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An Intuitionistic fuzzy count and cardinality of Intuitionistic fuzzy sets

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Abstract

The notion of Intuitonistic fuzzy sets was introduced by Atanassov [1] as an extension of the concept of fuzzy sets introduced by Zadeh such that it is applicable to more real life situations. In order to measure the cardinality of fuzzy sets several attempts have been made [4,6,8]. However, there are no such measures for intuitionistic fuzzy sets. In this paper we define the sigma count and relative sigma count for intuitionistic fuzzy sets and establish their properties. Also, we illustrate the genration of quantification rules.

Keywords: Fuzzy set, Intuitionistic Fuzzy set, Intuitionistic fuzzy count, Relative Intuitionistic fuzzy count.

2010 MSC: 57E05.

1 Introduction

The introduction of the fuzzy concept by Zadeh [7] is considered as a paradigm shift [5]. It introduces the concept of graded membership of elements instead of the binary membership used in Aristotelian logic. It is a very powerful modeling language that can cope with a large fraction of uncertainties of real life situations. Because of its generality it can be well adapted to different circumstances and contexts.

The cardinality of a set in the crisp sense plays an important role in Mathematics and its applications. Similarly it is worthwhile to think of cardinality of fuzzy sets, which is a measure. The concept of cardinality of a fuzzy set is an extension of the count of elements of a crisp set. A simple way of extending the concept of cardinality was suggested by Deluca and Termini [4]. This concept is related to the notion of the probability measure of a fuzzy set introduced by Zadeh [8] and is termed as the sigma count or the non-fuzzy cardinality of a set.

According to fuzzy set theory, the non-membership value of an element is one's complement of its membership value. However, in practical cases it is observed that this happens to be a serious constraint. So, Atanassov [1] introduced the notion of intuitionistic fuzzy sets as a generalisation of the concept of fuzzy sets which does not have the deficiency mentioned above. Unlike, the cardinality of a fuzzy set ([4],[6],[8]) there are no definitions of the cardinality of an intuitionistic fuzzy set in the literature. In this paper we introduce the sigma count as an extension of the notion of the corresponding notion for fuzzy sets and establish many properties. Also, we introduce the notion of relative sigma count and establish some properties. Finally we illustrate the generation of quantification rules.

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2 Definitions and Notations

In this section we shall provide some definitions and notations to be used in this paper. First we introduce the notion of a fuzzy set.

Definition 2.1. *Let X be a universal set. Then a fuzzy set A on X is defined through a membership function associated with A and denoted by μ_A as*

$$\mu_A : X \rightarrow [0,1], \tag{2.1}$$

such that every $x\epsilon X$ is associated with its membership value $\mu_A(x)$ lying in the interval $[0,1]$.

Clearly, the fuzzy set A is completely characterized by the set of points $\{(x, \mu_A(x)) : x\epsilon X\}$.

Definition 2.2. *For any two fuzzy sets A and B in X, we define the relationships between A and B as*

$$A = B \ iff \ \mu_A(x) = \mu_B(x), \forall x\epsilon X \tag{2.2}$$

$$A \subseteq B \ iff \ \mu_A(x) \leq \mu_B(x), \forall x\epsilon X \tag{2.3}$$

$$B \supseteq A \ iff \ A \subseteq B \tag{2.4}$$

Definition 2.3. *The union of the two fuzzy sets A and B is given by its membership function $\mu_{A\cup B}(x)$ defined by*

$$\mu_{A\cup B}(x) = max\{\mu_A(x), \mu_B(x)\}, \forall x\epsilon X. \tag{2.5}$$

Definition 2.4. *The intersection of the two fuzzy sets A and B is given by its membership function $\mu_{A\cap B}(x)$ defined by*

$$\mu_{A\cap B}(x) = min\{\mu_A(x), \mu_B(x)\}, \forall x\epsilon X. \tag{2.6}$$

Definition 2.5. *The complement $\bar{A}$ of the fuzzy set A with respect to universal set X is given by its membership function $\mu_{\bar{A}}(x)$ defined by*

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x), \forall x\epsilon X. \tag{2.7}$$

Definition 2.6. *Let X be an universal set. An intiitionistic fuzzy set or IFS A on X is defined through two functions μ_A and ν_A, called the membership and non-membership functions of A defined as*

$$\mu_A : X \rightarrow [0,1] \ and \ \nu_A : X \rightarrow [0,1] \tag{2.8}$$

such that every $x\epsilon X$ is associated with its membership value $\mu_A(x)$ and non-membership value $\nu_A(x)$ such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

Definition 2.7. *If A and B are two IFSs of the set X, then*

$$A \subset B \ iff \ \forall x\epsilon X, \mu_A(x) \leq \mu_B(x) \ and \ \nu_A(x) \geq \nu_B(x) \tag{2.9}$$

$$A \subset B \ iff \ B \supset A \tag{2.10}$$

$$A = B \ iff \ \forall x\epsilon X, [\mu_A(x) = \mu_B(x) \ and \ \nu_A(x) = \nu_B(x)] \tag{2.11}$$

$$\bar{A} = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x\epsilon X\} \tag{2.12}$$

$$A \cap B = \{\langle x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x))\rangle : x\epsilon X\} \tag{2.13}$$

$$A \cup B = \{\langle x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x))\rangle : x\epsilon X\} \tag{2.14}$$

$$A + B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x), \nu_A(x) \cdot \nu_B(x)\rangle : x\epsilon X\} \tag{2.15}$$

$$A \cdot B = \{\langle x, \mu_A(x) \cdot \mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x) \cdot \nu_B(x)\rangle : x \epsilon X\} \tag{2.16}$$

$$\Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle : x \epsilon X\} \tag{2.17}$$

$$*A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle : x \epsilon X\} \tag{2.18}$$

$$C(A) = \{\langle x, K, L\rangle : x \epsilon X\},\ where\ K = \max_{x \epsilon X} \mu_A(x)\ and\ L = \min_{x \epsilon X} \nu_A(x) \tag{2.19}$$

$$I(A) = \{\langle x, k, l\rangle : x \epsilon X\},\ where\ k = \min_{x \epsilon X} \mu_A(x)\ and\ l = \max_{x \epsilon X} \nu_A(x) \tag{2.20}$$

3 Cardinality of Intuitionistic Fuzzy Sets

In this section, we introduce the cardinality of intuitionistic fuzzy sets and establish some properties.

3.1 Definitions

The measure of fuzzy set is the form of its Σ count (sigma count) was introduced by Deluca and Termini [4] as a simple extension of the concept of cardinality of crisp sets. As mentioned above Intuitionistic fuzzy sets have better modeling power than those of fuzzy sets, by the way introducing the hesitation part. Here we define the cardinality of an Intuitionistic fuzzy set by extending the notion of Σ count stated above. Also, we establish some of their properties, and provide certain examples and application of these results.

Definition 3.1. *A fuzzy set A on X to be finite if $\mu_A(x) \neq 0$ for only a finite number of elements of X.*

Definition 3.2. *For any finite fuzzy set A on X, the sigma count of A, denoted by Σ count (A) is given by*

$$\Sigma \quad count \quad (A) = \sum_{x \epsilon X} \mu_A(x) \tag{3.21}$$

Definition 3.3. *For any IFS A on X we define cardinality of A (denoted by $\Sigma count(A)$) as*

$$\Sigma \quad count \quad (A) = \left[\sum_{i=1}^{n} \mu_A(x_i), \sum_{i=1}^{n} 1 - \nu_A(x_i)\right] \tag{3.22}$$

$$= [\Sigma\ count\ \Box A, \Sigma\ count\ *A] \tag{3.23}$$

It may be noted that when A is a fuzzy set on $X, \nu_A(x) = 1 - \mu_A(x)$, for all $x \epsilon X$, so that

$$\Sigma \quad count \quad (A) = \left[\sum_{i=1}^{n} \mu_A(x_i), \sum_{i=1}^{n} \mu_A(x_i)\right] = \sum_{i=1}^{n} \mu_A(x_i)$$

which is the definition of Σ count of a fuzzy set A defined above.

3.2 Properties of Σ count

We establish some properties of σ count of IFSs in this section.

Theorem 3.1. *For any two IFSs A and B on X*

(i) Σ count $(A \cup B) + \Sigma$ count $(A \cap B) = \Sigma$ count $(A) + \Sigma$ count(B)

(ii) Σ count $(A + B) + \Sigma$ count $(A \cdot B) = \Sigma$ count $(A) + \Sigma$ count(B)

Proof. We have

$$\Sigma \ \ count \ \ (A \cup B) = \left[\sum_{i=1}^{n} (\mu_A(x_i) \vee \mu_B(x_i)), \sum_{i=1}^{n} 1 - (\nu_A(x_i) \wedge \nu_B(x_i)) \right]$$

$$= \left[\sum_{i=1}^{n} (\mu_A(x_i) \vee \mu_B(x_i)), \sum_{i=1}^{n} \{(1 - \nu_A(x_i)) \vee (1 - \nu_B(x_i))\} \right]$$

and

$$\Sigma \ \ count \ \ (A \cap B) = \left[\sum_{i=1}^{n} (\mu_A(x_i) \wedge \mu_B(x_i)), \sum_{i=1}^{n} 1 - (\nu_A(x_i) \vee \nu_B(x_i)) \right]$$

$$= \left[\sum_{i=1}^{n} (\mu_A(x_i) \wedge \mu_B(x_i)), \sum_{i=1}^{n} \{(1 - \nu_A(x_i)) \wedge (1 - \nu_B(x_i))\} \right]$$

So,

$$\Sigma \ \ count \ \ (A \cup B) + \Sigma \ \ count \ \ (A \cap B) = \left[\sum_{i=1}^{n} (\mu_A(x_i) + \mu_B(x_i)), \sum_{i=1}^{n} \{(1 - \nu_A(x_i)) + (1 - \nu_B(x_i))\} \right]$$

$$= \left[\sum_{i=1}^{n} \mu_A(x_i), \sum_{i=1}^{n} 1 - \nu_A(x_i) \right] + \left[\sum_{i=1}^{n} \mu_B(x_i), \sum_{i=1}^{n} 1 - \nu_B(x_i) \right]$$

$$\Sigma \ \ count \ \ (A) + \Sigma \ \ count \ \ (B)$$

The proof of (ii) is similar to that of (i). $\square$

Theorem 3.2. *For any two IFSs A and B on X*
(i) Σ *count* $(\overline{A \cup B}) + \Sigma$ *count* $(\overline{A \cap B}) = \Sigma$ *count* $(\bar{A}) + \Sigma$ *count*$(\bar{B})$
(ii) Σ *count* $(\overline{A + B}) + \Sigma$ *count* $(\overline{A \cdot B}) = \Sigma$ *count* $(\bar{A}) + \Sigma$ *count*$(\bar{B})$

Proof.

$$A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \epsilon X\}$$

$$\overline{A \cup B} = \{\langle x, \nu_A(x) \wedge \nu_B(x), \mu_A(x) \vee \mu_B(x) \rangle : x \epsilon X\}$$

So,

$$\Sigma \ \ count \ \ (\overline{A \cup B}) = \left[\sum_{i=1}^{n} (\nu_A(x_i) \wedge \nu_B(x_i)), \sum_{i=1}^{n} 1 - (\mu_A(x_i) \vee \mu_B(x_i)) \right]$$

$$= \left[\sum_{i=1}^{n} (\nu_A(x_i) \wedge \nu_B(x_i)), \sum_{i=1}^{n} \{(1 - \mu_A(x_i)) \wedge (1 - \mu_B(x_i))\} \right]$$

similarly,

$$A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \epsilon X\}$$

$$\overline{A \cap B} = \{\langle x, \nu_A(x) \vee \nu_B(x), \mu_A(x) \wedge \mu_B(x) \rangle : x \epsilon X\}$$

and

$$\Sigma \ \ count \ \ (\overline{A \cap B}) = \left[\sum_{i=1}^{n} (\nu_A(x_i) \vee \nu_B(x_i)), \sum_{i=1}^{n} 1 - (\mu_A(x_i) \wedge \mu_B(x_i)) \right]$$

$$= \left[\sum_{i=1}^{n} (\nu_A(x_i) \vee \nu_B(x_i)), \sum_{i=1}^{n} \{(1 - \mu_A(x_i)) \vee (1 - \mu_B(x_i))\} \right]$$

Hence,

$$\Sigma \ \ count \ \ (\overline{A \cup B}) + \Sigma \ \ count \ \ (\overline{A \cap B}) = \left[\sum_{i=1}^{n}(\nu_A(x_i) + \nu_B(x_i)), \sum_{i=1}^{n}\{(1-\mu_A(x_i)) + (1-\mu_B(x_i))\}\right]$$

$$= \left[\sum_{i=1}^{n}\nu_A(x_i), \sum_{i=1}^{n} 1 - \mu_A(x_i)\right] + \left[\sum_{i=1}^{n}\nu_B(x_i), \sum_{i=1}^{n} 1 - \mu_B(x_i)\right]$$

$$= \Sigma \ \ count \ \ (\bar{A}) + \Sigma \ \ count \ \ (\bar{B})$$

The proof of (ii) is similar to that of (i) □

Next, by using the results of Atanassov [2,3], the following properties of Σ count of IFSs can be obtained.

Theorem 3.3. *For any IFSs A, we have:*
(i) Σ *count* $\Box A = \Sigma$ *count* $(\overline{*\bar{A}})$
(ii) Σ *count* $*A = \Sigma$ *count* $(\overline{\Box\bar{A}})$
(iii) Σ *count* $\Box\Box A = \Sigma$ *count* $\Box A$
(iv) Σ *count* $\Box * A = \Sigma$ *count* $*A$
(v) Σ *count* $*\Box A = \Sigma$ *count* $\Box A$
(vi) Σ *count* $** A = \Sigma$ *count* $*A$
(vii) Σ *count* $*\bar{A} = \Sigma$ *count* $\overline{\Box A}$
(viii) Σ *count* $\Box\bar{A} = \Sigma$ *count* $\overline{*A}$
(ix) Σ *count* $\bar{A} = \Sigma$ *count* A

It may be noted that from the definition of Σ *count of an IFS, it can be obtained directly that if* $A \subseteq B$ *then it is not always true that* Σ *count* $A \leq \Sigma$ *count* B*. Also*
(x) $\Sigma count \ \Box A \leq \Sigma count {*A}$

Proof. We have

$$\Sigma \ \ count \ \ \Box A = \left[\sum_{i=1}^{n}\mu_A(x_i), \sum_{i=1}^{n} 1 - (1-\mu_A(x_i))\right] = \sum_{i=1}^{n}\mu_A(x_i)$$

and

$$\Sigma \ \ count \ \ *A = \left[\sum_{i=1}^{n}(1-\nu_A(x_i)), \sum_{i=1}^{n}(1-\nu_A(x_i))\right] = \sum_{i=1}^{n}(1-\nu_A(x_i))$$

□

Also, by the definition of an IFS, $\mu_A(x_i) \leq 1 - \nu_A(x_i), i = 1, 2, \ldots, n$. So the claim follows.

Theorem 3.4. *For any two IFSs A and B,*
(i) $\Sigma count \ \Box(A \cup B) = \Sigma count \ (\Box A \cup \Box B)$
(ii) $\Sigma count \ *(A \cup B) = \Sigma count(*A \cup *B)$
(iii) $\Sigma count \ \Box(A \cap B) = \Sigma count(\Box A \cap \Box B)$
(iv) $\Sigma count \ *(A \cap B) = \Sigma count(*A \cap *B)$
(v) $\Sigma count \ (\overline{A \cup B}) = \Sigma count(\bar{A} \cap \bar{B})$
(vi) $\Sigma count \ (\overline{A \cap B}) = \Sigma count(\bar{A} \cup \bar{B})$

Theorem 3.5. *For any two IFSs A and B on X,*
(i) $\Sigma count \ \Box(A \cup B) + \Sigma count \ \Box(A \cap B) = \Sigma count \ \Box A + \Sigma count \ \Box B$
(ii) $\Sigma count \ *(A \cup B) + \Sigma count \ *(A \cap B) = \Sigma count \ *A + \Sigma count \ *B$
(iii) $\Sigma count \ \Box(A + B) + \Sigma count \ \Box(A \cdot B) = \Sigma count \ \Box A + \Sigma count \ \Box B$
(iv) $\Sigma count \ *(A + B) + \Sigma count \ *(A \cdot B) = \Sigma count \ *A + \Sigma count \ *B$

Proof. (i)Σcount$\Box(A \cup B) = \Sigma$count $(\Box A \cup \Box B)$ and Σcount$\Box(A \cap B) = \Sigma$count $(\Box A \cap \Box B)$ so,

$$\Sigma \; count \; \Box(A \cup B) + \Sigma \; count \; \Box(A \cap B) = \Sigma \; count \; (\Box A \cup \Box B) + \Sigma \; count \; (\Box A \cap \Box B)$$

$$= \Sigma \; count \; \Box A + \Sigma \; count \; \Box B$$

$\Box$

Similarly (ii) can be established.
(iii) Σcount $\Box(A + B) + \Sigma$count $\Box(A \cdot B)$

Proof.

$$= \sum_{i=1}^{n} (\mu_A(x_i) + \mu_B(x_i) - \mu_A(x_i) \cdot \mu_B(x_i)) + \sum_{i=1}^{n} \mu_A(x_i) \cdot \mu_B(x_i)$$

$$= \sum_{i=1}^{n} \mu_A(x_i) + \sum_{i=1}^{n} \mu_B(x_i) = \Sigma \; count \; \Box A + \Sigma \; count \; \Box B$$

$\Box$

Similarly (iv) can be established.

Note 3.2.1. By using Theorems 3.1, 3.2 and 3.5, we have the following

(i) $\Sigma\text{count}(A \cup B) + \Sigma\text{count}A \cap B = \Sigma\text{count}(A + B) + \Sigma\text{count}(A \cdot B)$

$$= \Sigma \; count \; A + \Sigma \; count \; B$$

(ii) $\Sigma\text{count}(\overline{A \cup B}) + \Sigma\text{count}\overline{A \cap B} = \Sigma\text{count}(\overline{A + B}) + \Sigma\text{count}(\overline{A \cdot B})$

$$= \Sigma \; count \; \bar{A} + \Sigma \; count \; \bar{B}$$

(iii) $\Sigma\text{count}\Box(A \cup B) + \Sigma\text{count}\Box(A \cap B) = \Sigma\text{count}\Box(A + B) + \Sigma\text{count}\Box(A \cdot B)$

$$= \Sigma \; count \; \Box A + \Sigma \; count \; \Box B$$

(iv) $\Sigma\text{count}{*}(A \cup B) + \Sigma\text{count}{*}(A \cap B) = \Sigma\text{count}{*}(A + B) + \Sigma\text{count}{*}(A \cdot B)$

$$= \Sigma \; count \; * A + \Sigma \; count \; * B$$

3.3 Relative Σ count

The notion of relative Σ count for fuzzy sets has been introduced by Zadeh [9].

Definition 3.4. *If A and B are two fuzzy sets, then we define the relative sigma count of A with respect to B as rel Σcount $(A/B) = (\Sigma count(A \cap B)) \cdot (\Sigma count(B))^{-1}$, if A and b are two IFSs, then*

$$\Sigma \quad count \quad (A \cap B) = \left[\sum_{i=1}^{n} \mu_A(x_i) \wedge \mu_b(x_i), \sum_{i=1}^{n} 1 - (\nu_A(x_i) \vee \nu_B(x_i)) \right]$$

$$(\Sigma \quad count \quad (B))^{-1} = \left[\sum_{i=1}^{n} \mu_B(x_i), \sum_{i=1}^{n} 1 - \nu_B(x_i) \right]^{-1} = \left[\frac{1}{\sum_{i=1}^{n}(1 - \nu_B(x_i))}, \frac{1}{\sum_{i=1}^{n} \mu_B(x_i)} \right]$$

Consequently,

$$(\Sigma \; count \; (A \cap B)) \cdot (\Sigma \; count \; (B))^{-1} = \left[\frac{\sum_{i=1}^{n} \mu_A(x_i) \wedge \mu_B(x_i)}{\sum_{i=1}^{n}(1 - \nu_B(x_i))}, \frac{\sum_{i=1}^{n} 1 - (\nu_A(x_i) \vee \nu_B(x_i))}{\sum_{i=1}^{n} \mu_B(x_i)} \right]$$

$$= \left[\frac{\sum_{i=1}^{n} \mu_A(x_i) \wedge \mu_B(x_i)}{\sum_{i=1}^{n}(1 - \nu_B(x_i))}, \frac{\sum_{i=1}^{n}\{1 - \nu_A(x_i)) \wedge (1 - \nu_B(x_i))\}}{\sum_{i=1}^{n} \mu_B(x_i)} \right]$$

It may be noted that the right hand expression of the above interval may be greater than 1. For example, taking $X = \{x_1, x_2\}$ and A and B two IFSs over X defined by $A = \{(.8, .1)/x_1, (.2, .6)/x_2\}, B = \{(.6, .2)/x_1, (.3, .6)/x_2\}$. Then $(1-\nu_A(x_1)) \wedge (1-\nu_B(x_1)) + (1-\nu_A(x_2)) \wedge (1-\nu_B(x_2)) = .8 + .4 = 1.2$.

In view of the above remark, we define rel Σ count (A/B) for intuitionistic fuzzy sets as

$$rel\Sigma count(A/B) = \left[\frac{\sum_{i=1}^{n}\mu_A(x_i)\wedge\mu_B(x_i)}{\sum_{i=1}^{n}(1-\nu_B(x_i))}, min\left(1, \frac{\sum_{i=1}^{n}\{(1-\nu_A(x_i))\wedge(1-\nu_B(x_i))\}}{\sum_{i=1}^{n}\mu_B(x_i)}\right)\right]$$

3.3.1 Some Pathological Cases

Case I: Suppose A and B are fuzzy sets. Then $A = \Box A, B = \Box B, 1-\nu_A = \mu_A$ and $1-\nu_B = \mu_B$. So

$$rel\Sigma count(A/B) = \frac{\sum_{i=1}^{n}\mu_A(x_i)\wedge\mu_B(x_i)}{\sum_{i=1}^{n}(1-\nu_B(x_i))}$$

which is same as the Prop (A/B) introduced by Zadeh.

Case II: Suppose A is an IFS and B is a fuzzy set. Then,$1-\nu_B = \mu_B$. So that

$$rel\Sigma count(A/B) = \left[\frac{\sum_{i=1}^{n}(\mu_A(x_i)\wedge\mu_B(x_i))}{\sum_{i=1}^{n}\mu_B(x_i)}, \frac{\sum_{i=1}^{n}((1-\nu_A(x_i))\wedge\mu_B(x_i)}{\sum_{i=1}^{n}\mu_B(x_i)}\right]$$

$$\frac{1}{\sum_{i=1}^{n}\mu_B(x_i)} = \left[\sum_{i=1}^{n}(\mu_A(x_i)\wedge\mu_B(x_i)), \sum_{i=1}^{n}((1-\nu_A(x_i)\wedge\mu_B(x_i))\right]$$

Also, in this case the right hand limit of the interval is less than or equal to 1. So, we need not impost this additional restriction.

Case III: If a is a fuzzy set and B is a crisp set, then

$$rel\ \Sigma count(A/B) = \left[\frac{\sum_{i=1}^{n}\mu_A(x_i)}{Card(B)}, min\left(1, \frac{\sum_{i=1}^{n}\mu_A(x_i)}{Card(B)}\right)\right]$$

In particular when $B = X = \{x_1, x_2, \ldots x_n\}$, we get

$$rel\ \Sigma count(A/B) = \left[\frac{\sum_{i=1}^{n}\mu_A(x_i)}{n}, min\left(1, \frac{\sum_{i=1}^{n}\mu_A(x_i)}{n}\right)\right]$$

$$= \frac{\sum_{i=1}^{n}\mu_A(x_i)}{n} = \frac{1}{n}\sum_{i=1}^{n}\mu_A(x_i)$$

4 Some Applications

Definition 4.1. *Let A and B be two IFSs on X. Then the rel $\sum count(A/B)$ is defined by the interval $[e_1, e_2]$, where*

$$e_1 = \frac{\sum_{i=1}^{n}(\mu_A(x_i)\wedge\mu_B(x_i))}{\sum_{i=1}^{n}(1-\nu_B(x_i))} = min\left(1, \frac{\sum_{i=1}^{n}((1-\nu_A(x_i))\wedge(1-\nu_B(x_i))}{\sum_{i=1}^{n}\mu_B(x_i)}\right)$$

Here e_1 indicates the minimum amount of similarity between A and B and e_2 indicates the maximum amount of similarity between a and B.

Clearly, rel $\Sigma count(A/B) \subseteq [0,1]$ and rel $\Sigma count(A/B) \neq rel\Sigma count(B/A)$ in general.

$$rel\Sigma count(A/A) = \left[\frac{\sum_{i=1}^{n}\mu_A(x_i)}{\sum_{i=1}^{n}(1-\nu_A(x_i))}, 1\right].$$

Definition 4.2. *For a given class $\{A_i\}i\epsilon\lambda$ of IFSs on X, the IFS $'S'$ on X is said to be the super IFS if $S = \{< x, \mu_s(x), \nu_s(x) >: x\epsilon X\}$ where*

$$\mu_S(x) = \sup_{i\epsilon\lambda}\mu_{A_i}(x) \quad and \quad \nu_S(x) = \inf_{i\epsilon\lambda}\nu_{A_i}(x).$$

Definition 4.3. *Let A and B two IFSs on X. Then we say A dominates B if*

$$mid\ \ value(rel\ \ \Sigma\ \ count(A/S)) \geq mid\ \ value\ \ (rel\ \ \Sigma\ \ count\ \ (B/S))$$

	C_1	C_2	C_3	C_4	C_5	C_6
A	$(.2, .7)$	$(.5, .2)$	$(.8, .1)$	$(.6, .3)$	$(.4, .5)$	$(.3, .6)$
B	$(.6, .2)$	$(.2, .7)$	$(.7, .3)$	$(.8, .2)$	$(.5, .3)$	$(.9, .1)$
C	$(.2, .7)$	$(.4, .5)$	$(.8, .2)$	$(.9, .1)$	$(.6, .3)$	$(.5, .2)$
D	$(.5, .4)$	$(.3, .5)$	$(.6, .3)$	$(.5, .3)$	$(.7, .2)$	$(.9, .0)$

S	$(.6, .2)$	$(.5, .2)$	$(.8, .1)$	$(.9, .1)$	$(.7, .2)$	$(.9, .0)$

4.1 Case Studies

Case Study 1: Consider the problem of gradation of students of a class. The Characteristics, which are to determine the gradation, may be some characteristics as

- Skill
- Knowledge
- Discipline in the school
- Punctually
- Efficiency in extracurricular activities
- Age

A selector may have to use the above characteristics and make their evaluation for each student in a class, considering all the information. The gradation list can be prepared basing upon the evaluation and some technique. We may use the technique of dominance as defined in definition 4.3 as the factor of gradation.

To make a case study, we assume that the number of characteristics be six. On the basis of these six characteristics which we denote by C_1, C_2, C_3, C_4, C_5 and C_6, suppose there are four students with the characteristics as mentioned above in the form of a matrix:

The super IFS $'S'$ will be given above in the form of matrix:

$$rel\Sigma count(A/S) = \left[\frac{\sum_{i=1}^{n}(\mu_A(x_i) \wedge \mu_S(x_i))}{\sum_{i=1}^{n}(1-\nu_s(x_i))}, min\left(\frac{1, \sum_{i=1}^{n}((1-\nu_A(x_i)) \wedge (1-\nu_B(x_i)))}{\sum_{i=1}^{n}\mu_S(x_i)}\right)\right]$$

$$= \left[\frac{2.8}{5.2}, min\left(1, \frac{3.6}{4.4}\right)\right] = \left[\frac{7}{13}, \frac{9}{11}\right] = [.54, .82]$$

$$rel\Sigma count(B/S) = \left[\frac{3.7}{5.2}, min\left(1, \frac{4.2}{4.4}\right)\right] = \left[\frac{37}{52}, \frac{21}{22}\right] = [.71, .95]$$

$$rel\Sigma count(C/S) = \left[\frac{3.4}{5.2}, min\left(1, \frac{4}{4.4}\right)\right] = \left[\frac{17}{26}, \frac{10}{11}\right] = [.65, .9]$$

$$rel\Sigma count(D/S) = \left[\frac{3.5}{5.2}, min\left(1, \frac{4.3}{4.4}\right)\right] = \left[\frac{35}{52}, \frac{43}{44}\right] = [.67, .98]$$

The mid values of rel$\Sigma count(A/S)$, rel$\Sigma count(B/S)$, rel$\Sigma count(C/S)$ and rel$\Sigma count(D/S)$ are .68,.83,.75, .825 respectively. So, the grading is B, D, C, A.

Definition 4.4. *Let A be an IFS on $X = \{x_1, x_2, \ldots, x_n\}$. Then depth of A denoted by depth (A) is given by*

$$depth(A) = [n, n] - \Sigma count A = [n, n] - [a_1, a_2]$$

where

$$a_1 = \sum_{i=1}^{n}\mu_A(x_i) \quad and \quad a_2 = \sum_{i=1}^{n}(1-\nu_A(x_i))) = [n - a_2, n - a_1]$$

Clearly, $depth(X) = 0$ and depth $(\phi) = n$.

Definition 4.5. *Let A_1 and A_2 be two IFSs over X, then we say A_2 is a better representative of X than A_1 denoted by $A_2 \supset A_1$, if and only if*

$$| \, depth(A_2) \, | < | \, depth(A_1) \, |$$

where $| \, [a, b] \, |$ is given by $max(| \, a \, |, | \, b \, |)$.

Using the above definitions a grading of IFSs defined over a set X can be made. The ordering being the A_i comes higher in the order the A_k if A_k is a better representative of X than A_i. We explain this by a case study.

Case Study 2: Consider four IFSs A_1, A_2, A_3 and A_4 defined over the finite set $X = \{x_1, x_2\}$ given by

$$A_1 = \{(.5, .4)/x_1, (.2, .8)/x_2\}$$

$$A_2 = \{(.1, .8)/x_1, (.9, 0)/x_2\}$$

$$A_3 = \{(.1, .9)/x_1, (0, 1)/x_2\} \ \ and$$

$$A_4 = \{(.2, .5)/x_1, (.1, .7)/x_2\} \ \ and$$

Here

$$|depth(A_1)| = |[2 - .8, 2 - .7]| = |[1.2, 1.3]| = 1.3$$

$$|depth(A_2)| = |[2 - 1.2, 2 - .1]| = |[.8, 1]| = 1$$

$$|depth(A_3)| = |[2 - .1, 2 - .1]| = |[1.9, 1.9]| = 1.9$$

$$|depth(A_4)| = |[2 - .9, 2 - .3]| = |[1.1, 1.7]| = 1.7$$

So, $A_2 \supset A_1 \supset A_4 \supset A_3$. Thus A_2 is the best representative of x.

5 Quantification Rules

If "x *is* A" be a proposition, then the proposition is modified by the modifier by $'m'$ as not , very, fairly etc. Hence the modifier proposition be "x is mA".

Similarly proposition may be quantified by intuitionistic fuzzy quantifiers such as usually, frequently, most etc. Quantifiers are denoted by Q. So, "$Qx's$ are $A's$" is a quantified proposition and "$QA's$ are $B's$" is known as extended quantified propositions. For example, 'most cars are fast' is a quantified proposition, where 'most fast cars are dangerous' is an extended quantified proposition.

The extended quantified proposition as "$QA's$ are $B's$", where Q is a intuitionistic fuzzy quantifier with membership function $\mu_Q(x)$ and the non-membership function $\nu_Q(x)$ and the IFSs A and B have membership and non-membership functions with the same argument on $x \epsilon U, (\mu_A(x), \nu_A(x))$ and$(\mu_B(x), \nu_B(x))$ correspondingly.

We have to find out the truth of the above quantified propositions.

Let A and B are two IFSs on a finite universe of discourse $U = \{x_1, x_2, \ldots, x_n\}$ then

$$\Sigma count A = \left[\sum_{i=1}^{n} \mu_A(x_i), \sum_{i=1}^{n} (1 - \nu_A(x_i)) \right]$$

$$\Sigma count B = \left[\sum_{i=1}^{n} \mu_B(x_i), \sum_{i=1}^{n} (1 - \nu_B(x_i)) \right]$$

In particular Σ count $X = [n, n] = n$, where each $x_i, i = 1, 2, \dots n$ has a membership value $'1'$ and non-membership value $'0'$.

The truth value of the proposition in a finite universe U is determined by truth $(QAs\ are\ Bs) = (\mu_Q(r), \nu_Q(r))$, where the value of $'r'$ is

$$r = rel\Sigma count(B/A) = \frac{\Sigma count(A \cap B)}{\Sigma count(A)}$$

$$= \left[\frac{\sum_{i=1}^{n}(\mu_A(x_i) \wedge \mu_B(x_i))}{\sum_{i=1}^{n}(1 - \nu_s(x_i))}, min\left(1, \frac{\sum_{i=1}^{n}((1 - \nu_A(x_i)) \wedge (1 - \nu_B(x_i))}{\sum_{i=1}^{n} \mu_A(x_i)}\right)\right]$$

The meaning of the coefficient $r = rel\Sigma count(B/A)$ is that it expresses the proportion of B in A.

In particular case, when A and B are fuzzy sets instead of IFSs, then the proposition "QA are B" reduced to the concept of Zadeh's sense.

Also, in the case, "Qxs are Bs" that is, when instead of an IFS A, we have a crisp set $\{x_i\} = U$, then truth value of "Qxs are B" be

$$truth(Qx's\ are\ B) = (\mu_Q(r_0), \nu_Q(r_0))$$

where

$$r_0 = rel\Sigma count(B/U) = \left[\frac{\sum_{i=1}^{n} \mu_B(x_i)}{n}, min\left(1, \frac{\sum_{i=1}^{n}(1 - \nu_B(x_i))}{n}\right)\right]$$

Example 5.1. *Consider the proposition "most cars are fast". Assume that cars, fast and most are defined as*

$$cars\underline{\Delta}y = \{y_1, y_2, y_3\}, U = \{y_1, y_2, y_3\}$$

$$cars\underline{\Delta}B = (.1, .8)/y_1 + (.6, .2)/y_2 + (.8, .2)/y_3$$

and most $= Q$, where

$$\mu_Q(x) = \begin{cases} 0 & 0 \le x \le .3; \\ 1 - \{1 + (2x - 0.6)^2\}^{-1} & .3 \le x \le .7; \\ 1 & .7 \le x. \end{cases}$$

and

$$\nu_Q(x) = \begin{cases} 1 & 0 \le x \le .4; \\ \{1 + (2x - 0.8)^2\}^{-1} & .4 \le x \le .8; \\ 0 & .8 \le x. \end{cases}$$

$$r_0 = rel\Sigma count(B/U) = \left[\frac{\sum_{i=1}^{n} \mu_B(x_i)}{n}, min\left(1, \frac{\sum_{i=1}^{n}(1 - \nu_B(x_i))}{n}\right)\right]$$

$$= \left[\frac{1.5}{3}, min\left(1, \frac{1.8}{3}\right)\right] = [.5, .6]$$

mid value $(r_0) = .55$, which is the average of the degree of car speed.

Now substituting $r_0 = .55$ for $'x'$, we have

$$\mu_Q(.55) = .2\ and\ \nu_Q(.55) = .53$$

The truth value depends on how both the quantifiers Q(most) and the set B(fast) are defined.

Example 5.2. *Let us consider the more general proposition, 'Most fast cars are dangerous', using the data in the above example for cars, fast and most.*

In addition, let dangerous be defined as

$$dangerous\ \underline{\Delta}A = (.2, .7)/x_1 + (.5, .4)/x_2 + (.6, .4)/x_3$$

to define $'r'$ we have to calculate

$$r = rel\Sigma count(B/A) = \left[\frac{1.2}{1.5}, min\left(1, \frac{1.4}{1.3}\right)\right] = \left[\frac{4}{5}, min\left(1, \frac{14}{13}\right)\right] = \left[\frac{4}{5}, 1\right]$$

$$mid\ value\ (r) = \frac{\frac{4}{5}+1}{2} = \frac{4+5}{5} \times \frac{1}{2} = \frac{9}{10} = .9$$

which represent the proportion of B in A.

Finally, substituting $'r'$ for $'x'$, we have

$$\mu_Q(.9) = 1\ and\ \nu_Q(.9) = 0.$$

6 Conclusion

In this paper a measure of cardinality of IFS, called $\Sigma count$ which generalizes the notion of Σcount of fuzzy sets introduced [4] has been put forth and studied. Many results involving Σ count of transformed IFSs by using modal operations have been established. A notion called relative Σ count is defined and as an application, a case study is made. Intuitionistic fuzzy quantifiers are discussed and illustrated by taking examples.

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