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On some fractional q-Integral inequalities

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Abstract In this paper, using the Riemann-Liouville fractional q-integral, we establish some new results of the Gruss and Chebyshev q-integral inequalities.

Keywords: Integral inequalities, Riemann-Liouville fractional integral, Gruss inequality, Chebyshev inequality.

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1 Introduction

Let us consider the functional (see[2]):

$$T(f,g) := \frac{1}{b-a}\int_a^b f(x)g(x)dx - \left(\frac{1}{b-a}\int_a^b f(x)dx\right)\left(\frac{1}{b-a}\int_a^b g(x)dx\right) \quad (1.1)$$

where f and g are two integrable functions on $[a,b]$.
In [11], Gruss proved the well known inequality:

$$|T(f,g)| \leq \frac{(\Phi-\varphi)(\Psi-\psi)}{4}, \quad (1.2)$$

where f and g are two integrable functions on $[a,b]$ satisfying the conditions

$$\varphi \leq f(x) \leq \Phi, \qquad \psi \leq g(x) \leq \Psi, \qquad \varphi,\Psi,\Phi,\psi \in \mathbb{R}, x \in [a,b]. \quad (1.3)$$

In the case of $f^{'}, g^{'} \in L_\infty(a,b)$, S. S. Dragomir (see[6]) proved that

$$|S(f,p,g)| \leq \|f^{'}\|_\infty \|g^{'}\|_\infty \left[\int_a^b p(x)dx \int_a^b x^2 p(x)dx - \left(\int_a^b xp(x)dx\right)^2\right], \quad (1.4)$$

where

$$S(p,f,g) := \frac{1}{2}T(f,g,p,q) = \int_a^b p(x)\int_a^b p(x)f(x)g(x)dx - \left(\int_a^b p(x)f(x)dx\right)\left(\int_a^b p(x)g(x)dx\right). \quad (1.5)$$

If f is M-g-Lipschitzian on $[a,b]$: i.e.

$$|f(x)-f(y)| \leq M|g(x)-g(y)|; M>0, x,y \in [a,b], \quad (1.6)$$

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Dragomir [6] proved that

$$|S(f,p,g)| \leq M\left[\int_a^b p(x)dx\int_a^b p(x)g^2(x)dx - \left(\int_a^b g(x)p(x)dx\right)^2\right], \tag{1.7}$$

and if f is an L_1- lipschitzian function on $[a,b]$ and g is an L_2-lipschitzian function on $[a,b]$, the author proved that [6]

$$|S(p,f,g)| \leq L_1L_2\left(\int_a^b p(x)\int_a^b x^2p(x)dx - \left(\int_a^b xp(x)\right)^2\right). \tag{1.8}$$

Using the Riemann-Liouville fractional integral, many authors have studied the fractional integral inequalities and their applications(see[1, 3, 4, 5, 6]).
In [5], Dahmani et al. gave the following fractional integral inequalities, using the Riemann-Liouville fractional integral :
let f and g be two integrable functions on $[0,\infty[$ and p,q two positive functions, then for all $t>0, \alpha>0$,

$$\begin{aligned}|J^\alpha q(t)J^\alpha pfg(t) + J^\alpha p(t)J^\alpha qfg(t) - J^\alpha pf(t)J^\alpha qg(t) - J^\alpha qf(t)J^\alpha pg(t)| \\ \leq J^\alpha p(t)J^\alpha q(t)(\Phi-\varphi)(\Psi-\psi)\end{aligned}$$

Morever, if f and g are two lipschitzian functions on $[0,\infty[$, we have

$$\begin{aligned}|J^\alpha q(t)J^\alpha pfg(t) + J^\alpha p(t)J^\alpha qfg(t) - J^\alpha qf(t)J^\alpha pg(t) - J^\alpha pf(t)J^\alpha qg(t)| \\ \leq L_1L_2(J^\alpha q(t)J^\alpha t^2p(t) + J^\alpha p(t)J^\alpha t^2q(t) - J^\alpha tq(t)J^\alpha tp(t)).\end{aligned}$$

In [4], Dahmani established a new class of inequalities for the extended Chebyshev functional as follows:
let f and g two differentiable functions on $[0,\infty[$ and p,q two positive functions. If $f', g' \in L_\infty([0,\infty[)$, then

$$\begin{aligned}&|J^\beta q(t)J^\alpha pfg(t) + J^\alpha p(t)J^\beta qfg(t) - J^\alpha pf(t)J^\beta qg(t) - J^\beta qf(t)J^\alpha pg(t)| \\ &\leq \|f'\|_\infty\|g'\|_\infty(J^\alpha p(t)J^\beta t^2q(t) + J^\beta q(t)J^\alpha t^2p(t) - 2(J^\alpha tp(t))(J^\beta tq(t)),\end{aligned}$$

for all $t>0, \alpha>0$, and $\beta>0$.
The main aim of this paper is to establish some generalization of these inequalities using q-fractional integrals.

2 Basic Definitions

Throughout this paper, we will fix $q\in(0,1)$. For the convenience of the reader, we provide in this section a summary of the mathematical notations and definitions used in this paper (see [7] and [9] and [12]). We write for $a,b\in\mathbb{C}$,

$$(a;q)_\infty = \prod_{k=0}^{\infty}(1-aq^k), \qquad (a-b)^{(\alpha)} = a^\alpha\frac{(\frac{b}{a};q)_\infty}{(q^\alpha\frac{b}{a};q)_\infty}.$$

The q-Jackson integral from 0 to a is defined by (see [8])

$$\int_0^a f(x)d_qx = (1-q)a\sum_{n=0}^{\infty}f(aq^n)q^n, \tag{2.1}$$

provided the sum converges absolutely.
The q-Jackson integral in a generic interval $[a,b]$ is given by (see [8])

$$\int_a^b f(x)d_qx = \int_0^b f(x)d_qx - \int_0^a f(x)d_qx. \tag{2.2}$$

The fractional q-integral of the Riemann-Liouville type is (see [12])

$$\left(J_{q,a}^\alpha f\right)(x) = \frac{1}{\Gamma_q(\alpha)}\int_a^x (x-qt)^{(\alpha-1)}f(t)d_qt; \quad \alpha>0 \tag{2.3}$$

where

$$\Gamma_q(\alpha) = \frac{1}{1-q}\int_0^1 \left(\frac{u}{1-q}\right)^{\alpha-1} e_q(qu)d_qu, \qquad \textit{and} \quad e_q(t) = \prod_{k=0}^{\infty}(1-q^kt).$$

The q-fractional integration has the following semi-group property

$$\left(J_{q,a}^{\beta} J_{q,a}^{\alpha} f\right)(x) = \left(J_{q,a}^{\alpha+\beta} f\right)(x); \quad \alpha > 0, \beta > 0. \tag{2.4}$$

Finally, for $b > 0$ and $a = bq^n$, $n = 1, 2, \ldots, \infty$, we write

$$[a, \quad b]_q = \{bq^k : \quad 0 \le k \le n\}.$$

3 Main results

Theorem 3.1. *Let f and g be two functions defined on $[a,b]_q$ satisfying the condition (1.3) and let v, w be two positive functions on $[a,b]_q$.*
Then

$$\begin{aligned}|J_{q,a}^{\alpha}w(b)J_{q,a}^{\alpha}vfg(b) + J_{q,a}^{\alpha}v(b)J_{q,a}^{\alpha}wfg(b) - J_{q,a}^{\alpha}vf(b)J_{q,a}^{\alpha}wg(b) - J_{q,a}^{\alpha}wf(b)J_{q,a}^{\alpha}vg(b)|\\ \le J_{q,a}^{\alpha}v(b)J_{q,a}^{\alpha}w(b)(\Phi-\varphi)(\Psi-\psi).\end{aligned} \tag{3.1}$$

Proof. From the condition (1.3), we have

$$|f(\tau) - f(\rho)| \le \Phi - \varphi, \quad |g(\tau) - g(\rho)| \le \Psi - \psi, \quad \tau, \rho \in [a,b]_q, \tag{3.2}$$

which implies that

$$|\left(f(\tau) - f(\rho)\right)\left(g(\tau) - g(\rho)\right)| \le (\Phi-\varphi)(\Psi-\psi). \tag{3.3}$$

Define

$$H(\tau,\rho) = f(\tau)g(\tau) + f(\rho)g(\rho) - f(\tau)g(\rho) - f(\rho)g(\tau), \quad \tau, \rho \in [a,b]_q. \tag{3.4}$$

Multiplying (3.4) by $\frac{(b-q\tau)^{(\alpha-1)}}{\Gamma_q(\alpha)}v(\tau)$ and integrating with respect to τ from a to b, we get

$$\begin{aligned}&\frac{1}{\Gamma_q(\alpha)}\int_a^b (b-q\tau)^{(\alpha-1)}v(\tau)H(\tau,\rho)d_q\tau\\ &= J_{q,a}^{\alpha}vfg(b) + f(\rho)g(\rho)J_{q,a}^{\alpha}v(b) - g(\rho)J_{q,a}^{\alpha}vf(b) - f(\rho)J_{q,a}^{\alpha}vg(b).\end{aligned} \tag{3.5}$$

Now, multiplying (3.5) by $\frac{(b-q\rho)^{(\alpha-1)}}{(\Gamma_q(\alpha))}w(\rho)$ and integrating with respect to ρ from a to b, we can state that

$$\begin{aligned}&\frac{1}{(\Gamma_q(\alpha))^2}\int_a^b\int_a^b (b-q\tau)^{(\alpha-1)}(b-q\rho)^{(\alpha-1)}v(\tau)w(\rho)H(\tau,\rho)d_q\tau d_q\rho\\ &= J_{q,a}^{\alpha}w(b)J_{q,a}^{\alpha}vfg(b) + J_{q,a}^{\alpha}v(b)J_{q,a}^{\alpha}wfg(b) - J_{q,a}^{\alpha}vf(b)J_{q,a}^{\alpha}wg(b) - J_{q,a}^{\alpha}wf(b)J_{q,a}^{\alpha}vg(b).\end{aligned} \tag{3.6}$$

Using (3.3), we can estimate (3.6) as follows

$$\begin{aligned}&|\frac{1}{(\Gamma_q(\alpha))^2}\int_a^b\int_a^b (b-q\tau)^{(\alpha-1)}(b-q\rho)^{(\alpha-1)}v(\tau)w(\rho)H(\tau,\rho)d_q\tau d_q\rho|\\ \le\;& \frac{(\Phi-\varphi)(\Psi-\psi)}{(\Gamma_q(\alpha))^2}\int_a^b\int_a^b (b-q\tau)^{\alpha-1}(b-q\rho)^{\alpha-1}v(\tau)w(\rho)d_q\tau d_q\rho.\end{aligned} \tag{3.7}$$

Consequently,

$$\begin{aligned}&|\frac{1}{(\Gamma_q(\alpha))^2}\int_a^b\int_a^b (b-q\tau)^{(\alpha-1)}(b-q\rho)^{(\alpha-1)}v(\tau)w(\rho)H(\tau,\rho)d_q\tau d_q\rho|\\ \le\;& J_{q,a}^{\alpha}v(b)J_{q,a}^{\alpha}w(b)(\Phi-\varphi)(\Psi-\psi).\end{aligned}$$

Theorem (3.1) is thus proved. □

Theorem 3.2. *Let f and g be two functions defined on $[a,b]_q$ satisfying the condition (1.3) and let v,w be two positive functions on $[a,b]_q$.*
Then

$$|J^{\beta}_{q,a}w(b)J^{\alpha}_{q,a}vfg(b)+J^{\alpha}_{q,a}v(b)J^{\beta}_{q,a}wfg(b)-J^{\alpha}_{q,a}vf(b)J^{\beta}_{q,a}wg(b)-J^{\beta}_{q,a}wf(b)J^{\alpha}_{q,a}vg(b)| \leq J^{\alpha}_{q,a}v(b)J^{\beta}_{q,a}w(b)(\Phi-\varphi)(\Psi-\psi). \tag{3.8}$$

Proof. Multiplying (3.5) by $\frac{(b-q\rho)^{(\beta-1)}}{\Gamma_q(\beta)}w(\rho)$ and integrating with respect to ρ from a to b, we get

$$\frac{1}{\Gamma_q(\alpha)\Gamma_q(\beta)}\int_a^b\int_a^b(b-q\tau)^{(\alpha-1)}(b-q\rho)^{(\beta-1)}v(\tau)w(\rho)H(\tau,\rho)d_q\tau d_q\rho = J^{\beta}_{q,a}w(b)J^{\alpha}_{q,a}vfg(b)+J^{\alpha}_{q,a}v(b)J^{\beta}_{q,a}wfg(b)-J^{\alpha}_{q,a}vf(b)J^{\beta}_{q,a}wg(b)-J^{\beta}_{q,a}wf(b)J^{\alpha}_{q,a}vg(b). \tag{3.9}$$

On the other hand

$$\frac{(\Phi-\varphi)(\Psi-\psi)}{\Gamma_q(\alpha)\Gamma_q(\beta)}\int_a^b\int_a^b(b-q\tau)^{(\alpha-1)}(b-q\rho)^{\beta-1}v(\tau)w(\rho)d_q\tau d_q\rho = J^{\alpha}_{q,a}v(b)J^{\beta}_{q,a}w(b)(\Phi-\varphi)(\Psi-\psi). \tag{3.10}$$

Hence

$$|\frac{1}{\Gamma_q(\alpha)\Gamma_q(\beta)}\int_a^b\int_a^b(b-q\tau)^{(\alpha-1)}(b-q\rho)^{(\beta-1)}v(\tau)w(\rho)H(\tau,\rho)d_q\tau d_q\rho| \leq J^{\alpha}_{q,a}v(b)J^{\beta}_{q,a}w(b)(\Phi-\varphi)(\Psi-\psi). \tag{3.11}$$

This ends the proof. □

Remark 3.1. *Applying Theorem (3.2) for $\alpha=\beta$, we obtain Theorem (3.1).*

Theorem 3.3. *Let f and g be two functions defined on $[a,b]_q$ satisfying the condition (1.6) and let v,w be two positive functions on $[a,b]_q$. Then the inequality*

$$|J^{\beta}_{q,a}w(b)J^{\alpha}_{q,a}vfg(b)+J^{\alpha}_{q,a}v(b)J^{\beta}_{q,a}wfg(b)-J^{\alpha}_{q,a}vf(b)J^{\beta}_{q,a}wg(b)-J^{\beta}_{q,a}wf(b)J^{\alpha}_{q,a}vg(b)| \leq M[J^{\alpha}_{q,a}v(b)J^{\beta}_{q,a}wg^2(b)+J^{\beta}_{q,a}w(b)J^{\alpha}_{q,a}vg^2(b)-2J^{\alpha}_{q,a}vg(b)J^{\beta}_{q,a}wg(b)] \tag{3.12}$$

is valid.

Proof. Multiplying (3.4) by $\frac{(b-q\tau)^{(\alpha-1)}v(\tau)}{\Gamma_q(\alpha)}$ and integrating the resulting identity with respect to τ from a to b, we obtain

$$\frac{1}{\Gamma_q(\alpha)}\int_a^b(b-q\tau)^{(\alpha-1)}v(\tau)H(\tau,\rho)d_q\tau = J^{\alpha}_{q,a}vfg(b)-f(\rho)J^{\alpha}_{q,a}vg(b)-g(\rho)J^{\alpha}_{q,a}vf(b)+f(\rho)g(\rho)J^{\alpha}_{q,a}v(b). \tag{3.13}$$

Multiplying (3.13) by $\frac{(b-q\rho)^{(\beta-1)}w(\rho)}{\Gamma_q(\beta)}$and integrating the resulting identity with respect to ρ from a to b, we get

$$\frac{1}{\Gamma_q(\alpha)\Gamma_q(\beta)}\int_a^b\int_a^b(b-q\tau)^{(\alpha-1)}(b-q\rho)^{(\beta-1)}v(\tau)w(\rho)H(\tau,\rho)d_q\tau d_q\rho = J^{\beta}_{q,a}w(b)J^{\alpha}_{q,a}vfg(b)-J^{\beta}_{q,a}wf(b)J^{\alpha}_{q,a}vg(b)-J^{\alpha}_{q,a}vf(b)J^{\beta}_{q,a}wg(b)+J^{\alpha}_{q,a}v(b)J^{\beta}_{q,a}wfg(b). \tag{3.14}$$

On the other hand, we have

$$|f(\tau)-f(\rho)|\leq M|g(\tau)-g(\rho)|. \tag{3.15}$$

This implies that

$$|H(\tau,\rho)|\leq M(g(\tau)-g(\rho))^2, \quad \tau,\rho\in[a,b]_q. \tag{3.16}$$

Hence, it follows that

$$\frac{1}{\Gamma_q(\alpha)}\int_a^b (b-q\tau)^{(\alpha-1)}v(\tau)|H(\tau,\rho)|d_q\tau$$
$$\leq M\left(J^{\alpha}_{q,a}vg^2(b)-2g(\rho)J^{\alpha}_{q,a}vg(b)+g^2(\rho)J^{\alpha}_{q,a}v(b)\right). \quad (3.17)$$

Consequently,

$$\frac{1}{\Gamma_q(\alpha)\Gamma_q(\beta)}\int_a^b\int_a^b (b-q\tau)^{(\alpha-1)}(b-q\rho)^{(\beta-1)}v(\tau)w(\rho)|H(\tau,\rho)|d_q\tau d_q\rho$$
$$\leq \frac{M}{\Gamma_q(\beta)}\int_a^b \left((b-q\rho)^{\beta-1}w(\rho)\left[J^{\alpha}_{q,a}vg^2(b)-2g(\rho)J^{\alpha}_{q,a}vg(b)+g^2(\rho)J^{\alpha}_{q,a}v(b)\right]\right)d_q\rho. \quad (3.18)$$

So,

$$\frac{1}{\Gamma_q(\alpha)\Gamma_q(\beta)}\int_a^b\int_a^b (b-q\tau)^{(\alpha-1)}(b-q\rho)^{(\beta-1)}v(\tau)w(\rho)|H(\tau,\rho)|d_q\tau d_q\rho$$
$$\leq M[J^{\alpha}_{q,a}v(b)J^{\beta}_{q,a}wg^2(b)+J^{\beta}_{q,a}w(b)J^{\alpha}_{q,a}vg^2(b)-2J^{\alpha}_{q,a}vg(b)J^{\beta}_{q,a}wg(b)]. \quad (3.19)$$

Theorem (3.3) is thus proved. □

In the particular case $\beta=\alpha$, we have the following result.

Corollary 3.1. *Under the assumptions of Theorem 3.3, we have*

$$|J^{\alpha}_{q,a}w(b)J^{\alpha}_{q,a}vfg(b)+J^{\alpha}_{q,a}v(b)J^{\alpha}_{q,a}wfg(b)-J^{\alpha}_{q,a}vf(b)J^{\alpha}_{q,a}wg(b)-J^{\alpha}_{q,a}wf(b)J^{\alpha}_{q,a}vg(b)|\leq$$
$$M[J^{\alpha}_{q,a}v(b)J^{\alpha}_{q,a}wg^2(b)+J^{\alpha}_{q,a}w(b)J^{\alpha}_{q,a}vg^2(b)-2J^{\alpha}_{q,a}vg(b)J^{\alpha}_{q,a}wg(b)]. \quad (3.20)$$

Theorem 3.4. *Let f and g be two lipschitzian functions on $[a,b]_q$ with the constants L_1 and L_2 and let v,w be two positive functions on $[a,b]_q$. Then, the inequality*

$$|J^{\beta}_{q,a}w(b)J^{\alpha}_{q,a}vfg(b)+J^{\alpha}_{q,a}v(b)J^{\beta}_{q,a}wfg(b)-J^{\alpha}_{q,a}vf(b)J^{\beta}_{q,a}wg(b)-J^{\alpha}_{q,a}vf(b)J^{\beta}_{q,a}wg(b)|$$
$$\leq L_1L_2(J^{\alpha}_{q,a}v(b)J^{\beta}_{q,a}(\tau^2w)(b)+J^{\beta}_{q,a}w(b)J^{\alpha}_{q,a}(\tau^2v)(b)-2J^{\alpha}_{q,a}(\tau v)(b)J^{\beta}_{q,a}(\tau w)(b))$$

is valid.

Proof. For all $\tau,\rho\in[a,b]_q$, we have

$$|f(\tau)-f(\rho)|\leq L_1|\tau-\rho|, \quad |g(\tau)-g(\rho)|\leq L_2|\tau-\rho|. \quad (3.21)$$

Hence

$$|H(\tau,\rho)|\leq L_1L_2(\tau-\rho)^2. \quad (3.22)$$

Setting

$$R(\tau,\rho):=L_1L_2(\tau-\rho)^2, \quad (3.23)$$

then, multiplying (3.23) by $\frac{(b-q\tau)^{(\alpha-1)}(b-q\rho)^{(\beta-1)}}{\Gamma_q(\alpha)\Gamma_q(\beta)}v(\tau)w(\rho)$ and integrating with respect to τ and ρ on $[a,b]^2_q$, we get

$$|\frac{1}{\Gamma_q(\alpha)\Gamma_q(\beta)}\int_a^b\int_a^b (b-q\tau)^{(\alpha-1)}(b-q\rho)^{(\beta-1)}v(\tau)w(\rho)R(\tau,\rho)d_q\tau d_q\rho|$$
$$=L_1L_2\left(J^{\alpha}_{q,a}v(b)J^{\beta}_{q,a}(\tau^2w)(b)+J^{\beta}_{q,a}w(b)J^{\alpha}_{q,a}(\tau^2v)(b)-2J^{\alpha}_{q,a}(\tau v)(b)J^{\beta}_{q,a}(\tau w)(b)\right).$$

The result is thus proved. □

Theorem 3.5. *Let f and g be two lipschitzian functions on $[a,b]_q$ with the constants L_1 and L_2 and let v,w be two positive functions on $[a,b]_q$. The inequality*

$$|J^{\alpha}_{q,a}w(b)J^{\alpha}_{q,a}vfg(b)+J^{\alpha}_{q,a}v(b)J^{\alpha}_{q,a}wfg(b)-J^{\alpha}_{q,a}wf(b)J^{\alpha}_{q,a}vg(b)-J^{\alpha}_{q,a}vf(b)J^{\alpha}_{q,a}wg(b)|$$
$$\leq L_1L_2(J^{\alpha}_{q,a}w(b)J^{\alpha}_{q,a}(\tau^2v)(b)+J^{\alpha}_{q,a}v(b)J^{\alpha}_{q,a}(\tau^2w)(b)-J^{\alpha}_{q,a}(\tau w)(b)J^{\alpha}_{q,a}(\tau v)(b)). \quad (3.24)$$

is valid.

Proof. same approach, we take $\alpha = \beta$ in Theorem 3.4. □

Corollary 3.2. *Let f and g be two functions defined on $[a,b]_q$ and let v,w be two positive functions on $[a,b]_q$. Then, the inequality*

$$\begin{aligned}&|J^{\beta}_{q,a}w(b)J^{\alpha}_{q,a}vfg(b) + J^{\alpha}_{q,a}v(b)J^{\beta}_{q,a}wfg(b) - J^{\alpha}_{q,a}vf(b)J^{\beta}_{q,a}wg(b) - J^{\alpha}_{q,a}vf(b)J^{\beta}_{q,a}wg(b)|\\ &\leq \|D_q f\|_\infty \|D_q g\|_\infty (J^{\alpha}_{q,a}v(b)J^{\beta}_{q,a}(\tau^2 w)(b) + J^{\beta}_{q,a}w(b)J^{\alpha}_{q,a}(\tau^2 v)(b) - 2J^{\alpha}_{q,a}(\tau v)(b)J^{\beta}_{q,a}(\tau w)(b))\end{aligned}$$

is valid, where $\|D_q h\|_\infty = \sup_{x\in[a,b]_q} |D_q h(x)|$.

Proof. We have

$$f(\tau) - f(\rho) = \int_\rho^\tau D_q f(t) d_q t, \quad g(\tau) - g(\rho) = \int_\rho^\tau D_q g(t) d_q t$$

so

$$|f(\tau) - f(\rho)| \leq \|D_q f\|_\infty |\tau - \rho| \quad and \quad |g(\tau) - g(\rho)| \leq \|D_q g\|_\infty |\tau - \rho|$$

and the result follows from Theorem 3.4. □

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