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On $\widetilde{g}_\alpha$-sets in bitopological spaces

M. Lellis Thivagar[a] and Nirmala Rebecca Paul[b,*]

[a] *School of Mathematics, Madurai Kamaraj University, Madurai–625021, Tamilnadu, India.*

[b] *Department of Mathematics, Lady Doak College, Madurai–625002, Tamilnadu, India.*

Abstract

The paper introduces $\widetilde{g}_\alpha$-closed sets in bitopological spaces and establishes the relationship between other existing sets. As an application $(i, j)\widetilde{g}_\alpha$-closure is introduced to define a new topology. We also derive a new decomposition of continuity.

Keywords: τ_j-open set, $\widetilde{g}_\alpha$-closed set, $\widetilde{g}_\alpha$-open set, $^{\#}gs$-open set.

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1 Introduction

The notion of generalised closed sets introduced by Levine[7] plays a significant role in general topology. A number of generalised closed sets have been introduced and their properties are investigated. Only a few of the class of generalised closed sets form a topology. The class of $\widetilde{g}_\alpha$-closed sets[4] is one among them. Kelly[5] introduced the concepts of bitopological spaces. Many topologists have introduced different types of sets in bitopological spaces. We have introduced $\widetilde{g}_\alpha$-closed sets in bitopological spaces and discussed their basic properties. We have introduced $(i, j)\widetilde{g}_\alpha$-closure and defined a new topology. We also introduced $(i, j)T_{\widetilde{g}_\alpha}$, $(i, j)^{\#}T_{\widetilde{g}_\alpha}$-spaces and derived a new decomposition of continuity in bitopological spaces.

2 Preliminaries

We list some definitions which are useful in the following sections. The interior and the closure of a subset A of (X, τ) are denoted by $Int(A)$ and $Cl(A)$, respectively. Throughout the paper, (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) (or simply X, Y and Z) represent bitopological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of X, A^c denote the complement of A.

Definition 2.1. *A subset A of a topological space (X, τ) is called*

(i) an ω-closed set [10] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in (X, τ),

(ii) a $^{*}g$-closed set [11] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is ω-open in (X, τ) ,

(iii) a $^{\#}g$-semi-closed set[13](briefly $^{\#}gs$-closed)[12] if $sCl(A) \subseteq U$ whenever $A \subseteq U$ and U is $^{*}g$-open in (X, τ) and

(iv) $\widetilde{g}_\alpha$ closed set[4] if $\alpha Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $^{\#}gs$-open in (X, τ)

The complement of ω-closed(resp $^{*}g$-closed,$^{\#}gs$-closed,$\widetilde{g}_\alpha$-closed)set is said to be ω-open(resp $^{*}g$-open,$^{\#}gs$-open,$\widetilde{g}_\alpha$-open)

*Corresponding author.

E-mail addresses: mlthivagar@yahoo.co.in (M. Lellis Thivagar) and nimmi_rebecca@yahoo.com (Nirmala Rebecca Paul)

Definition 2.2. *A subset A of a bitopological space (X, τ_1, τ_2) is called a*

(i)(τ_i, τ_j)-g-closed set[2] if τ_j-Cl(A)$\subseteq U$ whenever $A \subseteq U$ and $U \in \tau_i$,

(ii)(τ_i, τ_j)-gp-closed set[1] if τ_j-pCl(A)$\subseteq U$ whenever $A \subseteq U$ and $U \in \tau_i$,

(iii)(τ_i, τ_j)-gpr-closed set[3] if τ_j-pCl(A)$\subseteq U$ whenever $A \subseteq U$ and U is regular open in τ_i,

(iv)(τ_i, τ_j)-ω-closed set[3] if τ_j-Cl(A)$\subseteq U$ whenever $A \subseteq U$ and U is semi-open in τ_i,

(v)$(\tau_i, \tau_j)g^*$-closed set[9] if τ_j-Cl(A)$\subseteq U$ whenever $A \subseteq U$ and U is g-open in τ_i.

The family of all (τ_i, τ_j)-g-closed (resp (τ_i, τ_j)-gp-closed,(τ_i, τ_j)-gpr-closed and (τ_i, τ_j)-ω-closed) subsets of a botopological space (X, τ_1, τ_2) is denoted by $D(\tau_i, \tau_j)$ (resp $GPC(\tau_i, \tau_j)$,$\zeta(\tau_i, \tau_j)$,$C(\tau_i, \tau_j)$ and $D^*(\tau_i, \tau_j)$).

Definition 2.3. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be*

(i) (τ_i, τ_j)-gp-σ_k-continuous[1] if the inverse image of every σ_k-closed set in (Y, σ_1, σ_2) is (τ_i, τ_j)-gp-closed in (X, τ_1, τ_2),

(ii)$\zeta(\tau_i, \tau_j)$-σ_k-continuous[3] if the inverse image of every σ_k-closed set in (Y, σ_1, σ_2) is (τ_i, τ_j)-gpr-closed in (X, τ_1, τ_2),

(iii)$D^*(\tau_i, \tau_j)$-σ_k-continuous[9] if the inverse image of every σ_k-closed set in (Y, σ_1, σ_2) is (τ_i, τ_j)-g^*-closed in (X, τ_1, τ_2),

(iv)$D(\tau_i, \tau_j)$-σ_k-continuous[2] if the inverse image of every σ_k-closed set in (Y, σ_1, σ_2) is (τ_i, τ_j)-g-closed in (X, τ_1, τ_2),

(v)τ_j-σ_k-continuous[8] if the inverse image of every σ_k-closed set in (Y, σ_1, σ_2) is τ_j-closed in (X, τ_1, τ_2),

(vi) bi-continuous[8] if f is τ_1-σ_1-continuous and τ_2-σ_2-continuous,

(v) strongly bi-continuous[8] if f is bi-continuous,τ_1-σ_2-continuous and τ_2-σ_1-continuous.

Definition 2.4. *A topological space X is called a*

(i) $T_{\widetilde{g}_\alpha}$-space[4] if every $\widetilde{g}_\alpha$-closed set in it is α-closed.

(ii)$^{\#}T_{\widetilde{g}_\alpha}$-space[4] if every $\widetilde{g}_\alpha$-closed set in it is closed.

3 (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed sets

Definition 3.1. *A subset A of a bitopological space (X, τ_1, τ_2) is said to be (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed if τ_j-$\alpha Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $^{\#}gs$-open in τ_i .*
The collection of all (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed sets is denoted by $D\widetilde{G}_\alpha(\tau_i, \tau_j)$.

Remark 3.2. *If $\tau_1 = \tau_2$ then (τ_i, τ_j)-$\widetilde{g}_\alpha$ set reduces to $\widetilde{g}_\alpha$-closed set in a topological space.*

Proposition 3.3. *(i)Every τ_j-closed set is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed.*

(ii) Every τ_j-α-closed set is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed.

Proof. (i) Let A be a τ_j-closed set and U be any $^{\#}gs$-open set in (X, τ_i) containing A. Then τ_j-$\alpha Cl(A) \subseteq \tau_j$-$Cl(A) = A \subseteq U$. Hence A is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed.
(ii)Let Let A be a τ_j-α-closed set and U be any $^{\#}gs$-open set in (X, τ_i) containing A. Then τ_j-$\alpha Cl(A) = A \subseteq U$. Hence A is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed. □

Remark 3.4. *The converse of the proposition 3.3 need not be true.*

Example 3.5. *Let $X = \{a, b, c\}$,$\tau_1 = \{\phi, X, \{a\}\}$, $\tau_2 = \{\phi, X, \{b\}, \{c\}, \{b, c\}\}$, $D\widetilde{G}_\alpha(\tau_i, \tau_j) = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\},$*
The set $\{b\}$ is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed but not τ_j-α-closed and τ_j-closed.

Proposition 3.6. *Every (τ_i,τ_j)-$\widetilde{g}_\alpha$-closed set is $(\tau_i,\tau_j)gp$-closed and $(\tau_i,\tau_j)gpr$-closed.*

Proof. (i)Let A be a (τ_i,τ_j)-$\widetilde{g}_\alpha$-closed set and U be any τ_i-open set in X. Then τ_j-$pCl(A) \subseteq \tau_j$-$\alpha Cl(A) \subseteq U$. Since every τ_i-open set is $^{\#}gs$-open in (X,τ_i)[12]. Hence A is $(\tau_i,\tau_j)gp$-closed.
(ii)Let A be a (τ_i,τ_j)-$\widetilde{g}_\alpha$-closed set and U be any regular open set in (X,τ_i)-open set in X. Then τ_j-$pCl(A) \subseteq \tau_j$-$\alpha Cl(A) \subseteq U$. Since every regular open set is $^{\#}gs$-open in (X,τ_i).Hence A is $(\tau_i,\tau_j)gp$-closed. □

Remark 3.7. *The converse of the proposition 3.6 need not be true.*

Example 3.8. *Let $X=\{a,b,c,d\}$,$\tau_1=\{\phi,X,\{b,c\},\{b,c,d\},\{a,b,c\}\}$,*
$\tau_2=\{\phi,X,\{a\},\{c,d\},\{a,c,d\}\}$,
$D\widetilde{G}_\alpha(\tau_i,\tau_j)=\{\phi,X,\{a\},\{b\},\{d\},\{a,b\},\{a,d\},\{b,d\},\{c,d\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$,
$GPC(\tau_i,\tau_j)=\{\phi,X,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$.
The set $\{a,c\}$ is (τ_i,τ_j)-gp-closed but not (τ_i,τ_j)-$\widetilde{g}_\alpha$-closed.

Example 3.9. *Let $X=\{a,b,c,d\}$,$\tau_1=\{\phi,X,\{a\},\{b\},\{a,b\}\}$, $\tau_2=\{\phi,X,\{a\},\{c,d\},\{a,c,d\}\}$, $D\widetilde{G}_\alpha(\tau_i,\tau_j)=$*
$\{\phi,X,\{b\},\{a,b\},\{b,c,d\}\}$,
$\zeta(\tau_i,\tau_j)=\{\phi,X,\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{c,d\},\{b,c\},\{b,d\},\{a,b,c\},\{a,c,d\},\{a,b,d\},\{b,c,d\}\}$.
The set $\{c\}$ is (τ_i,τ_j)-gpr-closed but not $(\tau_i,\tau_j)\widetilde{g}_\alpha$-closed.

Remark 3.10. *(τ_i,τ_j)-$\widetilde{g}_\alpha$-closed sets are independent of (τ_i,τ_j)-ω-closed sets,(τ_i,τ_j)-g-closed sets and (τ_i,τ_j)-g^*-closed sets.*

Example 3.11. *Let $X=\{a,b,c,d\}$,$\tau_1=\{\phi,X,\{a\},\{c,d\},\{a,c,d\}\}$, $\tau_2=\{\phi,X,\{a\}\}$, $D\widetilde{G}_\alpha(\tau_i,\tau_j)=\{\phi,X,\{b\},\{c\},\{d\},\{b,c$*
$C(\tau_i,\tau_j)=\{\phi,X,\{b\},\{a,b\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\}\}$.
$D(\tau_i,\tau_j)=\{\phi,X,\{b\},\{a,b\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\}\}$
,$D^*(\tau_i,\tau_j=\{\phi,X,\{b\},\{a,b\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$
The set $\{b\}$ is (τ_i,τ_j)-$\widetilde{g}_\alpha$-closed but not (τ_i,τ_j)-ω-closed. The set $\{c\}$ is (τ_i,τ_j)-$\widetilde{g}_\alpha$-closed but not (τ_i,τ_j)-g-closed. The set $\{a,b\}$ is $(\tau_i,\tau_j$-ω-closed, (τ_i,τ_j)-g-closed and $(\tau_i,\tau_j$-g^*-closed but not (τ_i,τ_j)-$\widetilde{g}_\alpha$-closed.

Proposition 3.12. *Union of two (τ_i,τ_j)-$\widetilde{g}_\alpha$-closed sets is (τ_i,τ_j)-$\widetilde{g}_\alpha$-closed.*

Proof. Let A and B are (τ_i,τ_j)-$\widetilde{g}_\alpha$-closed sets and U be any $^{\#}gs$-open set in (X,τ_i) containing A and B. Then τ_j-$\alpha Cl(A) \subseteq U$,τ_j-$\alpha Cl(A) \subseteq U$, τ_j-$\alpha Cl(A\cup B)=\tau_j$-$\alpha Cl(A)\cup\tau_j$-$\alpha Cl(B)\subseteq U$. Hence $A\cup B$ is (τ_i,τ_j)-$\widetilde{g}_\alpha$-closed. □

Remark 3.13. *Intersection of two (τ_i,τ_j)-$\widetilde{g}_\alpha$-closed sets need not be a (τ_i,τ_j)-$\widetilde{g}_\alpha$-closed set. In example 3.11 $\{a,b,c\}$ and $\{a,b,d\}$ are (τ_i,τ_j)-$\widetilde{g}_\alpha$-closed but their intersection $\{a,b\}$ is not (τ_i,τ_j)-$\widetilde{g}_\alpha$-closed.*

Remark 3.14. *In general $D\widetilde{G}_\alpha(\tau_1,\tau_2)$ is not equal to $D\widetilde{G}_\alpha(\tau_2,\tau_1)$.*
In example 3.8
$D\widetilde{G}_\alpha(\tau_1,\tau_2)=\{\phi,X,\{a\},\{b\},\{d\},\{a,b\},\{a,d\},\{b,d\},\{c,d\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$,
$D\widetilde{G}_\alpha(\tau_2,\tau_1)=\{\phi,X,\{a\},\{d\},\{a,d\},\{a,b,c\},\{a,b,d\}\}$. *Hence they are not equal.*

Proposition 3.15. *If $\tau_1\subseteq\tau_2$ in (X,τ_1,τ_2) then $D\widetilde{G}_\alpha(\tau_1,\tau_2)\supseteq D\widetilde{G}_\alpha(\tau_2,\tau_1)$.*

Proof. Let A be $(\tau_2,\tau_1)\widetilde{g}_\alpha$-closed and U be any τ_1-$^{\#}gs$-open set containing A. Since $\tau_1\subseteq\tau_2$, U is τ_2-$^{\#}gs$-open, τ_2-$\alpha Cl(A)\subseteq\tau_1$-$\alpha Cl(A)\subseteq U$. Hence A is (τ_1,τ_2)-$\widetilde{g}_\alpha$-closed. □

Theorem 3.16. *For each point x of (X,τ_1,τ_2), $\{x\}$ is τ_i-$^{\#}gs$-closed or $\{x\}^c$ is (τ_1,τ_2)-$\widetilde{g}_\alpha$-closed for each fixed integer i,j of $\{1,2\}$.*

Proof. Suppose $\{x\}$ is not τ_i-$^{\#}gs$-closed. Then $\{x\}^c$ is not τ_i-$^{\#}gs$-open. Then the only τ_i-$^{\#}gs$-open set containing $\{x\}$ is the set X. τ_j-$\alpha Cl(\{x\}^c)\subseteq X$. Hence $\{x\}^c$ is (τ_1,τ_2)-$\widetilde{g}_\alpha$-closed. □

Proposition 3.17. *If a set A is a (τ_1,τ_2)-$\widetilde{g}_\alpha$-closed set in (X,τ_1,τ_2) then*
τ_j-$\alpha Cl(A)-A$ contains no non empty τ_i-$^{\#}gs$-closed set.

Proof. Let A be a (τ_1,τ_2)-$\widetilde{g}_\alpha$-closed and F be a τ_i-$^{\#}gs$-closed set contained in $\tau_j\alpha Cl(A)-A$. Since A is (τ_1,τ_2)-$\widetilde{g}_\alpha$-closed τ_j-$\alpha Cl(A)\subseteq F^c$. Hence $F\subseteq$
τ_j-$\alpha Cl(A)\cap(\tau_j$-$\alpha Cl(A))^c=\phi$. □

Remark 3.18. *The converse of the proposition 3.17 need not be true.*

Example 3.19. *Let* $X = \{a, b, c, d\}, \tau_1 = \{\phi, X, \{a, b\}\}, \tau_2 = \{\phi, X, \{a, b\}, \{c, d\}\}$. ${}^{\#}GSO(X, \tau_1) = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b$
$D\widetilde{G}_\alpha(\tau_1, \tau_2) = \{\phi, X, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$,
If $A = \{b\}, \tau_j\text{-}\alpha Cl(A) - A = \{a, b\} - \{b\} = \{a\}$. *But* $\{b\}$ *is not* (τ_1, τ_2)-$\widetilde{g}_\alpha$*-closed.*

Proposition 3.20. *If* A *is* (τ_i, τ_j)-$\widetilde{g}_\alpha$*-closed in* (X, τ_1, τ_2) *then* A *is* τ_j-α*-closed if and only if* $\tau_j \alpha Cl(A) - A$ *is* $\tau_i -^{\#} gs$*-closed.*

Proof. Necessity implies $\tau_j\text{-}\alpha Cl(A) - A = \phi$ and hence $\tau_j\text{-}\alpha Cl(A) - A$ is
$\tau_i -^{\#} gs$-closed.
Sufficiency. If $\tau_j\text{-}\alpha Cl(A) - A$ is $\tau_i -^{\#} gs$-closed then by proposition 3.17
$\tau_j\text{-}\alpha Cl(A) - A = \phi$. Therefore A is τ_j-α-closed. □

Proposition 3.21. *If* A *is* (τ_i, τ_j)-$\widetilde{g}_\alpha$*-closed then* $\tau_i\text{-}\alpha Cl(\{x\}) \cap A \neq \phi$ *for each* $x \in \tau_j\text{-}\alpha Cl(A)$.

Proof. If $\tau_i\text{-}\alpha Cl(\{x\}) \cap A = \phi$ for each $x \in \tau_j\text{-}\alpha Cl(A)$ then $A \subseteq (\tau_i \alpha Cl(\{x\}))^c$. Since A is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed, we have $\tau_j\text{-}\alpha Cl(A) \subseteq (\tau_i\text{-}\alpha Cl(\{x\})^c$.[Since $(\tau_j\text{-}\alpha Cl(\{x\})^c$ is τ_i-α-open and therefore τ_i-semi-open, τ_i-${}^{\#}$gs-open]. This gives $x \notin \tau_j \alpha Cl(A)$. A contradiction. □

Proposition 3.22. *If* A *is* (τ_i, τ_j)-$\widetilde{g}_\alpha$*-closed* $A \subseteq B \subseteq \tau_j\text{-}\alpha Cl(A)$ *then* B *is* (τ_i, τ_j)-$\widetilde{g}_\alpha$*-closed.*

Proof. Let $B \subseteq U$ where U is τ_i-${}^{\#}gs$-open. Then $A \subseteq B \subseteq U$.Since A is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed $\tau_j\text{-}\alpha Cl(A) \subseteq U$. Therefore $\tau_j\text{-}\alpha Cl(B) \subseteq \tau_j\text{-}\alpha Cl(\tau_j\text{-}\alpha Cl(A)) =$
$\tau_j\text{-}\alpha Cl(A)$. Thus $\tau_j\text{-}\alpha Cl(B) \subseteq U$. Therefore B is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed. □

Proposition 3.23. *If* $A \subseteq Y \subseteq X$ *and* A *is* (τ_i, τ_j)-$\widetilde{g}_\alpha$*-closed then* A *is* (τ_i, τ_j)-$\widetilde{g}_\alpha$*-closed relative to* Y.

Proof. Let S be any τ_i-${}^{\#}gs$-open set in Y such that $A \subseteq S$. Then $S = U \cap Y$ for some $U \in^{\#} GSO(X, \tau_i)$. Thus $A \subseteq U \cap Y$ and $A \subseteq U$. Since A is
(τ_i, τ_j)-$\widetilde{g}_\alpha$-closed in X, $\tau_j\text{-}\alpha Cl(A) \subseteq U$. Therefore $Y \cap \tau_j\text{-}\alpha Cl(A) \subseteq Y \cap U$.That is $\tau_j\text{-}\alpha Cl_Y(A) \subseteq S$. Since $\tau_j\text{-}\alpha Cl_Y(A) = Y \cap \tau_j\text{-}\alpha Cl(A)$. Hence A is
(τ_i, τ_j)-$\widetilde{g}_\alpha$-closed. □

Definition 3.24. *A subset* A *of a bitopological space* (X, τ_1, τ_2) *is said to be* (τ_i, τ_j)-$\widetilde{g}_\alpha$*-open if its complement is* (τ_i, τ_j)-$\widetilde{g}_\alpha$*-closed in* X.

Theorem 3.25. *In a bitopological space* (X, τ_1, τ_2)

(i) Every τ_j*-open set is* $(\tau_i, \tau_j)\widetilde{g}_\alpha$*-open.*

(ii)Every τ_j-α*-open set is* $(\tau_i, \tau_j)\widetilde{g}_\alpha$*-open.*

(iii)Every (τ_i, τ_j)-$\widetilde{g}_\alpha$*-open set is* $(\tau_i, \tau_j)gp$*-open.*

(iv)Every (τ_i, τ_j)-$\widetilde{g}_\alpha$*-open set is* $(\tau_i, \tau_j)gpr$*-open.*

Remark 3.26. *The converse of the theorem 3.25 need not be true.*

Theorem 3.27. *A subset* A *of a bitopological space* (X, τ_1, τ_2) *is* (τ_i, τ_j)-$\widetilde{g}_\alpha$*-open if and only if* $F \subseteq \tau_j\text{-}\alpha Int(A)$ *whenever* F *is* τ_i-${}^{\#}gs$*-closed and* $F \subseteq A$.

Proof. Necessity. Let A be a (τ_i, τ_j)-$\widetilde{g}_\alpha$-open set in X and F be a τ_i-${}^{\#}gs$-closed set such that $F \subseteq A$. Then $A^c \subseteq F^c$ where F^c is τ_i-${}^{\#}gs$-open and A^c is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed implies $\tau_j\text{-}\alpha Cl(A^c) \subseteq (F^c)$,$\tau_j$-$(\alpha Int(A))^c \subseteq F^c$. Hence $F \subseteq \tau_j\text{-}\alpha Int(A)$.
Sufficiency.Let $F \subseteq \tau_j\text{-}\alpha Int(A)$ where F is ${}^{\#}gs$-closed and $F \subseteq A$. Then $A^c \subseteq F^c = G$ and G is τ_i-${}^{\#}gs$-open. Then $G^c \subseteq A$ implies $G^c \subseteq \tau_j\text{-}\alpha Int(A)$ or $\tau_j\text{-}\alpha Cl(A^c)$= $(\tau_j\text{-}\alpha Int(A))^c \subseteq G$. Thus A^c is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed or A is
(τ_i, τ_j)-$\widetilde{g}_\alpha$-open. □

4 Applications

Definition 4.1. *A bitopological space (X, τ_1, τ_2) is said to be a $(i, j)T_{\widetilde{g}_\alpha}$-space if every (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed set in it is τ_j-α-closed.*

Remark 4.2. *If $\tau_1 = \tau_2$ then $(i, j)T_{\widetilde{g}_\alpha}$-space becomes a $T_{\widetilde{g}_\alpha}$-space.*

Theorem 4.3. *A bitopological space (X, τ_1, τ_2) is a $(i, j)T_{\widetilde{g}_\alpha}$-space if and only if each $\{x\}$ is τ_j-α-open or τ_i-$^{\#}gs$-closed for each $x \in X$.*

Proof. Suppose that $\{x\}$ is not τ_i-$^{\#}gs$-closed then by theorem 3.16, $\{x\}^c$ is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed. Since X is a $(i, j)T_{\widetilde{g}_\alpha}$-space, $\{x\}^c$ is τ_j-α-closed. Therefore $\{x\}$ is τ_j-α-open in X.
Conversely let F be a (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed set. By assumption $\{x\}$ is τ_j-α-open or τ_i-$^{\#}gs$-closed for any $x \in \tau_j$-$\alpha Cl(F)$.
case(i) $\{x\}$ is τ_j-α-open. Since $x \in \tau_j$-$\alpha Cl(F)$, $\{x\} \cap F \neq \phi$. Hence $x \in F$.
case(ii) Suppose $\{x\}$ is τ_i-$^{\#}gs$-closed. If $x \notin F$, then $\{x\} \subseteq \tau_j$-$\alpha Cl(F) - F$ which is a contradiction by proposition 3.20. Therefore $x \in F$. Hence F is a $\tau_j\alpha$-closed subset of X. □

Definition 4.4. *A bitopological space (X, τ_1, τ_2) is said to be $(i, j)^{\#}T_{\widetilde{g}_\alpha}$-space if every (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed set in it is τ_j-closed.*

Proposition 4.5. *Every $(i, j)^{\#}T_{\widetilde{g}_\alpha}$-space is a $(i, j)T_{\widetilde{g}_\alpha}$-space*

Proof. Since every τ_j-closed set is τ_j-α-closed, the proposition is valid. □

Remark 4.6. *The converse of the proposition 4.5 need not be true. Let $X = \{a, b, c\}, \tau_1 = \{\phi, X, \{a, \}, \{b, c\}\}, \tau_2 = \{\phi, X, \{a\}\}$. The space X is a $(i, j)T_{\widetilde{g}_\alpha}$-space but not a $(i, j)^{\#}T_{\widetilde{g}_\alpha}$-space.Since the set $\{b\}$ is not τ_j-closed.*

Definition 4.7. *For a subset A of a bitopological space (X, τ_1, τ_2), the $(i, j)\widetilde{g}_\alpha$-$Cl(A)$ is defined as $(i, j)\widetilde{g}_\alpha$-$Cl(A) = \bigcap\{F : A \subseteq F, F \in D\widetilde{G}_\alpha(\tau_i, \tau_j)\}$.*

Proposition 4.8. *Let A and B be two subsets of (X, τ_i, τ_j).*

(i) If $A \subseteq B$ then $(i, j)\widetilde{g}_\alpha$-$Cl(A) \subseteq (i, j)\widetilde{g}_\alpha$-$Cl(B)$,

(ii)If $\tau_1 \subseteq \tau_2$ then $(1, 2)\widetilde{g}_\alpha$-$Cl(A) \subseteq (2, 1)\widetilde{g}_\alpha$-$Cl(B)$.

Proof. It follows from proposition 3.15. □

Proposition 4.9. *For a subset A of (X, τ_1, τ_2),*

(i) $A \subseteq (i, j)\widetilde{g}_\alpha$-$Cl(A) \subseteq \tau_j$-$Cl(A)$,

(ii)If A is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed then $(i, j)\widetilde{g}_\alpha$-$Cl(A) = A$.

Proof. (i) By definition $A \subseteq (i, j)\widetilde{g}_\alpha$-$Cl(A)$. By Proposition 3.3 τ_j-$Cl(A)$ is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed. Therefore $A \subseteq (i, j)\widetilde{g}_\alpha$-$Cl(A) \subseteq \tau_j$-$Cl(A)$.
(ii) Follows from (i) and the definition 4.5. □

Remark 4.10. *The converse of the proposition 4.9 need not be true. In example 3.11 if $A = \{a, b\}$ then $(i, j)\widetilde{g}_\alpha$-$Cl(A) = \{a, b\}$. But A is not (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed.*

Theorem 4.11. *The closure operator $(i, j)\widetilde{g}_\alpha$-closure is the Kuratowski closure operator.*

Proof. (i)From proposition 3.3 and proposition 4.9 (ii) it follows that $(i, j)\widetilde{g}_\alpha Cl(\phi) = \phi$.

(ii) From proposition 4.9 (i) $A \subseteq (i, j)\widetilde{g}_\alpha$-$Cl(A)$.

(iii) If A and B are two subsets of X then $(i, j)\widetilde{g}_\alpha$-$Cl(A) \cup (i, j)\widetilde{g}_\alpha$-$Cl(B) \subseteq (i, j)\widetilde{g}_\alpha$-$Cl(A \cup B)$. If x does not belong to $(i, j)\widetilde{g}_\alpha$-$Cl(A) \cup (i, j)\widetilde{g}_\alpha$-$Cl(B)$, then there exist $C, D \in D\widetilde{G}_\alpha(\tau_i, \tau_j)$ such that $A \subseteq C, x \notin C, B \subseteq D, x \notin D$. Hence $A \cup B \subseteq C \cup D$ and $x \notin C \cup D$. Since $C \cup D$ is (τ_i, τ_j)-$\widetilde{g}$-closed by proposition 3.12, x does not belong to $(i, j)\widetilde{g}_\alpha$-$Cl(C \cup D)$. Hence $(i, j)\widetilde{g}_\alpha$-$Cl(A \cup B) \subseteq (i, j)\widetilde{g}_\alpha$-$Cl(A) \cup (i, j)\widetilde{g}_\alpha$-$Cl(B)$.Thus $(i, j)\widetilde{g}_\alpha$-$Cl(A \cup B) = (i, j)\widetilde{g}_\alpha$-$Cl(A) \cup (i, j)\widetilde{g}_\alpha$-$Cl(B)$.

(iv) Let E be a subset of X and A be a (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed set containing E, We have $(i,j)\widetilde{g}_\alpha$-$Cl(E) \subseteq A$, we have $(i,j)\widetilde{g}_\alpha$-$Cl(E) \supseteq (i,j)\widetilde{g}_\alpha$-$Cl((i,j)\widetilde{g}_\alpha$-$Cl(E))$. Conversely $(i,j)\widetilde{g}_\alpha$-$Cl(E) \subseteq (i,j)\widetilde{g}_\alpha$-$Cl((i,j)\widetilde{g}_\alpha$-$Cl(E))$.(By proposiotion 4.8 (i)). Hence $(i,j)\widetilde{g}_\alpha$-$Cl(E) = (i,j)\widetilde{g}_\alpha$-$Cl((i,j)\widetilde{g}_\alpha$-$Cl(E)$.

□

Definition 4.12. *$(i,j)\widetilde{g}_\alpha$-closure defines a new topology on X. The topology defined by $(i,j)\widetilde{g}_\alpha$-closure is defined as and denoted as $\widetilde{G}_\alpha(\tau_i, \tau_j) = \{E \subseteq X : (i,j)\widetilde{g}_\alpha\text{-}Cl(E^c) = E^c\}$.*

Example 4.13. *Let $X = \{a,b,c,d\}, \tau_1 = \{\phi, X, \{b,c\}, \{b,c,d\}, \{a,b,c\}\}, \tau_2 = \{\phi, X, \{a\}, \{c,d\}, \{a,c,d\}\}$.*
$D\widetilde{G}_\alpha(\tau_i, \tau_j) = \{\phi, X, \{a\}, \{b\}, \{d\}, \{a,b\}, \{a,d\}, \{b,d\}, \{c,d\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$.
$\widetilde{G}_\alpha(\tau_i, \tau_j) = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}\}$.

Proposition 4.14. *Let $i, j \in \{1,2\}$ be two fixed integers.*

(i)$\tau_j \subseteq \widetilde{G}_\alpha(\tau_i, \tau_j)$.

(ii) If a subset A of X is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed then A is $\widetilde{G}_\alpha(\tau_i, \tau_j)$-closed.

Proof. (i) Let U be any τ_j-open set, by proposition 3.3 and proposition 4.9 (ii) $(i,j)\widetilde{g}_\alpha$-$Cl(U^c) = U^c$. Hence $U \in \widetilde{G}_\alpha(\tau_i, \tau_j)$.

(ii) It follows from proposition 4.9(ii). If A is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed then $(i,j)\widetilde{g}_\alpha$-$Cl(A) = A$, this implies $A^c \in \widetilde{G}_\alpha(\tau_i, \tau_j)$ or A is $\widetilde{G}_\alpha(\tau_i, \tau_j)$-closed.

□

Proposition 4.15. *If $\tau_1 \subseteq \tau_2$ then $\widetilde{G}_\alpha(\tau_1, \tau_2) \supseteq \widetilde{G}_\alpha(\tau_2, \tau_1)$.*

Proof. Let $A \in \widetilde{G}_\alpha(\tau_2, \tau_1)$ then $(2,1)\widetilde{g}_\alpha$-$Cl(A^c) = A^c$. Since $\tau_1 \subseteq \tau_2$, $(1,2)\widetilde{g}_\alpha$-$Cl(A) \subseteq \widetilde{g}_\alpha$-$Cl(A)$, $A^c \subseteq (1,2)\widetilde{g}_\alpha$-Cl$(A^c) \subseteq (2,1)\widetilde{g}_\alpha$-Cl$(A^c) = A^c$. Thus $A^c = (1,2)\widetilde{g}_\alpha$-$Cl(A^c)$. Therefore $A \in \widetilde{G}_\alpha(1,2)$. □

5 $\widetilde{G}_\alpha(\tau_i, \tau_j)$-Continuity

Definition 5.1. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called $D\widetilde{G}_\alpha(\tau_i, \tau_j)$-$\sigma_k$-continuous if the inverse image of every σ_k-closed set in (Y, σ_1, σ_2) is a (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed set in (X, τ_1, τ_2).*

Remark 5.2. *If $\tau_1 = \tau_2 = \tau$ and $\sigma_1 = \sigma_2 = \sigma$ then the $D\widetilde{G}_\alpha(\tau_i, \tau_j)$-$\sigma_k$-continuous function coincides with $\widetilde{g}_\alpha$-continuous function.[6]*

Definition 5.3. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called τ_j-α-σ_k-continuous if the inverse image of every σ_k-closed set in (Y, σ_1, σ_2) is a τ_j-α-closed set in (X, τ_1, τ_2).*

Proposition 5.4. *If a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is τ_j-σ_k(resp τ_j-α-σ_k-continuous) continuous then it is $D\widetilde{G}_\alpha(\tau_i, \tau_j)$-$\sigma_k$-continuous.*

Proof. Let V be any σ_k-closed set in (Y, σ_1, σ_2) then $f^{-1}(V)$ is τ_j-closed (resp τ_j-α-closed). Since every τ_j-closed (resp τ_j-α-closed) set is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed. Hence f is $D\widetilde{G}_\alpha(\tau_i, \tau_j)$-$\sigma_k$-continuous. □

Remark 5.5. *The converse of the proposition 5.4 need not be true.*

Example 5.6. *Let $X = Y = \{a,b,c\}, \tau_1 = \{\phi, X, \{a\}\}, \tau_2 = \{\phi, X, \{b\}\}, \sigma_1 = \{\phi, Y, \{a,b\}\}, \sigma_2 = \{\phi, Y, \{a\}\}$. The function f is defined as $f(a) = b, f(b) = a, f(c) = c$. Here f is $D\widetilde{G}_\alpha(\tau_i, \tau_j)$-$\sigma_k$-continuous but not τ_j-σ_k(resp τ_j-α-σ_k)continuous.*

Proposition 5.7. *If a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $D\widetilde{G}_\alpha(\tau_i, \tau_j)$-$\sigma_k$-continuous then it is (τ_i, τ_j)-gp-σ_k continuous and $\zeta(\tau_i, \tau_j)$-σ_k continuous.*

Proof. Let V be any σ_k-closed set in (Y, σ_1, σ_2) then $f^{-1}(V)$ is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed which is (τ_i, τ_j)-gp-closed and (τ_i, τ_j)-gpr-closed. Hence f is (τ_i, τ_j)-gp-σ_k continuous and $\zeta(\tau_i, \tau_j)$-σ_k continuous. □

Remark 5.8. *The converse of the proposition 5.7 need not be true.*

Example 5.9. *Let $X = Y = \{a, b, c\}, \tau_1 = \{\phi, X, \{a\}\}, \tau_2 = \{\phi, X, \{a\}, \{b, c\}\}$, $\sigma_1 = \{\phi, Y, \{a, b\}\}, \sigma_2 = \{\phi, Y, \{a, c\}\}$. The function f is the identity map. Here f is (τ_i, τ_j)-gp-σ_k-continuous but not $D\widetilde{G}_\alpha(\tau_i, \tau_j)$-$\sigma_k$-continuous.*

Example 5.10. *Let $X = Y = \{a, b, c\}, \tau_1 = \{\phi, X, \{a\}\}, \tau_2 = \{\phi, X, \{a, b\}\}$, $\sigma_1 = \{\phi, Y, \{a, b\}\}, \sigma_2 = \{\phi, Y, \{a, c\}\}$. The function f is the identity map. Here f is $\zeta(\tau_i, \tau_j)$-σ_k-continuous but not $D\widetilde{G}_\alpha(\tau_i, \tau_j)$-$\sigma_k$-continuous.*

Remark 5.11. *$D\widetilde{G}_\alpha(\tau_i, \tau_j)$-$\sigma_k$-coninuity is independent of $D(\tau_i, \tau_j)$-σ_k-coninuity and $C(\tau_i, \tau_j)$-σ_k-continuity.*

Example 5.12. *Let $X = Y = \{a, b, c\}, \tau_1 = \{\phi, X, \{a, b\}\}, \tau_2 = \{\phi, X, \{a\}\}$, $\sigma_1 = \{\phi, Y, \{a\}, \{a, b\}, \{a, c\}\}, \sigma_2 = \{\phi, Y, \{a, c\}\}$. The function f is the identity map. Here f is $D\widetilde{G}_\alpha(\tau_i, \tau_j)$-$\sigma_k$-continuous but not $D(\tau_i, \tau_j)$-σ_k-continuous and $C(\tau_i, \tau_j)$-gp-σ_k-continuous.*

Example 5.13. *Let $X = Y = \{a, b, c\}, \tau_1 = \{\phi, X, \{a\}, \{b, c\}\}, \tau_2 = \{\phi, X, \{a, b\}\}$, $\sigma_1 = \{\phi, Y, \{b\}\}, \sigma_2 = \{\phi, Y, \{c\}\}$. The function f is the identity map. Here f is $D(\tau_i, \tau_j)$-σ_k-continuous and $C(\tau_i, \tau_j)$-σ_k-continuous but not $D\widetilde{G}_\alpha(\tau_i, \tau_j)$-$\sigma_k$-continuous.*

Definition 5.14. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called $\widetilde{g}_\alpha$-bi continuous if f is both $D\widetilde{G}_\alpha(2, 1)$-$\sigma_1$-continuous and $D\widetilde{G}_\alpha(1, 2)$-$\sigma_2$-continuous.*

Definition 5.15. *A function $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called strongly $\widetilde{g}_\alpha$-bi continuous if it is $\widetilde{g}_\alpha$-bi continuous,$D\widetilde{G}_\alpha(2, 1)$-$\sigma_2$-continuous and $D\widetilde{G}_\alpha(1, 2)$-$\sigma_1$-continuous.*

Proposition 5.16. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a function.*
(i) If f is bi-continuous then it is $\widetilde{g}_\alpha$-bi continuous.
(ii) If f is strongly -bi-continuous then it is strongly $\widetilde{g}_\alpha$-bi continuous.

Proof. (i)Let f be bi-continuous. Then f is τ_1-σ_1-continuous and τ_2-σ_2-continuous. By proposition 5.4 f is $\widetilde{g}_\alpha$-bi continuous.
(ii)Let f be strongly bi-continuous then f is τ_1-σ_2-continuous and τ_2-σ_1-continuous. Then by proposition 5.4 f is strongly $\widetilde{g}_\alpha$-bi continuous. □

Example 5.17. *Let $X = \{a, b, c\}, \tau_1 = \{\phi, X, \{a\}\}, \tau_2 = \{\phi, X, \{b, c\}\}$, $Y = \{p, q\}, \sigma_1 = \{\phi, Y, \{p\}\}, \sigma_2 = \{\phi, Y, \{p\}, \{q\}\}$.The function f is defined as $f(a) = p, f(b) = f(c) = q$. Here f is $\widetilde{g}_\alpha$-bi continuous and strongly $\widetilde{g}_\alpha$-bi continuous but not bi-continuous and strongly bi-continuous.*

Proposition 5.18. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is strongly $\widetilde{g}_\alpha$-bi continuous then it is $\widetilde{g}_\alpha$-bi continuous.*

Proof. It follows from the definitions. □

Example 5.19. *Let $X = \{a, b, c\}, \tau_1 = \{\phi, X, \{a\}\}, \tau_2 = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$, $Y = \{p, q\}, \sigma_1 = \{\phi, Y, \{p\}\}, \sigma_2 = \{\phi, Y, \{q\}\}$.The function f is defined as $f(a) = p, f(b) = f(c) = q$. Here f is $\widetilde{g}_\alpha$-bi continuous but not strongly $\widetilde{g}_\alpha$-bi continuous.*

Proposition 5.20. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $D\widetilde{G}_\alpha(\tau_i, \tau_j)\sigma_k$-continuous if and only if $f^{-1}(U)$ is (τ_i, τ_j)-$\widetilde{g}_\alpha$-open in (X, τ_1, τ_2) for every σ_k-open set in (Y, σ_1, σ_2).*

Proof. Let f be $D\widetilde{G}_\alpha(\tau_i, \tau_j)\sigma_k$-continuous and U be a σ_k-open set in (Y, σ_1, σ_2).Then $f^{-1}(U^c)$ is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed. But $f^{-1}(U^c) = (f^{-1}(U))^c$ and so $f^{-1}(U)$ is (τ_i, τ_j)-$\widetilde{g}_\alpha$-open.
Conversely let for any σ_k-open set U in (Y, σ_1, σ_2) then $f^{-1}(U)$ be (τ_i, τ_j)-$\widetilde{g}_\alpha$-open in (X, τ_1, τ_2).Let F be a σ_k-closed set in (Y, σ_1, σ_2) But $f^{-1}(F^c) = (f^{-1}(F))^c$ and so $f^{-1}(F)$ is $(\tau_i, \tau_j)\widetilde{g}_\alpha$-closed. Hence f is $D\widetilde{G}_\alpha(\tau_i, \tau_j)$-$\sigma_k$-continuous. □

Theorem 5.21. *If a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is $D\widetilde{G}_\alpha(\tau_i, \tau_j)$-$\sigma_k$-continuous and if (X, τ_1, τ_2) is a $(i, j)^{\#}T_{\widetilde{g}_\alpha}$-space then f is*
(i) τ_j-σ_k-continuous.
(ii)$D(\tau_i, \tau_j)$-σ_k-continuous.
(iii)$C(\tau_i, \tau_j)$-σ_k-continuous.

Proof. Let V be any σ_k-closed set in Y then $f^{-1}(V)$ is (τ_i, τ_j)-$\widetilde{g}_\alpha$-closed in X. Since X is a $(i, j)^{\#}T_{\widetilde{g}_\alpha}$-space $f^{-1}(V)$ is τ_j-closed in X. Hence f is τ_j-σ_k-continuous. Since τ_j-closed set is τ_j-g-closed and τ_j-ω-closed, f is $D(\tau_i, \tau_j)$-σ_k-continuous and $C(\tau_i, \tau_j)$-σ_k-continuous. □

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