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On mild solutions of nonlocal semilinear functional integro-differential equations

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Abstract

In the present paper, we investigate the existence, uniqueness and continuous dependence on initial data of mild solutions of first order nonlocal semilinear functional integro-differential equations of more general type with delay in Banach spaces. Our analysis is based on semigroup theory and modified version of Banach contraction theorem.

Keywords: Existence, delay, functional, integro-differential equation, fixed point, semigroup theory.

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1 Introduction

In the present paper we consider semilinear functional integro-differential equation of first order of the type:

$$x'(t) = Ax(t) + f(t, x_t, \int_0^t k(t,s)h(s,x_s)ds), \quad t \in [0,T], \tag{1.1}$$

$$x(t) + (g(x_{t_1}, ..., x_{t_p}))(t) = \phi(t), \quad -r \le t \le 0, \tag{1.2}$$

where $0 < t_1 < t_2 < ... < t_p \le T$, $p \in N$; A is the infinitesimal generator of a strongly continuous semigroup of bounded linear operators $T(t), t \ge 0$ on X; f, g, h, k and ϕ are given functions satisfying some assumptions and $x_t(\theta) = x(t+\theta)$, for $\theta \in [-r, 0]$ and $t \in [0,T]$.

Equations of the form (1.1)-(1.2) or their special forms serve as an abstract formulation of partial integro-differential equations which arise in the problems with memory visco-elasticity and many other physical phenomena, see [1],[5],[8],[15] and the references given therein. The problems of existence, uniqueness and other qualitative properties of solutions for semilinear differential equations in Banach spaces has been studied extensively in the literature for last many years, see[1]-[11],[14],[15]. On the other hand, as nonlocal condition is more precise to describe natural phenomena than classical initial condition, the Cauchy problem with nonlocal condition also received much attention in recent years, see [2]-[4],[9],[10],[12],[17],[18].

L.Byzewski and H.Acka[3] studied existence,uniqueness and continuous dependence of a mild solution on initial data of problem (1.1)-(1.2) by Banach contraction theorem. The objective of this paper is to generalize and improve their results. We are achieving the same results with less restrictions by using modified version of Banach contraction principle.

The paper is organized as follows: Section 2 presents preliminaries and hypotheses. In section 3, we prove existence and uniqueness of solutions. Section 4, deals with continuous dependence on initial data of mild solutions . Finally in section 5, we give application based on our result.

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2 Preliminaries and Hypotheses

Let X be a Banach space with the norm $\|\cdot\|$. Let $C = \mathcal{C}([-r, 0], X), 0 < r < \infty$, be the Banach space of all continuous functions $\psi : [-r, 0] \to X$ endowed with supremum norm

$$\|\psi\|_C = \sup\{\|\psi(t)\| : -r \le t \le 0\}.$$

Let $B = \mathcal{C}([-r, T], X), T > 0$, be the Banach space of all continuous functions $x : [-r, T] \to X$ with the supremum norm $\|x\|_B = \sup\{\|x(t)\| : -r \le t \le T\}$. For any $x \in B$ and $t \in [0, T]$, we denote x_t the element of C given by $x_t(\theta) = x(t + \theta)$, for $\theta \in [-r, 0]$ and ϕ is a given element of C.

In this paper, we assume that, there exist positive constant $K \ge 1$ such that $\|T(t)\| \le K$, for every $t \in [0, T]$. Also $k : [0, T] \times [0, T] \to \mathbb{R}$ and since k is continuous on compact set $[0, T] \times [0, T]$, there is constant $L_1 > 0$ such that $|k(t, s)| \le L_1$, for $0 \le s \le t \le T$.

Definition 2.1. *A function $x \in B$ satisfying the equations:*

$$x(t) = T(t)\phi(0) - T(t)(g(x_{t_1}, ..., x_{t_p}))(0) + \int_0^t T(t-s) f(s, x_s, \int_0^s k(s, \tau) h(\tau, x_\tau) d\tau) ds, \quad t \in [0, T],$$
$$x(t) + (g(x_{t_1}, ..., x_{t_p}))(t) = \phi(t), \quad -r \le t \le 0,$$

is said to be the mild solution of the initial value problem (1.1)-(1.2).

The following Lemma is known as Pachpatte's inequality .

Lemma 2.1. *[13, p.33] Let u, f and g be nonnegative continuous functions defined on $\mathbb{R}^+$, for which the inequality*

$$u(t) \le u_0 + \int_0^t f(s) u(s) ds + \int_0^t f(s) (\int_0^s g(\sigma) u(\sigma) d\sigma) ds, \quad t \in \mathbb{R}^+,$$

holds, where u_0 is nonnegative constant. Then

$$u(t) \le u_0 [1 + \int_0^t f(s) exp(\int_0^s [f(\sigma) + g(\sigma)] d\sigma) ds], \quad t \in \mathbb{R}^+$$

Our results are based on the modified version of Banach contraction principle.

Lemma 2.2. *[16, p.196] Let X be a Banach space. Let D be an operator which maps the elements of X into itself for which D^r is a contraction, where r is a positive integer. Then D has a unique fixed point.*

We list the following hypotheses for our convenience.

(H_1) Let $f : [0, T] \times C \times X \to X$ such that for every $w \in B$,$x \in X$ and $t \in [0, T]$, $f(., w_t, x) \in B$ and there exists a constant $L > 0$ such that

$$\|f(t, \psi, x) - f(t, \phi, y)\| \le L(\|\psi - \phi\|_C + \|x - y\|), \qquad \phi, \psi \in C, \quad x, y \in X.$$

(H_2) Let $h : [0, T] \times C \to X$ such that for every $w \in B$ and $t \in [0, T]$, $h(., w_t) \in B$ and there exists a constant $H > 0$ such that

$$\|h(t, \psi) - h(t, \phi)\| \le H\|\psi - \phi\|_C, \qquad \phi, \psi \in C.$$

(H_3) Let $g : C^p \to C$ such that exists a constant $G \ge 0$ satisfying

$$\|(g(x_{t_1}, x_{t_2}, ..., x_{t_p}))(t) - (g(y_{t_1}, y_{t_2}, ..., y_{t_p}))(t)\| \le G\|x - y\|_B, \quad t \in [-r, 0].$$

3 Existence and Uniqueness

Theorem 3.1. *Suppose that the hypotheses* (H_1) - (H_3) *are satisfied. Then the initial-value problem* (1.1)-(1.2) *has a unique mild solution* x *on* $[-r, T]$.

Proof. Let $x(t)$ be a mild solution of the problem (1.1)-(1.2) then it satisfies the equivalent integral equation

$$x(t) = T(t)\phi(0) - T(t)(g(x_{t_1}, ..., x_{t_p}))(0) + \int_0^t T(t-s)f(s, x_s, \int_0^s k(s,\tau)h(\tau, x_\tau)d\tau)ds, \quad t \in [0,T], \tag{3.1}$$

$$x(t) + (g(x_{t_1}, ..., x_{t_p}))(t) = \phi(t), \quad -r \leq t \leq 0. \tag{3.2}$$

Now, we rewrite solution of initial value problem (1.1)-(1.2) as follows: For $\phi \in C$, define $\widehat{\phi} \in B$ by

$$\widehat{\phi}(t) = \begin{cases} \phi(t) - (g(x_{t_1}, ..., x_{t_p}))(t) & \text{if } -r \leq t \leq 0 \\ T(t)[\phi(0) - (g(x_{t_1}, ..., x_{t_p}))(0)] & \text{if } 0 \leq t \leq T \end{cases}$$

If $y \in B$ and $x(t) = y(t) + \widehat{\phi}(t)$, $t \in [-r, T]$, then it is easy to see that y satisfies

$$y(t) = 0; \quad -r \leq t \leq 0 \quad \text{and} \tag{3.3}$$

$$y(t) = \int_0^t T(t-s)f\left(s, y_s + \widehat{\phi}_s, \int_0^s k(s,\tau)h(\tau, y_\tau + \widehat{\phi}_\tau)d\tau\right)ds, \quad t \in [0,T] \tag{3.4}$$

if and only if $x(t)$ satisfies the equations (3.1)-(3.2).
We define the operator $F : B \to B$, by

$$(Fy)(t) = \begin{cases} 0 \quad \text{if} \quad -r \leq t \leq 0 \\ \int_0^t T(t-s)f\left(s, y_s + \widehat{\phi}_s, \int_0^s k(s,\tau)h(\tau, y_\tau + \widehat{\phi}_\tau)d\tau\right)ds & \text{if } t \in [0,T]. \end{cases} \tag{3.5}$$

From the definition of an operator F defined by the equation (3.5), it is to be noted that the equations (3.3)-(3.4) can be written as

$$y = Fy.$$

Now we show that F^n is a contraction on B for some positive integer n. Let $y, w \in B$ and using hypotheses (H_1) - (H_3) , we get,

$$\begin{aligned}
\|(Fy)(t) - (Fw)(t)\| &\leq \int_0^t \|T(t-s)\| \|f(s, y_s + \widehat{\phi}_s, \int_0^s k(s,\tau)h(\tau, y_\tau + \widehat{\phi}_\tau)d\tau) \\
&\quad - f(s, w_s + \widehat{\phi}_s, \int_0^s k(s,\tau)h(\tau, y_\tau + \widehat{\phi}_\tau)d\tau)\|ds \\
&\leq \int_0^t KL[\|(y_s + \widehat{\phi}_s) - (w_s + \widehat{\phi}_s)\|_C + L_1 \int_0^s \|h(\tau, y_\tau + \widehat{\phi}_\tau) - h(\tau, w_\tau + \widehat{\phi}_\tau)\|d\tau]ds \\
&\leq KL \int_0^t \|y_s - w_s\|_C ds + KL \int_0^t L_1 H \int_0^s \|y_\tau - w_\tau\|_C d\tau ds \\
&\leq KL \int_0^t \|y - w\|_B ds + KL \int_0^t L_1 H \int_0^s \|y - w\|_B d\tau ds \\
&\leq KL\|y - w\|_B t + KLL_1H\|y - w\|_B \frac{t^2}{2} \\
&\leq KL\|y - w\|_B t + KLL_1HT\|y - w\|_B \frac{t}{2} \\
&\leq KL\|y - w\|_B t + KLL_1HT\|y - w\|_B t \\
&\leq KL(1 + L_1HT)\|y - w\|_B t
\end{aligned}$$

$$
\begin{aligned}
&\|(F^2y)(t)-(F^2w)(t)\|\\
&=\|(F(Fy))(t)-(F(Fw))(t)\|\\
&=\|(F(y_1))(t)-(F(w_1))(t)\|\\
&\le \int_0^t \|T(t-s)\|\|f(s,y_{1_s}+\widehat{\phi}_s,\int_0^s k(s,\tau)h(\tau,y_{1_\tau}+\widehat{\phi}_\tau))\\
&\quad -f(s,w_{1_s}+\widehat{\phi}_s,\int_0^s k(s,\tau)h(\tau,w_{1_\tau}+\widehat{\phi}_\tau))\|ds\\
&\le \int_0^t KL\|y_{1_s}-w_{1s}\|_C+KL\int_0^t L_1H\|y_{1_\tau}-w_{1\tau}\|_C d\tau ds\\
&\le KL\int_0^t \|y_1-w_1\|_{\mathcal{C}([-r,s],X)}ds+KL\int_0^t L_1H\int_0^s \|y_1-w_1\|_{\mathcal{C}([-r,\tau],X)}d\tau ds\\
&\le KL\int_0^t \sup_{\tau\in[-r,s]}\|y_1(\tau)-w_1(\tau)\|ds+KLL_1H\int_0^t\int_0^s \sup_{\eta\in[-r,\tau]}\|y_1(\eta)-w_1(\eta)\|d\tau ds\\
&\le KL\int_0^t \sup_{\tau\in[-r,s]}\|Fy(\tau)-Fw(\tau)\|ds+KLL_1H\int_0^t\int_0^s \sup_{\eta\in[-r,\tau]}\|Fy(\eta)-Fw(\eta)\|d\tau ds\\
&\le KL\int_0^t \sup_{\tau\in[-r,s]}(KL[1+L_1HT]\|y-w\|_B\tau)ds\\
&\quad +KLL_1H\int_0^t\int_0^s \sup_{\eta\in[-r,\tau]}(KL[1+L_1HT]\|y-w\|_B\eta)d\tau ds\\
&\le K^2L^2[1+L_1HT]\|y-w\|_B[\int_0^t(\sup_{\tau\in[-r,s]}\tau)ds+\int_0^t L_1H\int_0^s(\sup_{\eta\in[-r,\tau]}\eta)d\tau ds]\\
&\le K^2L^2[1+L_1HT]\|y-w\|_B[\int_0^t sds+\int_0^t L_1H\int_0^s \tau d\tau ds]\\
&\le K^2L^2[1+L_1HT]\|y-w\|_B[\frac{t^2}{2}+L_1H\frac{t^3}{3!}]\\
&\le K^2L^2[1+L_1HT]^2\|y-w\|_B[\frac{t^2}{2}+L_1HT\frac{t^2}{3!}]\\
&\le K^2L^2[1+L_1HT]^2\|y-w\|_B[\frac{t^2}{2!}+L_1HT\frac{t^2}{2!}]\\
&\le \frac{(KL[1+L_1HT]t)^2}{2!}\|y-w\|_B
\end{aligned}
$$

Continuing in this way, we get,

$$\|(F^ny)(t)-(F^nw)(t)\|\le \frac{(KL[1+L_1HT]t)^n}{n!}\|y-w\|_B.$$

For n large enough, $\frac{(KL[1+L_1HT]t)^n}{n!}<1$. Thus there exist a positive integer n such that F^n is a contraction in B. By virtue of Lemma 2.2, the operator F has a unique fixed point $\tilde{y}$ in B. Then $\tilde{x}=\tilde{y}+\widehat{\phi}$ is a solution of the Cauchy problem (1.1)-(1.2). This completes the proof. □

4 Continuous Dependence on Initial Data

Theorem 4.1. *Suppose that the functions f, h and g satisfies the hypotheses (H_1) - (H_3). Then for each $\phi_1,\phi_2\in C$ and for the corresponding mild solutions x_1,x_2 of the problems*

$$x'(t)=Ax(t)+f(t,x_t,\int_0^t k(s,t)h(t,x_t)dt),\quad t\in[0,T], \tag{4.1}$$

$$x(t)+(g(x_{t_1},...,x_{t_p}))(t)=\phi_i(t),\quad -r\le t\le 0,\quad (i=1,2) \tag{4.2}$$

the inequality

$$\|x_1-x_2\|_B\le[K\|\phi_1-\phi_2\|_C+KG\|x_1-x_2\|_B]\left[1+KLTe^{(KL+L_1H)T}\right]. \tag{4.3}$$

is true.

Moreover, if $G = 0$, then it reduces to classical inequality

$$\|x_1 - x_2\|_B \leq K\left[1 + KLTe^{(KL+L_1H)T}\right]\|\phi_1 - \phi_2\|_C. \tag{4.4}$$

.

Proof. Let $\phi_i (i = 1, 2)$ be arbitrary functions in C and let $x_i (i = 1, 2)$ be the mild solutions of the problem (4.1)-(4.2).
Then for $t \in [-r, 0]$,

$$x_1(t) - x_2(t) = \phi_1(t) - (g(x_{1t_1}, ..., x_{1t_p}))(t) - \phi_2(t) + (g(x_{2t_1}, ..., x_{2t_p}))(t) \tag{4.5}$$

and for $t \in [0, T]$,

$$\begin{aligned} x_1(t) - x_2(t) =& T(t)[\phi_1(0) - \phi_2(0) - (g(x_{1t_1}, ..., x_{1t_p}))(0) + (g(x_{2t_1}, ..., x_{2t_p}))(0)] \\ &+ \int_0^t T(t-s)[f(s, x_{1s}, \int_0^s k(s,\tau)h(\tau, x_{1\tau})d\tau) \\ &- f(s, x_{2s}, \int_0^s k(s,\tau)h(\tau, x_{2\tau})d\tau)]ds \end{aligned} \tag{4.6}$$

From (4.6) and hypotheses $(H1)$ - $(H3)$, we get, for $t \in [0, t]$,

$$\begin{aligned} \|x_1(t) - x_2(t)\| =& \|T(t)\|\|\phi_1 - \phi_2\|_C + G\|T(t)\|\|x_1 - x_2\|_B \\ &+ \int_0^t \|T(t-s)\|\|f(s, x_{1s}, \int_0^s k(s,\tau)h(\tau, x_{1\tau})d\tau) \\ &- f(s, x_{2s}, \int_0^s k(s,\tau)h(\tau, x_{2\tau})d\tau)\|ds \\ &\leq K\|\phi_1 - \phi_2\|_C + KG\|x_1 - x_2\|_B \\ &+ \int_0^t KL\left[\|x_{1s} - x_{2s}\|_C + L_1H\int_0^s \|x_{1\tau} - x_{2\tau}\|_C d\tau\right]ds \end{aligned} \tag{4.7}$$

Define the function $z : [-r, T] \to \mathbb{R}$ by $z(t) = \sup\{\|x_1(s) - x_2(s)\| : -r \leq s \leq t\}, t \in [0, T]$. Let $t^* \in [-r, t]$ be such that $z(t) = \|x_1(t^*) - x_2(t^*)\|$. If $t^* \in [0, t]$, then from inequality (4.7),we have

$$\begin{aligned} z(t) = \|x_1(t^*) - x_2(t^*)\| &\leq K\|\phi_1 - \phi_2\|_C + KG\|x_1 - x_2\|_B \\ &+ \int_0^{t^*} KL\left[\|x_{1s} - x_{2s}\|_C + L_1H\int_0^s \|x_{1\tau} - x_{2\tau}\|_C d\tau\right]ds \\ &\leq K\|\phi_1 - \phi_2\|_C + KG\|x_1 - x_2\|_B \\ &+ \int_0^t KL\left[\|x_{1s} - x_{2s}\|_C + L_1H\int_0^s \|x_{1\tau} - x_{2\tau}\|_C d\tau\right]ds \\ &\leq K\|\phi_1 - \phi_2\|_C + KG\|x_1 - x_2\|_B + \int_0^t KL\left[z(s) + L_1H\int_0^s z(\tau)d\tau\right]ds \end{aligned} \tag{4.8}$$

If $t^* \in [-r, 0]$ then $z(t) \leq \|\phi_1 - \phi_2\|_C + G\|x_1 - x_2\|_B$ and since $K > 1$ the inequality (4.8) holds good. Thus $t^* \in [-r, T]$ the inequality (4.8) holds good. Thanks to Pachpatte's inequality given in Lemma 2.1 and applying it to inequality (4.8) we get,

$$\begin{aligned} z(t) &\leq [K\|\phi_1 - \phi_2\|_C + KG\|x_1 - x_2\|_B]\left[1 + \int_0^t KLe^{\int_0^s (KL+L_1H)d\tau}ds\right] \\ &\leq [K\|\phi_1 - \phi_2\|_C + KG\|x_1 - x_2\|_B]\left[1 + \int_0^t KLe^{(KL+L_1H)T}ds\right] \\ &\leq [K\|\phi_1 - \phi_2\|_C + KG\|x_1 - x_2\|_B]\left[1 + KLTe^{(KL+L_1H)T}\right] \end{aligned}$$

Consequently,

$$\|x_1 - x_2\|_B \leq [K\|\phi_1 - \phi_2\|_C + KG\|x_1 - x_2\|_B]\left[1 + KLTe^{(KL+L_1H)T}\right]. \tag{4.9}$$

Hence the inequality (4.3) holds. Finally inequality (4.4) is a consequence of the inequality (4.9). Hence the proof is complete. □

5 Applications

To illustrate the application of our result proved in section 3, consider the following semilinear partial functional differential equation of the form

$$\frac{\partial}{\partial t}w(u,t) = \frac{\partial^2}{\partial u^2}w(u,t) + H\left(t, w(u,t-r), \int_0^t k(t,s)P(s,w(s-r))ds\right), 0 \le u \le \pi, t \in [0,T] \quad (5.1)$$

$$w(0,t) = w(\pi,t) = 0, \quad 0 \le t \le T, \quad (5.2)$$

$$w(u,t) + \sum_{i=1}^{p} w(u,t_i+t) = \phi(u,t), \quad 0 \le u \le \pi, \quad -r \le t \le 0, \quad (5.3)$$

where $0 < t_1 \le t_2 \le t_p \le T$, the function $H : [0,T] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is continuous . We assume that the functions H and P satisfy the following conditions:
For every $t \in [0,T]$ and $u, v, x, y \in \mathbb{R}$, there exists a constant $l, p > 1$ such that

$$|H(t,u,x) - H(t,v,y)| \le l(|u-v| + |x-y|)$$
$$|P(t,u) - P(t,v)| \le p(|u-v|).$$

Let us take $X = L^2[0,\pi]$. Define the operator $A : X \to X$ by $Az = z''$ with domain $D(A) = \{z \in X : z, \ z'$ are absolutely continuous, $z'' \in X$ and $z(0) = z(\pi) = 0\}$. Then the operator A can be written as

$$Az = \sum_{n=1}^{\infty} -n^2(z,z_n)z_n, \quad z \in D(A)$$

where $z_n(u) = (\sqrt{2/\pi})\sin nu, \ n = 1, 2, ..$ is the orthogonal set of eigenvectors of A and A is the infinitesimal generator of an analytic semigroup $T(t), \ t \ge 0$ and is given by

$$T(t)z = \sum_{n=1}^{\infty} exp(-n^2 t)(z,z_n)z_n, \quad z \in X.$$

Now, the analytic semigroup $T(t)$ being compact, there exists constant K such that

$$|T(t)| \le K, \quad \text{for each} \quad t \in [0,T].$$

Define the function $f : [0,T] \times C \times X \to X$, as follows

$$f(t,\psi,x)(u) = H(t,\psi(-r)u, x(u)),$$
$$h(t,\phi)(u) = P(t,\phi(-r)u)$$

for $t \in [0,T], \psi, \phi \in C$, $x \in X$ and $0 \le u \le \pi$. With these choices of the functions the equations (5.1)-(5.3) can be formulated as an abstract integro-differential equation in Banach space X:

$$x'(t) = Ax(t) + f\left(t, x_t, \int_0^t k(t,s)h(s,x_s)ds\right), \quad t \in [0,T]$$
$$x(t) + (g(x_{t_1}, ..., x_{t_p}))(t) = \phi(t), \quad t \in [-r,0].$$

Since all the hypotheses of the theorem 3.1 are satisfied, the theorem 3.1, can be applied to guarantee the existence of mild solution $w(u,t) = x(t)u, \quad t \in [0,T], u \in [0,\pi]$, of the semilinear partial integro-differential equation (5.1)-(5.3).

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