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Fuzzy boundedness and contractiveness on intuitionistic 2-fuzzy 2-normed linear space

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Abstract

The concepts of fuzzy boundedness, fuzzy continuity and intuitionistic fuzzy 2- contractive mapping on intutionistic 2-fuzzy 2-normed linear space are introduced. Using these concepts some theorems are proved.

Keywords: Intuitionistic 2-fuzzy 2-normed linear space, convergent and Cauchy sequences , intuitionistic 2-fuzzy 2-Banach space.

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1 Introduction

In 1965, the theory of fuzzy sets was introduced by L. Zadeh [9]. In 1964, a satisfactory theory of 2-norm on a linear space has been introduced and developed by Gahler [2]. In 2003, the concepts fuzzy norm and α-norm were introduced by Bag and Samanta [1]. Jialu Zhang [3] has defined fuzzy linear space in a different way. The notion of 2-fuzzy 2-normed linear space of the set of all fuzzy sets of a set was introduced by R.M. Somasundaram and Thangaraj Beaula [6]. The concept of intuitionistic 2fuzzy 2-normed linear space of the set of all fuzzy sets of a set was introduced by Thangaraj Beaula and Lilly Esthar Rani [7].

We have introduced the concepts of fuzzy boundedness, fuzzy continuity and intuitionistic fuzzy 2 contractive mapping on intutionistic 2-fuzzy 2-normed linear space. Using these concepts some theorems are proved.

2 Preliminaries

For the sake of completeness, we reproduce the following definitions due to Gahler [2], Bag and Samanta [1] and Jialu Zhang [3].

Definition 2.1. *[2] Let X be a real linear space of dimension greater than one and let $\|\cdot,\cdot\|$ be a real valued function on $X \times X$ satisfying the following conditions:*

1. $\|x, y\| = 0$ *if and only if x and y are linearly dependent,*
2. $\|x, y\| = \|y, x\|$,
3. $\|\alpha x, y\| = |\alpha|\ \|x, y\|$, *where α is real,*
4. $\|x, y+z\| \leq \|x, y\| + \|x, z\|$.

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$\|\cdot,\cdot\|$ *is called a 2-norm on X and the pair (X,* $\|\cdot,\cdot\|$*) is called a 2-normed linear space.*

Definition 2.2. *[1] Let X be a linear space over K(the field of real or complex numbers). A fuzzy subset N of X × R (R, the set of real numbers) is called a fuzzy norm on X if and only if for all x, u ∈ X and c ∈ K.*

(N1) for all $t \in R$ *with* $t \leq 0$*, N(x, t)=0.*

(N2) for all $t \in R$ *with* $t > 0$*, N(x, t)=1 if and only if x = 0.*

(N3) for all $t \in R$ *with* $t > 0$*,* $N(cx, t) = N(x, \frac{t}{|c|})$*, if* $c \neq 0$*.*

(N4) for all s, t ∈ R, x, u ∈ X, $N(x+u, s+t) \geq \min\{N(x, s), N(u, t)\}$ *.*

(N5) N(x, ·) is a non decreasing function of R and $\lim_{t\to\infty} N(x, t) = 1$*.*

The pair (X, N) will be referred to as a fuzzy normed linear space.

Definition 2.3. *[3] Let X be any non - empty set and F(X) be the set of all fuzzy sets on X. Then for U, V ∈ F(X) and k ∈ K the field of real numbers, define*

$$\begin{aligned} U+V &= \{ (x+y, \lambda\wedge\mu) | (x,\lambda)\in U, (y,\mu)\in V\} , \\ kU &= \{ (kx, \lambda | (x,\lambda)\in U\} . \end{aligned}$$

Definition 2.4. *[3] A fuzzy linear space* $\tilde{X} = X \times (0,1]$ *over the number field K, where the addition and scalar multiplication operation on* $\tilde{X}$ *are defined by*

$(x, \lambda)+(y, \mu)=(x+y, \lambda\wedge\mu)$, $k(x, \lambda) = (kx, \lambda)$

is a fuzzy normed space if to every $(x, \lambda) \in \tilde{X}$ *there is associated a non-negative real number,* $\|(x, \lambda)\|$*, called the fuzzy norm of* (x, λ) *, in such a way that*

1. $\|(x, \lambda)\| = 0$ *if and only if x=0 the zero element of X,* $\lambda \in (0,1]$*.*
2. $\|k(x, \lambda)\| = |k|\ \|(x, \lambda)\|$ *for all* $(x, \lambda) \in \tilde{X}$ *and all k ∈ K.*
3. $\|(x, \lambda)+(y, \mu)\| \leq \|(x, \lambda\wedge\mu\| + \|(y, \lambda\wedge\mu)\|$ *for all* (x, λ) *and* $(y, \mu) \in \tilde{X}$*.*
4. $\|(x, \vee\lambda_t)\| = \vee\|(x, \lambda_t)\|$ *for* $\lambda_t \in (0,1]$*.*

Definition 2.5. *[6] Let X be a non empty and F(X) be the set of all fuzzy sets in X. If f ∈ F(X) then f=*$\{ (x, \mu) \mid x \in X$ *and* $\mu \in (0,1]\}$ *. Clearly f is a bounded function for* $|f(x)| \leq 1$*. Let K be the space of real numbers, then F(X) is a linear space over the field K where the addition and scalar multiplication are defined by*

$$\begin{aligned} f+g &= \{ (x,\mu)+(y,\eta) = \{ (x+y, \mu\wedge\eta) | (x,\mu)\in f, \text{ and } (y,\eta)\in g\} \\ kg &= \{ (kf, \mu \mid (x,\mu)\in f\} \text{ where } k\in K. \end{aligned}$$

The linear space F(X) is said to be normed space if to every f∈F(X), there is associated a non-negative real number $\|f\|$ *called the norm of f in such a way that*

1. $\|f\| = 0$ *if and only if* $f = 0$

For, $\|f\| = 0 \Leftrightarrow \{ \|(x, \mu)\| \mid (x,\mu) \in f \} = 0$

$\Leftrightarrow x = 0,\ \mu \in (0,1]$

$\Leftrightarrow f = 0.$

2. $\|kf\| = |k|\ \|f\|$, $k \in K$

For, $\|kf\| = \{ \|k(x, \mu)\| \mid (x, \mu) \in f, k \in K\}$

$= \{ |k|\ \|x, \mu\| \mid (x, \mu) \in f\}$

$= |k|\ \|f\|.$

3. $\|f+g\| \leq \|f\| + \|g\|$ *for every f , g ∈ F(X)*

For, $\|f+g\| = \{ \|(x, \mu) + (y, \eta)\| \mid x,y \in X, \mu, \eta \in (0,1]\}$

$= \{ \|(x+y), (\mu\wedge\eta)\| \mid x,y \in X, \mu, \eta \in (0,1]\}$

$\leq \{ \|x, \mu\wedge\eta\| + \|y, \mu\wedge\eta\| \mid (x, \mu) \in f \text{ and } (y, \eta) \in g\}$

$= \|f\| + \|g\|.$

And so (F(X), $\|\cdot\|$*) is a normed linear space.*

Definition 2.6. *[6] A 2-fuzzy set on X is a fuzzy set on F(X).*

Definition 2.7. *[6] Let F(X) be a linear space over the real field K. A fuzzy subset N of F(X)× R,(R, the set of real numbers) is called a 2-fuzzy 2-norm on X(or fuzzy 2-norm on F(X)) if and only if,*

(N1) for all $t \in R$ with $t \leq 0$, $N(f_1, f_2, t)=0$.

(N2) for all $t \in R$ with $t>0$, $N(f_1, f_2, t)=1$ if and only if f_1 and f_2 are linearly dependent.

(N3) $N(f_1, f_2, t)$ is invariant under any permutation of f_1, f_2.

(N4) for all $t \in N$ with $t \geq 0$,
$N(f_1, cf_2, t) = N(f_1, f_2, \frac{t}{|c|})$ if $c \neq 0$, $c \in K$ (field).

(N5) for all $s, t \in R$, $N(f_1, f_2 + f_3, s+t) \geq \min\{N(f_1, f_2, s), N(f_1, f_3, t)\}$.

(N6) $N(f_1, f_2, \cdot) : (0, \infty) \to [0, 1]$ is continuous.

(N7) $\lim_{t\to\infty} N(f_1, f_2, t) = 1$.

Then the pair (F(X), N) is a fuzzy 2-normed linear space or (X, N) is a 2-fuzzy 2-normed linear space.

Definition 2.8. *A binary operation* $* : [0, 1] \times [0, 1] \to [0, 1]$ *is a continuous t-norm if it satisfies the following conditions:*

1. $*$ is commutative and associative.
2. $*$ is continuous.
3. $a * 1 = a$, for all $a \in [0, 1]$.
4. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ and $a, b, c, d \in [0, 1]$.

Definition 2.9. *A binary operation* $\Diamond : [0, 1] \times [0, 1] \to [0, 1]$ *is a continuous t - conorm if it satisfies the following conditions:*

1. $\Diamond$ is commutative and associative.
2. $\Diamond$ is continuous.
3. $a \Diamond 0 = a$, for all $a \in [0, 1]$.
4. $a \Diamond b \leq c \Diamond d$ whenever $a \leq c$ and $b \leq d$ and $a, b, c, d \in [0, 1]$.

Remark 2.1. (1) For any $r_1, r_2 \in (0, 1)$ with $r_1 > r_2$ there exist $r_3, r_4 \in (0, 1)$ such that $r_1 * r_3 \geq r_2$ and $r_1 \geq r_4 \Diamond r_2$.

(2) For any $r_5 \in (0,1)$, there exist $r_6, r_7 \in (0,1)$ such that $r_6 * r_6 \geq r_5$ and $r_7 \Diamond r_7 \geq r_5$.

Definition 2.10. *An intuitionistic fuzzy 2- normed linear space (I-F-2-NLS) is of the form* $A = \{ F(X), N(f_1, f_2, t), M(f_1, f_2, t) \mid (f_1, f_2) \in F[(X)]^2\}$ *where F(X) is a linear space over a field K, * is a continuous t-norm,* $\Diamond$ *is a continuous t-conorm, N and M are fuzzy sets on* $[F(X)]^2 \times (0,\infty)$ *such that N denotes the degree of membership and M denotes the degree of non-membership of* $(f_1, f_2, t) \in [F(X)]^2 \times (0,\infty)$ *satisfying the following conditions:*

(1) $N(f_1, f_2, t) + M(f_1, f_2, t) \leq 1$.

(2) $N(f_1, f_2, t) > 0$.

(3) $N(f_1, f_2, t) = 1$ if and only if f_1, f_2 are linearly dependent.

(4) $N(f_1, f_2, t)$ is invariant under any permutation of f_1, f_2.

(5) $N(f_1, f_2, t) : (0, \infty) \to [0,1]$ is continuous in t.

(6) $N(f_1, cf_2, t) = N(f_1, f_2, \frac{t}{|c|})$, if $c \neq 0$, $c \in K$.

(7) $N(f_1, f_2, s) * N(f_1, f_3, t) \leq N(f_1, f_2 + f_3, s + t)$.

(8) $M(f_1, f_2, t) > 0$.

(9) $M(f_1, f_2, t) = 0$ if and only if f_1, f_2 are linearly dependent.

(10) $M(f_1, f_2, t)$ is invariant under any permutation of f_1, f_2.

(11) $M(f_1, cf_2, t) = M(f_1, f_2, \frac{t}{|c|})$ if $c \neq 0$, $c \in k$.

(12) $M(f_1, f_2, s) \lozenge M(f_1, f_3, t) \geq M(f_1, f_2 + f_3, s + t)$.

(13) $M(f_1, f_2, t) : (0, \infty) \to [0,1]$ is continuous in t.

3 Fuzzy boundedness and fuzzy continuity on intuitionistic fuzzy 2- normed linear space

Definition 3.1. *A sequence $\{f_n\}$ in an (IF 2-NLS) is said to converge to f if for given $r > 0$, $t > 0$, $0 < r < 1$, there exists an integer $n_0 \in N$ such that*

$N(f_n - f, g_1, t) > 1 - r$, $N(f_n - f, g_2, t) > 1 - r$

$M(f_n - f, g_1, t) < r$, $M(f_n - f, g_2, t) < r$

where g_1, g_2 are linearly independent (or) $N(f_n - f, g_i, t) \to 1$ as $n \to \infty$ for $i = 1, 2$ and $M(f_n - f, g_i, t) \to 0$ as $n \to \infty$ for $i = 1, 2$.

Definition 3.2. *A sequence $\{f_n\}$ is a cauchy sequence if for given $\in > 0$,*
$N(f_n - f_m, g_i, t) > 1 - \in$, $M(f_n - f_m, g_i, t) < \in$, $0 < \in < 1$, $t > 0$, g_i's are linearly independent, for $i = 1, 2$.

Definition 3.3. *Let $A = \{ (F(X), N(f_1, f_2, t), M(f_1, f_2, t) \mid (f_1, f_2) \in [F(X)]^2) \}$ be an intuitionistic fuzzy 2-normed linear space then*

$$N((f_1, f_2), (f_1', f_2'), t) = N((f_1 - f_1'), (f_2 - f_2'), t)$$

$$M((f_1, f_2), (f_1', f_2'), t) = M((f_1 - f_1'), (f_2 - f_2'), t)$$

*are intuitionistic 2-fuzzy metrics defined on A and (A, N, M, *) is an intuitionistic 2-fuzzy metric space (i-2-f-m-s).*

Definition 3.4. *Let (A, N, M, *) be an intuitionistic 2-fuzzy normed linear space. For $t > 0$, define the openball $B((f_1, f_2), r, t)$ with center $(f_1, f_2) \in A$ and radius $0< r<1$ as*

$$B((f_1, f_2), r, t)=\{ (g_1, g_2) \in A : N((f_1, g_1), (f_2, g_2), t)> 1 - r$$

$$M(f_1 - g_1), (f_2 - g_2)< r\} .$$

Definition 3.5. *A subset $G \subset A$ is said to be open if for each $(f_1, f_2) \in G$, there exists $t > 0$ and $0 < r < 1$ such that $B((f_1, f_2), r, t), r, t) \subset G$.*

Definition 3.6. *Let $\Im$ be the set of all open subsets of A, then it is called the intuitionistic 2-fuzzy topology induced by the intuitionistic 2-fuzzy norm.*

Definition 3.7. *Let (A, N, M, *) be an i-2-f-m-s then a subset D of A is said to be intuitionistic 2- fuzzy bounded if there exists* $t > 0$ *and* $0 < r < 1$ *such that*

$M((f_1, f_2), (g_1, g_2), t) > 1 - r$, $N((f_1, f_2), (g_1, g_2), t) < r$
for each
$((f_1, f_2), (g_1, g_2)) \in [F(X)]^2$.

Definition 3.8. *Let* $(A, N_1, M_1, *)$ $(B, N_2, M_2, *)$ *be an intuitionistic 2-fuzzy normed linear space , a mapping* $T : A \to B$ *is said to be an intuitionistic fuzzy 2 - bounded if there exist constants* $m_1, m_2 \in R^+$ *such that for every* $f \in A$ *and for each* $t > 0$,

$N_2(Tf,\ Tg,\ t) > N_1(f,\ g,\ \frac{t}{m_1})$

$M_2(Tf,\ Tg,\ t) > M_1(f,\ g,\ \frac{t}{m_2})$.

Definition 3.9. *Let* $T : A \to B$ *be a linear operator from IF 2-Banach Space A to IF 2- Banach space B. Then T is said to be an intuitionistic 2 -fuzzy continuous if for each* $\in$ *with* $0 < \in < 1$, *there exists* δ , $0 < \delta < 1$, *such that*

$N_1\ (f,\ g,\ t) \geq 1 - \delta$ *and* $M_1\ (f,\ g,\ t) \leq \delta$, *implies*

$N_2\ (Tf,\ Tg,\ t) \geq 1 - \in$ *and* $M_2\ (Tf,\ Tg,\ t) \leq \in$.

Theorem 3.1. *A linear operator* $T : (A, N_1, M_1, *) \to (B, N_2, M_2, *)$ *is an intuitionistic 2- fuzzy bounded if and only if it is an intuitionistic 2- fuzzy continuous.*

Proof. Assume $T : A \to B$ is an intuitionistic 2-fuzzy bounded. Then there exist constants m_1, $m_2 \in R^+$ such that for every $f \in A$ and for each $t > 0$,

$$N_2(Tf, Tg, t) \geq N_1(f, g, \frac{t}{m_1})$$
$$M_2(Tf, Tg, t) \leq M_1(f, g, \frac{t}{m_2}). \tag{3.1}$$

Suppose for $\in$, with $0 < \in < 1$, choose δ, with $0 < \delta < 1$

such that $N_1(f, g, t) \geq 1 - \delta$ and $M_1\ (f, g, t) \leq$ for any $t > 0$
and $N_1\ (f, g, \frac{t}{m_1}) \leq 1 - \in$

$$M_1(f, g, \frac{t}{m_2}) < \in \text{ (because } m_1, m_2 > 0). \tag{3.2}$$

Using (3.2) in (3.1) we get

$N_2\ (Tf, Tg, t) \geq 1 - \in$ and $M_2\ (Tf, Tg, t) \leq \in$

Hence T is an intuitionistic 2- fuzzy continuous.
Conversely,

Suppose T is an intuitionistic 2- fuzzy continuous.

For $\in$ with $0 < \in < 1$, there exists δ with $0 < \delta < 1$

such that $N_1\ (f, g, t) < 1 - \delta$, $M_1\ (f, g, t) < \delta$ implies that

$$N_2(Tf, Tg, t) > 1 - \in, M_2(Tf, Tg, t) < \in . \tag{3.3}$$

Choose m_1, $m_2 \in R^+$ such that

N_1 (f, g, $\frac{t}{m_1}$) $\leq$ 1 - $\in$ for given N_1(f, g, t)>1-δ and

$$M_1(f, g, \frac{t}{m_2}) \geq \in \text{ for given } \quad M_1(f, g, t) < \delta. \tag{3.4}$$

Then applying (3.4) on (3.3) we get

N_2(Tf, Tg, t) $>$ 1 - $\in \geq N_1$(f, g, $\frac{t}{m_1}$)

M_2 (Tf, Tg, t) $< \delta \leq M_1$ (f, g, $\frac{t}{m_2}$)

Therefore T is intuitionistic 2- fuzzy bounded. □

4 Intuitionistic 2-fuzzy contraction on intuitionistic 2-fuzzy metric space

Definition 4.1. *Let (A, N, M, *) be an intuitionistic 2-fuzzy metric space then*
$T : A \rightarrow A$ is said to be intuitionistic 2- fuzzy contraction if there exists $C \in$ (0, 1) such that CN_2 (Tf, Tg, t) $\geq N_1$ (f, g, t) and $\frac{1}{C}$ M_2 (Tf, Tg, t) $\leq M_1$ (f, g, t).

Theorem 4.1. *Let (A, N, M, *) be a intuitionistic 2-fuzzy metric space. If $T : A \rightarrow A$ is an intuitionistic 2-fuzzy contractive mapping then T is an intuitionistic 2- fuzzy uniformly continuous.*

Proof. Assume T : A $\rightarrow$ A is an intuitionistic 2- fuzzy contractive mapping. Then there exists C $\in$ (0, 1) such that CN_2(Tf, Tg, t) $\geq N_1$(f, g, t) and

$\frac{1}{C}$ M_2 (Tf, Tg, t) $\leq M_1$ (f, g, t) for every t $<$ 0

Assume for a given $\in$ with $0 < \in < 1$ there exists $0 < \delta < 1$ such that
N_1 (f, g, t) $\geq$ 1-δ and M_1 (f, g, t) $< \delta$

Then CN_2 (Tf, Tg, t) $\geq$ 1 -δ implies N_2 (Tf, Tg, t) $\geq \frac{1-\delta}{C}$ and M_2 (Tf, Tg, t)$\leq \delta$ implies M_2 (Tf, Tg, t) $\leq \delta C$

Choose C and δ in such a way that $\delta = \frac{1}{1+C}$.
Then we can define $\in$ so that it satisfies the relationship $\frac{1-\delta}{C} \geq 1- \in$ and $\delta C \leq \in$.
Thus N_2 (Tf, Tg, t)$\geq$ 1-$\in$ and M_2 (Tf, Tg, t)$\leq\in$. Therefore, T is an intuitionistic 2- fuzzy uniformly continuous. □

Definition 4.2. *Let (F(X), N, M) be an intuitionistic 2-fuzzy normed linear space. S is said to be is intuitionistic 2- fuzzy closed if and only if any sequence $\{ f_n \}$ in S converges to $f \in S$.*
(ie) $\lim_{n\rightarrow\infty}$ N (f_n - f, g_i, t) = 1 and $\lim_{n\rightarrow\infty}$ M (f_n-f, g_i, t) = 0 for i = 1, 2 implies $f \in S$.

Definition 4.3. *Let (F(X), N, M) be an intuitionistic 2 -fuzzy normed linear space. $\bar{B}$ (f, $\in$, t) = $\{ g \in F(X)$ | N (f, g, t) > 1 -$\in$, M (f, g, t) < $\in \}$ is said to be a closed ball centered at f of radius $\in$ w.r.to t if and only if any sequence $\{ f_n \}$ in $\bar{B}$(f, $\in$, t) converges to g then $g \in \bar{B}$ (f, $\in$, t).*

Theorem 4.2. *Suppose A = (F(X), N, M) is an intuitionistic 2-fuzzy Banach space. Let $T : A \rightarrow A$ be an intuitionistic 2- fuzzy contractive mapping on $\bar{B}$(f, $\in$, t) with contraction constant C and C N (f, Tf, t) > 1 - $\in$ and $\frac{1}{C}$ M(f, Tf, t) < $\in$ Then there exists a sequence $\{ f_n \}$ in F(X) such that N(f, f_n, t) >1-$\in$ and M (f, f_n, t) < $\in$.*

Proof. Assume f_1 = T(f), f_2 = T(f_1) = T (T(f_1)) = T^2 (f_1)
therefore f_n = T(f_{n-1}) = T^n (f) for all n $\in$ N.
Then C N (f, Tf, t) > 1-$\in$ implies N (f, Tf, t)>$\frac{1-\in}{C}$ > 1 -$\in$

Therefore N (f, f_1, t) > 1 -$\in$

Also $\frac{1}{C}$M (f, Tf, t) $<\in$ implies M(f, Tf, t) $<$ C $\in$ $<\in$

Thus M(f, Tf, t) $<\in$ and so f$\in \bar{B}$ (f,$\in$, t)

Now assume $f_1, f_2, ., f_{n-1} \in \bar{B}$ (f, $\in$, t)

Let us show that $f_n \in \bar{B}$ (f, $\in$, t)

C N (f_1, f_2, t) = C N (Tf, Tf_1, t)

$\geq$ N (f, f_1, t)

$>$ 1$-\in$

So, N (f_1, f_2 t) $> \frac{1-\in}{C} >$ 1 -$\in$

C N (f_2, f_3, t) = C N (T(f_1), T(f_2), t)

$\geq$ N (f_1, f_2, t)

$>$ 1$-\in$

therefore N (f_2, f_3, t) $> \frac{1-\in}{C} >$ 1 -$\in$

Again $\frac{1}{C}$M (f_1, f_2, t) = $\frac{1}{C}$ M (T(f), T(f_1), t) $\leq$ M (f, f_1, t)

Thus M(f_1, f_2, t) $\leq$ C M (f, f_1, t) $<$ C $\in$ $<\in$

Again $\frac{1}{C}$M (f_2, f_3, t) = $\frac{1}{C}$ M(T(f_1), T(f_2), t) $\leq$ M(f_1, f_2, t)

So,
M(f_2, f_3, t) $\leq$ C M (f_1, f_2, t)$<$ C $\in$ $<\in$

Thus we obtain
N(f_3, f_4, t) $>$ 1 -$\in$, M (f_3, f_4, t) $<\in$,..., N(f_{n-1}, f_n, t) $>$ 1 -$\in$ M (f_{n-1}, f_n, t) $<\in$

Thus we obtain N(f, f_n, t) $\geq$ N(f, f_1, $\frac{t}{n}$) * N(f_1, f_2, $\frac{t}{n}$) * ,..., N(f_{n-1}, f_n, $\frac{t}{n}$)

$> (1-\in) * (1-\in) * .... * (1-\in)$

$= 1-\in$

Therefore , N(f, f_n, t) $>$ 1 -$\in$

M(f, f_n, t) $\leq$ M (f,f_1,$\frac{t}{n}$) $\diamond$ ... $\diamond$ M(f_{n-1}, f_n, $\frac{t}{n}$)

= r $\diamond$ r $\diamond$ ... $\diamond$ r = r
Thus N (f, f_n, t) $>$ 1- $\in$ and M (f, f_n, t) $<\in$. □

Lemma 4.1. *Let (F(X), N, M, *) be an intuitionistic 2-fuzzy normed linear space. Let T : F(X) $\to$ F(X) be an intuitionistic 2- fuzzy continuous. If $f_n \to f$ then*
T(f_n) $\to$ T(f) as n$\to \infty$.

Proof. Given $f_n \to f$ in (F(X), N, M, *). Then for given $\in > 0$, $t > 0$, $0 < t < 1$ there exists an integer $n_0 \in N$ such that $N(f_n\text{-}f, g_i, t) > 1 - \in$ and $M(f_n\text{-}f, g_i, t) < \in$

where g_i's are linearly independent for all $n \geq n_0$, i = 1, 2.

Since T is is intuitionistic 2- fuzzy continuous,

$N(T(f_n - f), Tg_i, t) > 1 - \in$ and $M(T(f_n - f), Tg_i, t) < \in$ implies

$N(Tf_n - Tf, g'_i, t) > 1 - \in$ and $M(Tf_n - Tf, g'_i, t) < \in$

Thus $Tf_n \to Tf$ as $n \to \infty$. □

Lemma 4.2. *Let (F(X), N, M, *) be an intuitionistic 2-fuzzy normed linear space then N and M are jointly continuous.*

Proof. If $f_n \to f$ and $g_n \to g$ in (F(X), N, M, *)
we have to prove that $N(f_n\text{-}f, g_n\text{-} g, t) > 1 - \in$ and $M(f_n\text{-}f, g_n - g, t) < \in$ as $n \to \infty$.

We know that

$\lim_{n\to\infty} N(f_n\text{-}f, f'_i, t) = 1$ or $> 1\text{-}\in$, $\lim_{n\to\infty} N(g_n\text{-}g, f'_i, t) = 1 > 1 - \in$ and

$\lim_{n\to\infty} M(f_n - f, f'_i, t) = 0 < \in$, $\lim_{n\to\infty} M(g_n\text{-}g, f'_i, t) = 0 < \in$

$$N(f_n\text{-}f, g_n\text{-}g, t) \geq N(f_n\text{-}f, f'_i, \tfrac{t}{2}) * N(g_n\text{-}g, f'_i, \tfrac{t}{2})$$

$$> (1- \in) * (1- \in)$$

$$= 1- \in$$

And, $(f_n\text{-}f, g_n\text{-}g, t) \leq M(f_n\text{-}f, f'_i, \tfrac{t}{2}) \diamond M(g_n\text{-}g, f'_i, \tfrac{t}{2})$

$$< \in \diamond \in = \in.$$ □

Definition 4.4. *Let (F(X), N, M, *) be an intuitionistic 2-fuzzy normed linear space. A subset A of F (X) is said to be an intuitionistic 2- fuzzy bounded if N(f, g, t) ≥ 1 - M and M(f, g,t) ≤ M where M is a positive constant.*

References

[1] T. Bag and S.K. Samanta, Finite dimensional fuzzy normed linear space, *J. Fuzzy Math.*, 11(3)(2003), 687-705.

[2] S. Gahler, Lineare 2 - normierte Raume, *Math. Nachr.*, 28(1964), 1-43.

[3] Jialu Zhang, The continuity and Boundedness of Fuzzy linear operators in Fuzzy normed space, *J. Fuzzy Math.*, 13(2005), 519-536.

[4] A.R. Meenakshi and R. Cokilavany, On fuzzy 2- normed linear spaces, *J. Fuzzy Math.*, 9(2)(2001), 345- 351.

[5] R. Saadati and S.M. Vaezpour, Some results on Fuzzy Banach spaces, *J. Appl. Math and Comput.*, 17(12)(2005), 475-484.

[6] R.M. Somasundaram and Thangaraj Beaula, Some Aspects of 2-Fuzzy 2-Normed Linear Spaces, *Bulletin of Malaysian Mathematical Society*, 32(2)(2009), 211-222.

[7] Thangaraj Beaula and Lilly Esthar Rani Some Aspects of Intuitionistic 2-Fuzzy 2-Normed Linear Spaces, *J. Fuzzy Math.*, 20(2)(2012), 371-378.

[8] A. White , 2-Banach spaces, *Math. Nachr.*, 42(1969), 43-60.

[9] A. Zadeh, Fuzzy sets, *Inform and control*, 8(1965), 338-353.

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