

Obviously, the operator Φ having a fixed point is equivalent to $\tilde{\Phi}$ having one. Now, we shall show that the operator $\tilde{\Phi}_1, \tilde{\Phi}_2$ satisfy all the conditions of Lemma 2.2.

Theorem 3.1. *If assumption* $(\mathbf{H_1})-(\mathbf{H_8})$ *hold, then* $\tilde{\Phi}_1$ *is a contraction and* $\tilde{\Phi}_2$ *is completely continuous.*

Proof. Let $u, v \in \mathcal{B}_h''$. Then, we have to show that $\tilde{\Phi}_1$ is a contraction on $\mathcal{B}_h''$, we have

$$\begin{aligned} &E\|\tilde{\Phi}_1u(t)-\tilde{\Phi}_1v(t)\|^2 \\ &\le E\|g(t,u_t+\tilde{\phi}_t)-g(t,v_t+\tilde{\phi}_t)\|^2+E\|\int_0^t AT(t-s)\Big[g(s,u_s+\tilde{\phi}_s)-g(s,v_s+\tilde{\phi}_s)\Big]ds\|^2 \\ &\quad+E\|\int_0^t AT(t-s)\Big[e\Big(s,u_s+\tilde{\phi}_s,\int_0^s h_1(s,\tau,u_\tau+\tilde{\phi}_\tau)d\tau\Big)-e\Big(s,v_s+\tilde{\phi}_s,\int_0^s h_1(s,\tau,v_\tau+\tilde{\phi}_\tau)d\tau\Big)\Big]ds\|^2 \end{aligned}$$

$$\leq 16\Big\{E\|g(t,u_t+\tilde{\phi}_t)-g(t,v_t+\tilde{\phi}_t)\|^2+E\|\int_0^t AT(t-s)[g(s,u_s+\tilde{\phi}_s)-g(s,v_s+\tilde{\phi}_s)]ds\|^2$$
$$+E\|\int_0^t AT(t-s)\Big[e\Big(s,u_s+\tilde{\phi}_s,\int_0^s h_1(s,\tau,u_\tau+\tilde{\phi}_\tau)d\tau\Big)-e\Big(s,v_s+\tilde{\phi}_s,\int_0^s h_1(s,\tau,v_\tau+\tilde{\phi}_\tau)d\tau\Big)\Big]ds\|^2\Big\}$$
$$\leq 16\Big\{M_g\|(-A)^{-\beta}\|^2E\|u_t-v_t\|^2_{\mathcal{B}_h}+M_gE\|u_t-v_t\|^2_{\mathcal{B}_h}\frac{(M_{1-\beta}b^\beta)^2}{2\beta-1}$$
$$+\frac{(M_{1-\beta}b^\beta)^2}{2\beta-1}M_e[E\|u_t-v_t\|^2_{\mathcal{B}_h}+M_{h_1}E\|u_t-v_t\|^2_{\mathcal{B}_h}]\Big\}$$
$$\leq 16\Big\{M_gM_0+\Big[M_g+M_e(1+M_{h_1})\Big]\frac{(M_{1-\beta}b^\beta)^2}{2\beta-1}\Big\}E\|u_t-v_t\|^2_{\mathcal{B}_h}$$
$$\leq 16\Big\{M_gM_0+\Big[M_g+M_e(1+M_{h_1})\Big]\frac{(M_{1-\beta}b^\beta)^2}{2\beta-1}\Big\}\Big[2l^2\sup_{s\in[0,t]}E\|u(s)-v(s)\|^2+2\|u_0\|^2_{\mathcal{B}_h}+2\|v_0\|^2_{\mathcal{B}_h}\Big]$$
$$\leq 32l^2\Big\{M_gM_0+\Big[M_g+M_e(1+M_{h_1})\Big]\frac{(M_{1-\beta}b^\beta)^2}{2\beta-1}\Big\}E\|u(s)-v(s)\|^2$$
$$\leq \sup_{s\in[0,b]}C_0E\|u(s)-v(s)\|^2.$$

Since, $\|u_0\|^2_{\mathcal{B}_h}=0$, $\|v_0\|^2_{\mathcal{B}_h}=0$. Taking the supremum over t,

$$\|\tilde{\Phi}_1u-\tilde{\Phi}_1v\|^2\leq C_0\|u-v\|^2,$$

and so, by assumption $0\leq C_0\leq 1$, we see that $\tilde{\Phi}_1$ is a contraction on $\mathcal{B}''_h$.

Now, we show that the operator $\tilde{\Phi}_2$ is completely continuous. First, we show that $\tilde{\Phi}_2$ maps bounded sets into bounded sets in $\mathcal{B}''_h$. It is enough to show that there exists a positive constants r such that for each $y\in B_q=\{y\in\mathcal{B}''_h:\|y\|^2_b\leq q\}$ one has $E\|\tilde{\Phi}_2y\|^2_b\leq r$. Now for $t\in J$,

$$\tilde{\Phi}_2y(t)=\int_0^t T(t-s)f(s,y_s+\tilde{\phi}_s)ds+\int_0^t T(t-s)\sigma\Big(s,y_s+\tilde{\phi}_s,\int_0^s h_2(s,\tau,y_\tau+\tilde{\phi}_\tau)d\tau\Big)dw(s)$$
$$+\sum_{0<t_k<t}T(t-t_k)I_k(y(t_k^-)+\tilde{\phi}(t_k^-)),\quad t\in J.$$

Therefore, by the assumption, for each $t\in J$, we have

$$E\|\tilde{\Phi}_2y(t)\|^2\leq 9M_1\int_0^b m(s)\Omega_1(E\|y_s+\tilde{\phi}_s\|^2_{\mathcal{B}_h})ds+9M_1Tr(Q)\int_0^t p(s)\Omega_2\Big(E\|y_s+\tilde{\phi}_s\|^2_{\mathcal{B}_h}$$
$$+\int_0^s\gamma m(\tau)\Omega_3(E\|y_\tau+\tilde{\phi}_\tau\|^2_{\mathcal{B}_h})d\tau\Big)ds+9M_1\sum_{k=1}^m d_k$$
$$\leq 9M_1\Omega_1(q')\int_0^b m(s)ds+9M_1Tr(Q)\Omega_2(q'+L_0\Omega_3(q'))\int_0^b p(s)ds+9M_1\sum_{k=1}^m d_k$$
$$=r.$$

Then, for each $y\in\tilde{\Phi}_2y(B_q)$, we have $\|\tilde{\Phi}_2y\|^2_b\leq r$.

Next, we show that $\tilde{\Phi}_2$ maps bounded set into equicontinuous sets of $\mathcal{B}''_h$.
Let $0<\tau_1<\tau_2\leq b$. Then for each $y\in\mathcal{B}_q=\{y\in\mathcal{B}''_h:\|y\|_b\leq q\}$ and $y\in\tilde{\Phi}_2y$. Then for each $t\in J$, we have

$$\tilde{\Phi}_2y(t)=\int_0^t T(t-s)f(s,y_s+\tilde{\phi}_s)ds+\int_0^t T(t-s)\sigma\Big(s,y_s+\tilde{\phi}_s,\int_0^s h_2(s,\tau,y_\tau+\tilde{\phi}_\tau)d\tau\Big)dw(s)$$
$$+\sum_{0<t_k<t}T(t-t_k)I_k(y(t_k^-)+\tilde{\phi}(t_k^-)),\quad t\in J.$$

Let $\tau_1,\tau_1\in J-\{t_1,t_2,\cdots,t_m\}$. Then, we have

$$E\|\tilde{\Phi}_2y(\tau_2)-\tilde{\Phi}_2y(\tau_1)\|^2$$
$$\leq 9E\|\int_0^{\tau_1-\epsilon}[T(\tau_2-s)-T(\tau_1-s)]f(s,y_s+\tilde{\phi}_s)ds\|^2$$

$$
\begin{aligned}
&+ 9E\Big\| \int_{\tau_1-\epsilon}^{\tau_1} [T(\tau_2 - s) - T(\tau_1 - s)] f(s, y_s + \tilde{\phi}_s) ds \Big\|^2 \\
&+ 9E\Big\| \int_{\tau_1}^{\tau_2} [T(\tau_2 - s)] f(s, y_s + \tilde{\phi}_s) ds \Big\|^2 \\
&+ 9E\Big\| \int_{0}^{\tau_1-\epsilon} [T(\tau_2 - s) - T(\tau_1 - s)] \sigma\Big(s, y_s + \tilde{\phi}_s, \int_0^s h_2(s, \tau, y_\tau + \tilde{\phi}_\tau) d\tau\Big) dw(s) \Big\|^2 \\
&+ 9E\Big\| \int_{\tau_1-\epsilon}^{\tau_1} [T(\tau_2 - s) - T(\tau_1 - s)] \sigma\Big(s, y_s + \tilde{\phi}_s, \int_0^s h_2(s, \tau, y_\tau + \tilde{\phi}_\tau) d\tau\Big) dw(s) \Big\|^2 \\
&+ 9E\Big\| \int_{\tau_1}^{\tau_2} [T(\tau_2 - s)] \sigma\Big(s, y_s + \tilde{\phi}_s, \int_0^s h_2(s, \tau, y_\tau + \tilde{\phi}_\tau) d\tau\Big) dw(s) \Big\|^2 \\
&+ 9E\Big\| \sum_{0<t_k<\tau_1} \|[T(\tau_2 - t_k) - T(\tau_1 - t_k)] I_k(y(t_k^-) + \tilde{\phi}(t_k^-)) \Big\|^2 \\
&+ 9E\Big\| \sum_{\tau_1 \le t_k < \tau_2} \| T(\tau_2 - t_k) I_k(y(t_k^-) + \tilde{\phi}(t_k^-)) \Big\|^2 \\
\le\ & 9 \int_0^{\tau_1-\epsilon} E\|T(\tau_2 - s) - T(\tau_1 - s)\|^2 h_{q'}(s) + 9 \int_{\tau_1-\epsilon}^{\tau_1} E\|T(\tau_2 - s) - T(\tau_1 - s)\|^2 h_{q'}(s) ds \\
&+ 9 \int_{\tau_1}^{\tau_2} E\|T(\tau_2 - s)\|^2 h_{q'}(s) ds + 9bTr(Q) \int_0^{\tau_1-\epsilon} E\|T(\tau_2 - s) - T(\tau_1 - s)\|^2 p(s)\Omega_2(q') ds \\
&+ 9bTr(Q) \int_{\tau_1-\epsilon}^{\tau_1} E\|T(\tau_2 - s) - T(\tau_1 - s)\|^2 p(s)\Omega_2(q') ds \\
&+ 9bTr(Q) \int_{\tau_1}^{\tau_2} E\|T(\tau_2 - s)\|^2 p(s)\Omega_2(q') ds + 9 \sum_{0<t_k<\tau_1} E\|T(\tau_2 - t_k) - T(\tau_1 - t_k)\|^2 d_k \\
&+ 9M_1 \sum_{\tau_1 \le t_k < \tau_2} d_k.
\end{aligned}
$$

The right-hand side of the above inequality is independent of $y \in B_q$ tends to zero as $\tau_2 - \tau_1 \to 0$, and for ϵ sufficiently small, since the compactness of $\{T(t)\}_{t\geq 0}$ implies the continuity in the uniform operator topology. Thus the set $\{\tilde{\Phi}_2 y : y \in B_q\}$ is equicontinuous. Here we consider only the case $0 < \tau_1 \le \tau_2 \le b$, since the other cases $\tau_1 \le \tau_2 \le 0$ 0r $\tau_1 \le 0 \le \tau_2 \le b$ are very simple.

Next, we show that $\tilde{\Phi}_2 : \mathcal{B}_h'' \to \mathcal{B}_h''$ is continuous.

Let $\{y^{(n)}(t)\}_{n=0}^{\infty} \subseteq \mathcal{B}_h''$, with $y^{(n)} \to y$ in $\mathcal{B}_h''$. Then, there is a number $q \ge 0$ such that $|y^{(n)}(t)| \le q$ for all n and a.e. $t \in J$, so $y^{(n)} \in B_q$ and $y \in B_q$. Using (3.12), we have $\|y_t^{(n)} + \tilde{\phi}_t\|_{\mathcal{B}_h}^2 \le q', t \in J$. By Definition 2.1, $(\mathbf{H_8})$, I_k, $k = 1, 2, \cdots, m$, is continuous

$$
\begin{gathered}
f(t, y_t^{(n)} + \tilde{\phi}_t) \to f(t, y_t + \tilde{\phi}_t), \\
\sigma\Big(t, y_t^{(n)} + \tilde{\phi}_t, \int_0^t h_2(s, \tau, y_\tau^{(n)} + \tilde{\phi}_\tau) d\tau\Big) \to \sigma\Big(t, y_t + \tilde{\phi}_t, \int_0^t h_2(s, \tau, y_\tau + \tilde{\phi}_\tau) d\tau\Big),
\end{gathered}
$$

for each $t \in J$, and since

$$
\begin{aligned}
&E\|f(t, y_t^{(n)} + \tilde{\phi}_t) - f(t, y_t + \tilde{\phi}_t)\|^2 \le 2\alpha_{q'}(t), \\
&E\Big\|\sigma\Big(t, y_t^{(n)} + \tilde{\phi}_t, \int_0^t h_2(s, \tau, y_\tau^{(n)} + \tilde{\phi}_\tau) d\tau\Big) - \sigma\Big(t, y_t + \tilde{\phi}_t, \int_0^t h_2(s, \tau, y_\tau + \tilde{\phi}_\tau) d\tau\Big)\Big\|^2 \le 2p(t)\Omega_2(q').
\end{aligned}
$$

By the dominated convergence theorem that,

$$
\begin{aligned}
E\|\tilde{\Phi}_2 y^{(n)} - \tilde{\Phi}_2 y\|^2 = \sup_{t \in J} E\Big\| &\int_0^t T(t - s)\Big[\sigma\Big(t, y_s^{(n)} + \tilde{\phi}_s, \int_0^t h_2(s, \tau, y_\tau + \tilde{\phi}_\tau) d\tau\Big) \\
&- \sigma\Big(t, y_s + \tilde{\phi}_s, \int_0^t h_2(s, \tau, y_\tau + \tilde{\phi}_\tau) d\tau\Big)\Big] dw(s) \\
&+ \int_0^t T(t - s)\Big[f(t, y_s^{(n)} + \tilde{\phi}_s) - f(t, y_s + \tilde{\phi}_s)\Big] ds \\
&+ \sum_{0 \le t_k < t} T(t - t_k)\Big[I_k(y^n(t_k^-) + \tilde{\phi}(t_k^-)) - I_k(y(t_k^-) + \tilde{\phi}(t_k^-))\Big]\Big\|^2
\end{aligned}
$$

$$
\begin{aligned}
&\le M_1 Tr(Q)\int_0^t E\|\sigma\Big(t, y_s^{(n)}+\tilde{\phi}_s, \int_0^t h_2(s,\tau,y_\tau+\tilde{\phi}_\tau)d\tau\Big)\\
&\quad -\sigma\Big(t, y_s+\tilde{\phi}_s, \int_0^t h_2(s,\tau,y_\tau+\tilde{\phi}_\tau)d\tau\Big)\|^2 ds\\
&\quad + M_1\int_0^t E\|f(t,y_s^{(n)}+\tilde{\phi}_s)-f(t,y_s+\tilde{\phi}_s)\|^2 ds\\
&\quad + \sum_{0\le t_k<t}\|T(t-t_k)\|^2 E\|I_k(y^n(t_k^-)+\tilde{\phi}(t_k^-))-I_k(y(t_k^-)+\tilde{\phi}(t_k^-))\|^2\\
&\quad \to 0 \quad \text{as} \quad n\to\infty.
\end{aligned}
$$

Thus, $\tilde{\Phi}_2$ is continuous.

Next, we show that $\tilde{\Phi}_2$ maps B_q into a precompact set in H. Let $0<t\le b$ be fixed and ϵ be a real number satisfying $0<\epsilon\le t$. For $y\in B_q$, we define

$$
\begin{aligned}
(\tilde{\Phi}_2^\epsilon y)(t) &= \int_0^{t-\epsilon} T(t-s)\sigma\Big(t, y_s+\tilde{\phi}_s, \int_0^t h_2(s,\tau,y_\tau+\tilde{\phi}_\tau)d\tau\Big)dw(s)\\
&\quad + \int_0^{t-\epsilon} T(t-s)f(t,y_s+\tilde{\phi}_s)ds + \sum_{0\le t_k<t-\epsilon} T(t-t_k)I_k(y(t_k^-)+\tilde{\phi}(t_k^-))\\
&= T(\epsilon)\int_0^{t-\epsilon} T(t-s-\epsilon)\sigma\Big(t, y_s+\tilde{\phi}_s, \int_0^t h_2(s,\tau,y_\tau+\tilde{\phi}_\tau)d\tau\Big)dw(s)\\
&\quad + T(\epsilon)\int_0^{t-\epsilon} T(t-s-\epsilon)f(t,y_s+\tilde{\phi}_s)ds\\
&\quad + T(\epsilon)\sum_{0\le t_k<t-\epsilon} T(t-t_k-\epsilon)I_k(y(t_k^-)+\tilde{\phi}(t_k^-)).
\end{aligned}
$$

Since $T(t)$ is a compact operator, the set $V_\epsilon(t)=\{(\tilde{\Phi}_2^\epsilon y)(t): y\in B_q\}$ is relatively compact in H for every ϵ, for every $0<\epsilon<t$. Moreover, for each $y\in B_q$, we have

$$
\begin{aligned}
&E\|(\tilde{\Phi}_2 y)(t)-(\tilde{\Phi}_2^\epsilon y)(t)\|^2\\
&\le \int_{t-\epsilon}^t \|T(t-s)\|^2 E\|\sigma\Big(t, y_s+\tilde{\phi}_s, \int_0^t h_2(s,\tau,y_\tau+\tilde{\phi}_\tau)d\tau\Big)\|^2 dw(s)\\
&\quad + \int_{t-\epsilon}^t \|T(t-s)\|^2 E\|f(t,y_s+\tilde{\phi}_s)\|^2 ds + \sum_{t-\epsilon\le t_k<t}\|T(t-t_k)\|^2 E\|I_k(y(t_k^-)+\tilde{\phi}(t_k^-))\|^2\\
&\le M_1 Tr(Q)\int_{t-\epsilon}^t p(s)\Omega_2(q')d(s) + M_1\int_{t-\epsilon}^t \alpha_{q'}(s)ds + M_1\sum_{t-\epsilon\le t_k<t} d_k.
\end{aligned}
$$

Therefore,

$$
E\|(\tilde{\Phi}_2 y)(t)-(\tilde{\Phi}_2^\epsilon y)(t)\|^2 \to 0, \quad \text{as} \quad \epsilon\to 0.
$$

and there are precompact sets arbitrarily close to the set $\{(\tilde{\Phi}_2 y)(t): y\in B_q\}$. Thus, the set $\{(\tilde{\Phi}_2^\epsilon y)(t): y\in B_q\}$ is precompact in H. Therefore, from Arzela- Ascoli theorem, the operator $\tilde{\Phi}_2$ is completely continuous.

In order to study the existence results for the problem (1.1)-(1.3), we consider the following nonlinear operator equation,

$$
\begin{aligned}
x(t) &= \lambda T(t)[\phi(0)-g(0,\phi)] + \lambda g(t,x_t) + \lambda\int_0^t AT(t-s)g(s,x_s)ds\\
&\quad + \lambda\int_0^t T(t-s)f(s,x_s)ds + \lambda\int_0^t AT(t-s)e(s,x_s,\int_0^s h_1(s,\tau,x_\tau)d\tau)ds\\
&\quad + \lambda\int_0^t T(t-s)\sigma(s,x_s,\int_0^s h_2(s,\tau,x_\tau)d\tau)dw(s)\\
&\quad + \lambda\sum_{0<t_k<t} T(t-t_k)I_k(x(t_k^-)), \quad t\in J, \qquad (3.13)
\end{aligned}
$$

for some $0<\lambda<1$. The following lemma proves that an a priori bound exists for the solution of the above equation. □

Theorem 3.2. *If hypothesis* $(\mathbf{H_1})-(\mathbf{H_8})$ *are satisfied, then there exist an a priori bound* $K\geq 0$ *such that* $\|x_t\|^2_{\mathcal{B}_h}\leq K, t\in J$*, where* K *depends only on* b *and on the function* $\Omega_1,\Omega_2,\hat{m}$ *and* Ω_3*.*

Proof. From (3.13), we have

$$\begin{aligned}E\|x(t)\|^2 &\leq 64M_1\|\phi\|^2_{\mathcal{B}_h}+64\|(-A)^{-\beta}\|^2M_1(c_1\|\phi\|^2_{\mathcal{B}_h}+c_2)\\&+64\|(-A)\|^{-\beta}\|^2M_g(c_1\|x_t\|^2_{\mathcal{B}_h}+c_2)+64\left[\int_0^t\frac{M^2_{1-\beta}bc_1}{(t-s)^{2(1-\beta)}}\|x_s\|^2_{\mathcal{B}_h}ds+\frac{M^2_{1-\beta}c_2b^{2\beta}}{2\beta-1}\right]\\&+64M_1\int_0^t m(s)\Omega_1(\|x_s\|^2_{\mathcal{B}_h})ds+64(M_e\tilde{c}_1+\tilde{c}_2)\frac{M^2_{1-\beta}b^{2\beta}}{2\beta-1}\\&+64bM^2_{1-\beta}M_e(1+M_{h_1})\int_0^t\frac{\|x_s\|^2_{\mathcal{B}_h}}{(t-s)^{2(1-\beta)}}ds\\&+64M_1Tr(Q)\int_0^t p(s)\Omega_2\Big(\|x_s\|^2_{\mathcal{B}_h}+\int_0^s\gamma m(\tau)\Omega_3(\|x_\tau\|^2)d\tau\Big)ds+64\ M_1\sum_{k=1}^m d_k.\end{aligned}$$

Now, we consider the function μ defined by

$$\mu(t)=\sup_{0\leq s\leq t}E\|x(s)\|^2,\quad 0\leq t\leq b.$$

From, Lemma 2.1 and the above inequality, we have

$$E\|x(t)\|^2=2\|\phi\|^2_{\mathcal{B}_h}+2l^2\sup_{0\leq s\leq t}(E\|x(s)\|^2).$$

Therefore, we get

$$\begin{aligned}\mu(t)&\leq 2\|\phi\|^2_{\mathcal{B}_h}+2l^2\Big\{\overline{F}+64\|(-A)^{-\beta}\|^2c_1\mu(t)+64bM^2_{1-\beta}c_1\int_0^t\frac{\mu(s)}{(t-s)^{2(1-\beta)}}ds\\&+64\ M_1\int_0^t m(s)\Omega_1\mu(s)ds+64\|(-A)^{-\beta}\|^2M_e(1+M_{h_1})\mu(t)\\&+64bM^2_{1-\beta}M_e(1+M_{h_1})\int_0^t\frac{\mu(s)}{(t-s)^{2(1-\beta)}}ds\\&+64\ M_1Tr(Q)\int_0^t p(s)\Omega_2\Big(\mu(s)+\int_0^s\gamma m(\tau)\Omega_3(\mu(\tau))d\tau\Big)ds\Big\},\end{aligned}$$

where $\overline{F}$ is given in (3.10). Thus, we have

$$\begin{aligned}\mu(t)\leq K_1&+K_2\int_0^t\frac{\mu(s)}{(t-s)^{2(1-\beta)}}ds+K_3\int_0^t m(s)\Omega_1\mu(s)ds\\&+K_4\int_0^t p(s)\Omega_2\Big(\mu(s)+\int_0^s\gamma m(\tau)\Omega_3(\mu(\tau))d\tau\Big)ds,\end{aligned}$$

where K_1,K_2,K_3 are given in (3.9). By Lemma 2.3, we have

$$\begin{aligned}\mu(t)\leq B_0\Bigg(&K_1+K_3\int_0^t m(s)\Omega_1\mu(s)ds\\&+K_4\int_0^t p(s)\Omega_2\Big(\mu(s)+\int_0^s\gamma m(\tau)\Omega_3\mu(\tau)d\tau\Big)ds\Bigg),\end{aligned}$$

where

$$B_0=e^{K_2^n(\Gamma(2\beta-1))^nb^{n2\beta-1}/\Gamma(n(2\beta-1)}\sum_{j=0}^{n-1}\Big(\frac{K_2b^{2\beta-1}}{2\beta-1}\Big)^j.$$

Let us take the right-hand side of the above inequality as $v(t)$. Then $v(0)=B_0K_1,\ \mu(t)\leq v(t), 0\leq t\leq b$, and

$$v'(t)\leq B_0\Big[K_3m(t)\Omega_1\mu(t)+K_4p(t)\Omega_2\Big(\mu(t)+\int_0^t\gamma m(s)\Omega_3(\mu(s))ds\Big)\Big].$$

Since, ψ is nondecreasing,

$$v'(t) \leq B_0\Big[K_3 m(t)\Omega_1 v(t) + K_4 p(t)\Omega_2\Big(v(t) + \int_0^t \gamma m(s)\Omega_3(v(s))ds\Big)\Big].$$

Let $w(t) = v(t) + \int_0^t \gamma m(s)\Omega_3(v(s))ds$. Then $w(0) = v(0)$ and $v(t) \leq w(t)$.

$$\begin{aligned} w'(t) &= v'(t) + \gamma m(t)\Omega_3(v(t)) \\ &\leq B_0 K_3 m(t)\Omega_1(w(t)) + B_0 K_4 p(t)\Omega_2(w(t)) + \gamma m(t)\Omega_3(w(t)) \\ &\leq \widehat{m}(t)[\Omega_1(w(t)) + \Omega_2(w(t)) + \Omega_3(w(t))]. \end{aligned}$$

This implies that,

$$\int_{w(0)}^{w(t)} \frac{ds}{\Omega_1(s) + \Omega_2(s) + \Omega_3(s)} \leq \int_0^b \widehat{m}(s) \leq \int_{B_0 K_1}^{\infty} \frac{ds}{\Omega_1(s) + \Omega_2(s) + \Omega_3(s)}.$$

This implies that $v(t) < \infty$. So the inequality shows that there is a constant K such that $v(t) \leq K,\ t \in J$. So, $\|x_t\|^2_{\mathcal{B}_h} \leq \mu(t) \leq v(t) \leq K,\ t \in J$, where K depends only on b and on the functions $\Omega_1, \Omega_2, \Omega_3$ and $\widehat{m}$. $\square$

Theorem 3.3. *Assume that the hypotheses* $(\mathbf{H_1}) - (\mathbf{H_8})$ *hold. Then problem (1.1)-(1.3) has at least one mild solution on* J.

Proof. Let us take the set,

$$G(\tilde{\Phi}) = \{y \in \mathcal{B}''_h : y = \lambda\tilde{\Phi}_1(\frac{y}{x}) + \lambda\tilde{\Phi}_2 y, \quad \text{for some} \quad \lambda \in (0,1)\}. \tag{3.14}$$

Then, for any $y \in G(\tilde{\Phi})$, we have by Theorem 3.2 that $\|x_t\|^2_{\mathcal{B}_h} \leq K, t \in J$, and hence

$$\begin{aligned} \|y\|_b^2 &= \|y_0\|^2_{\mathcal{B}_h} + \sup\{E\|y(t)\|^2 : 0 \leq t \leq b\} \\ &= \sup\{E\|y(t)\|^2 : 0 \leq t \leq b\} \\ &\leq \sup\{E\|x(t)\|^2 : 0 \leq t \leq b\} + \sup\{\|\tilde{\phi}(t)\|^2 : 0 \leq t \leq b\} \\ &\leq \sup\{l^-\|x_t\|^2_{\mathcal{B}_h} : 0 \leq t \leq b\} + \sup\{\|T(t)\phi(0)\|^2 : 0 \leq t \leq b\} \\ &\leq l^- K + M_1\|\phi(0)\|^2. \end{aligned}$$

This implies that G is bounded on J. Consequently, by the Krasnoselski-Schaefer type fixed point theorem the operator $\tilde{\Phi}$ has a fixed point $y^* \in \mathcal{B}''_h$. Since $x(t) = y^*(t) + \tilde{\phi}(t)$, $t \in (-\infty, b]$, x is a fixed point of the operator Φ which is a mild solution of problem (1.1)-(1.3). $\square$

4 Example

In this, we present the application for the problem (1.1)-(1.3), we consider the following impulsive neutral stochastic partial integrodifferential equation of the form

$$\begin{aligned} &\frac{\partial}{\partial t}\Big[v(t,y) - \int_{-\infty}^t \int_0^\pi a(s-t,\eta,y)d\eta ds\Big] = \frac{\partial^2}{\partial y^2}\Big[v(t,y) + \int_0^t a_1(t,y,s-t)P_1(v(s,y))ds \\ &\quad + \int_0^t \int_{-\infty}^s k(s-\tau)P_2(v(\tau,y))d\tau\Big]ds + k_0(y)v(t,y) + \int_0^t a_2(t,y,s-t)Q_1(v(s,y))ds \\ &\quad + \int_0^t \int_{-\infty}^s k(s-\tau)Q_2(v(\tau,y))d\tau d\beta(s), \quad y \in [0,\pi], \qquad t \in [0,b], \quad t \neq t_k. \end{aligned} \tag{4.1}$$

$$v(t,0) = v(t,\pi) = 0, \qquad t \geq 0, \tag{4.2}$$

$$v(t,y) = \phi(t,y), \quad t \in (-\infty, 0], \quad y \in [0,\pi], \tag{4.3}$$

$$\Delta v(t_i)(y) = \int_{-\infty}^{t_i} q_i(t_i - s)v(s,y)ds, \quad y \in [0,\pi], \tag{4.4}$$

where $0 < t_1 < \cdots < t_n < b$ are prefixed numbers and $\psi \in \mathcal{B}_h$ and $\beta(t)$ is a one-dimensional standard Wiener process. Let us take $H = L^2[0,\pi]$ with the norm $\|\cdot\|$. Define $A : H \to H$ by $A(t)z = -a(t,y)z''$ with domain,

$$D(A) = \{z(\cdot) \in H : z, z', \text{ are absolutely continuous, } z'' \in H, z(0) = z(\pi) = 0\},$$

Then

$$Az = \sum_{n=1}^{\infty} n^2 \langle z, z_n \rangle z_n, \quad z \in D(A),$$

where $z_n(s) = \sqrt{\frac{2}{\pi}} \sin(ns)$, $n = 1, 2, \cdots$ is the orthonormal set of eigenvector of A. It is well known that A is the infinitesimal generator of an analytic semigroup $T(t),\ t \geq 0$ in H and is given by

$$T(t)z = \sum_{n=1}^{\infty} exp^{-n^2 t} \langle z, z_n \rangle z_n, \quad z \in H.$$

For every $z \in H$, $(-A)^{\frac{1}{2}} z = \sum_{n=1}^{\infty} \frac{1}{n} \langle z, z_n \rangle z_n$, and $\|(-A)^{\frac{1}{2}}\|^2 = 1$. The operator $(-A)^{\frac{1}{2}}$ is given by

$$(-A)^{\frac{1}{2}} z = \sum_{n=1}^{\infty} n \langle z, z_n \rangle z_n,$$

on the space $D((-A)^{\frac{1}{2}}) = \{z \in H : \sum_{n=1}^{\infty} n \langle z, z_n \rangle z_n \in H\}$. Since, the analytic semigroup $T(t)$ is compact [38], there exists a constant $M_1 \geq 0$ such that $\|T(t\|^2 \leq M_1$ and satisfies (H_2).

Now, we give a special $\mathcal{B}_h$- space. Let $h(s) = e^{2s}, s \leq 0$, then $l = \displaystyle\int_{-\infty}^{0} h(s)ds = \frac{1}{2}$ and let

$$\|\phi\|_{\mathcal{B}_h} = \int_{-\infty}^{0} h(s) \sup_{s \leq \theta \leq 0} E\Big(\|\phi(\theta)\|^2\Big)^{\frac{1}{2}} ds.$$

It follows from [30], that $(\mathcal{B}_h, \|\cdot\|_{\mathcal{B}_h})$ is a Banach space.
Hence, for $(t,\phi) \in [0,b] \times \mathcal{B}_h$, where $\phi(\theta)(y) = \phi(\theta, y), (\theta, y) \in (-\infty, 0] \times [0,\pi]$. Set

$$\begin{aligned} v(t)(y) = v(t,y), \quad g(t,\phi)y &= \int_{-\infty}^{0} \int_{0}^{\phi} a(s-t,\eta,y)d\eta ds, \\ f(t,\phi)(y) &= \ k_0(y)\phi(t,y), \\ b(t,\phi,B_1\phi)(y) &= \int_{-\infty}^{0} a_1(t,y,\theta)P_1(\phi(\theta)(y))d\theta + B_1\phi(y), \end{aligned}$$

and

$$\sigma(t,\phi,B_2\phi)(y)) = \int_{-\infty}^{0} a_2(t,y,\theta)Q_1(\phi(\theta)(y))d\theta + B_2\phi(y),$$

where

$$\begin{aligned} B_1\phi(y) &= \int_0^t \int_{-\infty}^{0} k(s-\theta)P_2(\phi(\theta)(y))d\theta ds, \\ B_2\phi(y) &= \int_0^t \int_{-\infty}^{0} k(s-\theta)Q_2(\phi(\theta)(y))d\theta d\beta(s). \end{aligned}$$

Then, the above equation can be written in the abstract form as system (1.1)-(1.3). The function a_1, k and P_1, P_2 are assumed to satisfy the conditions of [27] and $q_i : R \to R$ are continuous and $d_i = \displaystyle\int_{-\infty}^{0} h(s)q_i^2(s)ds < \infty$ for $i = 1, 2, \cdots, n$. Moreover, $e([0,b] \times \mathcal{B}_h \times L^2) \subseteq D((-A)^{\frac{1}{2}})$ and $\|(-A)^{\frac{1}{2}} e(t,\phi_1,u_1)(y) - (-A)^{\frac{1}{2}} e(t,\phi_2,u_2)(y)\|^2 \leq M_e[\|\phi_1 - \phi_2\|^2_{\mathcal{B}_h} + |u_1 - u_2|^2]$ for some constants $M_e > 0$ depending on a_1, k, P_1, P_2 and $|u_1 - u_2|^2 = \|B_1\phi_1 - B_1\phi_2\|^2 \leq M_{h_1}\|\phi_1 - \phi_2\|^2_{\mathcal{B}_h}$ for $M_{h_1} > 0$ such that $\frac{1}{2}M_e(1 + M_{h_1})(1 + 2C_{\frac{1}{2}}\sqrt{e}) < 1$.
Suppose further that :

(i) The function $a_2(t,y,\theta)$ is continuous in $[0,b]\times[0,\pi]\times(-\infty,0]$ and $a_2(t,y,\theta)\geq 0$, $\int_{-\infty}^{0} a_2(t,y,\theta)d\theta = p_1(t,y)<\infty$.

(ii) The function $k(t-s)$ is continuous in $[0,b]$ and $k(t-s)\geq 0$, $\int_0^t\int_{-\infty}^{0} k(s-\theta)d\theta ds = p_2(t)<\infty$.

(iii) The function $Q_i(\cdot), i=1,2$ are continuous and for each $(\theta,y)\in(-\infty,0]\times[0,\pi], 0\leq Q_i(v(\theta)(y))\leq \Phi(\int_{-\infty}^{0} e^{2s}\|v(s,\cdot)\|_{L_2}ds)$, where $\Phi:[0,+\infty)\to(0,+\infty)$ is a continuous and nondecreasing function.

Now, we can see that,

$$
\begin{aligned}
&E|\sigma(t,\phi,B_2\phi)|_{L_2}\\
&=\Big[\int_0^{\pi}\Big(\int_{-\infty}^{0} a_2(t,y,\theta)Q_1(\phi(\theta)(y))d\theta + B_2\phi(\theta)(y)\Big)^2 dy\Big]^{\frac{1}{2}}\\
&\quad\leq\sqrt{2}\Big[\int_0^{\pi}\Big(\int_{-\infty}^{0} a_2(t,y,\theta)\Phi\Big(\int_{-\infty}^{0} e^{2s}\|\phi(s),(\cdot)\|_{L_2}ds\Big)d\theta\Big)^2 dy\Big]^{\frac{1}{2}}\\
&\quad+\sqrt{2}\Big[\int_0^{\pi}\Big(\int_0^t\int_{-\infty}^{0} k(\tau-\theta)\Phi\Big(\int_{-\infty}^{0} e^{2s}\|\phi(s),(\cdot)\|_{L_2}ds\Big)d\theta d\beta(\tau)\Big)^2 dy\Big]^{\frac{1}{2}}\\
&\leq\sqrt{2}\Big[\int_0^{\pi}\Big(\int_{-\infty}^{0} a_2(t,y,\theta)\Phi\Big(\int_{-\infty}^{0} e^{2s}\sup_{s\in[\theta,0]}\|\phi(s)\|_{L_2}ds\Big)d\theta\Big)^2 dy\Big]^{\frac{1}{2}}\\
&\quad+\sqrt{2}\;Tr(Q)\Big[\int_0^{\pi}\Big(\int_0^t\int_{-\infty}^{0} k(\tau-\theta)\Phi\Big(\int_{-\infty}^{0} e^{2s}\sup_{s\in[\theta,0]}\|\phi(s)\|_{L_2}ds\Big)d\theta d\tau\Big)^2 dy\Big]^{\frac{1}{2}}\\
&=\sqrt{2}\Big[\int_0^{\pi}\Big(\int_{-\infty}^{0} a_2(t,y,\theta)d\theta\Big)^2 dy\Big]^{\frac{1}{2}}\Phi(\|\phi\|_h^2)\\
&\quad+\sqrt{2}\;Tr(Q)\Big[\int_0^{\pi}\Big(\int_0^t\int_{-\infty}^{0} k(s-\theta)d\theta ds\Big)^2 dy\Big]^{\frac{1}{2}}\Phi(\|\phi\|_h^2)\\
&=\sqrt{2}\bigg(\Big[\int_0^{\pi}(p_1(t,y))^2dy\Big]^{\frac{1}{2}}+Tr(Q)\Big[\int_0^{\pi}(p_2(t,y))^2dy\Big]^{\frac{1}{2}}\bigg)\Phi(\|\phi\|_h^2)\\
&=\sqrt{2}[\bar{p}_1(t)+\sqrt{\pi}Tr(Q)\bar{p}_2(t)]\Phi(\|\phi\|_h^2).
\end{aligned}
$$

Since, $\Phi:[0,+\infty)\to[0,+\infty)$ is continuous and nondecreasing functions, we can take $p(t)=\sqrt{2}[\bar{p}_1(t)+\sqrt{\pi}Tr(Q)\bar{p}_2(t)]$ and $\Omega_2(r)=\Omega_3(r)=\Phi(r)$ in $(\mathbf{H_8})$. If $(\mathbf{H_5})$, $(\mathbf{H_7})$ and the bounds in $(\mathbf{H_8})$ are satisfied then equations (4.1)-(4.4) have a mild solution on $[0,b]$.

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