

## Prime cordial labeling of some wheel related graphs

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### Abstract

A *prime cordial labeling* of a graph  $G$  with the vertex set  $V(G)$  is a bijection  $f : V(G) \rightarrow \{1, 2, 3, \dots, |V(G)|\}$  such that each edge  $uv$  is assigned the label 1 if  $\gcd(f(u), f(v)) = 1$  and 0 if  $\gcd(f(u), f(v)) > 1$ , then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph which admits prime cordial labeling is called *prime cordial graph*. In this paper we prove that the gear graph  $G_n$  admits prime cordial labeling for  $n \geq 4$ . We also show that the helm  $H_n$  for every  $n$ , the closed helm  $CH_n$  (for  $n \geq 5$ ) and the flower graph  $Fl_n$  (for  $n \geq 4$ ) are prime cordial graphs.

*Keywords:* Prime cordial labeling, gear graph, helm, closed helm, flower graph.

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## 1 Introduction

We begin with simple, finite, connected and undirected graph  $G = (V(G), E(G))$  with  $p$  vertices and  $q$  edges. For standard terminology and notations we follow Gross and Yellen [5]. We will provide brief summary of definitions and other information which are necessary for the present investigations.

**Definition 1.1.** *If the vertices are assigned values subject to certain condition(s) then it is known as graph labeling.*

*Any graph labeling will have following three common characteristics:*

1. a set of numbers from which vertex labels are chosen;
2. a rule that assigns a value to each edge;
3. a condition that this value has to satisfy.

According to Beineke and Hegde [1] graph labeling serves as a frontier between number theory and structure of graphs. Graph labelings have many applications within mathematics as well as to several areas of computer science and communication networks. According to Graham and Sloane [4] the harmonious labellings are closely related to problems in error correcting codes while odd harmonious labeling is useful to solve undetermined equations as described by Liang and Bai [6]. The optimal linear arrangement concern to wiring network problems in electrical engineering and placement problems in production engineering can be formalised as a graph labeling problem as stated by Yegnanaryanan and Vaidhyathan [13]. The watershed transform is an important morphological tool used for image segmentation. An improved algorithm using Graceful labeling for watershed image segmentation is also proposed by Sridevi *et al.*[7]. For a dynamic survey on various graph labeling problems along with an extensive bibliography we refer to Gallian [3].

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**Definition 1.2.** A mapping  $f : V(G) \rightarrow \{0, 1\}$  is called binary vertex labeling of  $G$  and  $f(v)$  is called the label of the vertex  $v$  of  $G$  under  $f$ .

**Definition 1.3.** If for an edge  $e = uv$ , the induced edge labeling  $f^* : E(G) \rightarrow \{0, 1\}$  is given by  $f^*(e) = |f(u) - f(v)|$ . Then

$$\left. \begin{aligned} v_f(i) &= \text{number of vertices of } G \text{ having label } i \text{ under } f \\ e_f(i) &= \text{number of edges of } G \text{ having label } i \text{ under } f^* \end{aligned} \right\} \text{ where } i = 0 \text{ or } 1$$

**Definition 1.4.** A binary vertex labeling  $f$  of a graph  $G$  is called a cordial labeling if  $|v_f(0) - v_f(1)| \leq 1$  and  $|e_f(0) - e_f(1)| \leq 1$ . A graph  $G$  is cordial if it admits cordial labeling.

The concept of cordial labeling was introduced by Cahit [2]. Some labeling schemes are also introduced with minor variations in cordial theme. Product cordial labeling, total product cordial labeling and prime cordial labeling are among mention a few. The present work is focused on prime cordial labeling.

**Definition 1.5.** A prime cordial labeling of a graph  $G$  with vertex set  $V(G)$  is a bijection  $f : V(G) \rightarrow \{1, 2, 3, \dots, |V(G)|\}$  and the induced function  $f^* : E(G) \rightarrow \{0, 1\}$  is defined by

$$\begin{aligned} f^*(e = uv) &= 1, & \text{if } \gcd(f(u), f(v)) = 1; \\ &= 0, & \text{otherwise.} \end{aligned}$$

satisfies the condition  $|e_f(0) - e_f(1)| \leq 1$ . A graph which admits prime cordial labeling is called a prime cordial graph.

The concept of prime cordial labeling was introduced by Sundaram *et al.*[8] and in the same paper they have investigated several results on prime cordial labeling. Vaidya and Vihol [9] as well as Vaidya and Shah [12] have discussed prime cordial labeling in the context of some graph operations. Prime cordial labeling for some cycle related graphs have been discussed by Vaidya and Vihol in [10]. Vaidya and Shah [11] have investigated many results on prime cordial labeling. Same authors in [12] have proved that the wheel graph  $W_n$  admits prime cordial labeling for  $n \geq 8$ . The present work is aimed to investigate some new results on prime cordial labeling for some wheel related graphs.

**Definition 1.6.** The wheel  $W_n$  is defined to be the join  $K_1 + C_n$ . The vertex corresponding to  $K_1$  is known as apex and vertices corresponding to cycle are known as rim vertices while the edges corresponding to cycle are known as rim edges. We continue to recognize apex of wheel as the apex of respective graphs corresponding to definitions 1.6 to 1.9.

**Definition 1.7.** The gear graph  $G_n$  is obtained from the wheel by subdividing each of its rim edge.

**Definition 1.8.** The helm  $H_n$  is the graph obtained from a wheel  $W_n$  by attaching a pendant edge to each rim vertex. It contains three types of vertices: an apex of degree  $n$ ,  $n$  vertices of degree 4 and  $n$  pendant vertices.

**Definition 1.9.** The closed helm  $CH_n$  is the graph obtained from a helm  $H_n$  by joining each pendant vertex to form a cycle. It contains three types of vertices: an apex of degree  $n$ ,  $n$  vertices of degree 4 and  $n$  vertices degree 3.

**Definition 1.10.** The flower  $Fl_n$  is the graph obtained from a helm  $H_n$  by joining each pendant vertex to the apex of the helm. It contains three types of vertices: an apex of degree  $2n$ ,  $n$  vertices of degree 4 and  $n$  vertices of degree 2.

## 2 Main Results

**Theorem 2.1.** Gear graph  $G_n$  is a prime cordial graph for  $n \geq 4$ .

*Proof.* Let  $W_n$  be the wheel with apex vertex  $v$  and rim vertices  $v_1, v_2, \dots, v_n$ . To obtain the gear graph  $G_n$  subdivide each rim edge of wheel by the vertices  $u_1, u_2, \dots, u_n$ . Where each  $u_i$  is added between  $v_i$  and  $v_{i+1}$  for  $i = 1, 2, \dots, n-1$  and  $u_n$  is added between  $v_1$  and  $v_n$ . Then  $|V(G_n)| = 2n + 1$  and  $|E(G_n)| = 3n$ . To define  $f : V(G) \rightarrow \{1, 2, 3, \dots, 2n + 1\}$ , we consider following four cases.

**Case 1:**  $n = 3$ 

In  $G_3$  to satisfy the edge condition for prime cordial labeling it is essential to label four edges with label 0 and five edges with label 1 out of nine edges. But all the possible assignments of vertex labels will give rise to 0 labels for at most three edges and 1 labels for at least six edges. That is,  $|e_f(0) - e_f(1)| = 3 > 1$ . Hence,  $G_3$  is not prime cordial graph.

**Case 2:**  $n = 4$  to 9, 11, 14, 19

For  $n = 4$ ,  $f(v) = 6, f(v_1) = 3, f(v_2) = 9, f(v_3) = 4, f(v_4) = 8$  and  $f(u_1) = 1, f(u_2) = 7, f(u_3) = 2, f(u_4) = 5$ . Then  $e_f(0) = 6 = e_f(1)$ .

For  $n = 5$ ,  $f(v) = 6, f(v_1) = 9, f(v_2) = 5, f(v_3) = 4, f(v_4) = 8, f(v_5) = 3$  and  $f(u_1) = 7, f(u_2) = 10, f(u_3) = 2, f(u_4) = 1, f(u_5) = 11$ . Then  $e_f(0) = 8, e_f(1) = 7$ .

For  $n = 6$ ,  $f(v) = 6, f(v_1) = 9, f(v_2) = 8, f(v_3) = 4, f(v_4) = 11, f(v_5) = 1, f(v_6) = 10$  and  $f(u_1) = 12, f(u_2) = 2, f(u_3) = 13, f(u_4) = 5, f(u_5) = 7, f(u_6) = 3$ . Then  $e_f(0) = 9 = e_f(1)$ .

For  $n = 7$ ,  $f(v) = 2, f(v_1) = 7, f(v_2) = 4, f(v_3) = 6, f(v_4) = 12, f(v_5) = 8, f(v_6) = 10, f(v_7) = 14$  and  $f(u_1) = 5, f(u_2) = 3, f(u_3) = 9, f(u_4) = 15, f(u_5) = 1, f(u_6) = 11, f(u_7) = 13$ . Then  $e_f(0) = 10, e_f(1) = 11$ .

For  $n = 8$ ,  $f(v) = 2, f(v_1) = 1, f(v_2) = 4, f(v_3) = 6, f(v_4) = 12, f(v_5) = 8, f(v_6) = 10, f(v_7) = 14, f(v_8) = 16$  and  $f(u_1) = 7, f(u_2) = 3, f(u_3) = 9, f(u_4) = 15, f(u_5) = 5, f(u_6) = 11, f(u_7) = 13, f(u_8) = 17$ . Then  $e_f(0) = 12 = e_f(1)$ .

For  $n = 9$ ,  $f(v) = 2, f(v_1) = 4, f(v_2) = 5, f(v_3) = 6, f(v_4) = 12, f(v_5) = 18, f(v_6) = 8, f(v_7) = 10, f(v_8) = 14, f(v_9) = 16$  and  $f(u_1) = 7, f(u_2) = 3, f(u_3) = 9, f(u_4) = 15, f(u_5) = 1, f(u_6) = 11, f(u_7) = 13, f(u_8) = 17, f(u_9) = 19$ . Then  $e_f(0) = 13, e_f(1) = 14$ .

For  $n = 11$ ,  $f(v) = 2, f(v_1) = 4, f(v_2) = 5, f(v_3) = 6, f(v_4) = 12, f(v_5) = 18, f(v_6) = 8, f(v_7) = 10, f(v_8) = 14, f(v_9) = 16, f(v_{10}) = 20, f(v_{11}) = 22$  and  $f(u_1) = 7, f(u_2) = 3, f(u_3) = 9, f(u_4) = 15, f(u_5) = 21, f(u_6) = 11, f(u_7) = 13, f(u_8) = 17, f(u_9) = 19, f(u_{10}) = 23, f(u_{11}) = 1$ . Then  $e_f(0) = 16, e_f(1) = 17$ .

For  $n = 14$ ,  $f(v) = 2, f(v_1) = 4, f(v_2) = 5, f(v_3) = 6, f(v_4) = 12, f(v_5) = 18, f(v_6) = 24, f(v_7) = 8, f(v_8) = 10, f(v_9) = 14, f(v_{10}) = 16, f(v_{11}) = 20, f(v_{12}) = 22, f(v_{13}) = 26, f(v_{14}) = 28$  and  $f(u_1) = 7, f(u_2) = 3, f(u_3) = 9, f(u_4) = 15, f(u_5) = 21, f(u_6) = 27, f(u_7) = 11, f(u_8) = 13, f(u_9) = 17, f(u_{10}) = 19, f(u_{11}) = 23, f(u_{12}) = 25, f(u_{13}) = 29, f(u_{14}) = 1$ . Then  $e_f(0) = 21 = e_f(1)$ .

For  $n = 19$ ,  $f(v) = 2, f(v_1) = 4, f(v_2) = 5, f(v_3) = 6, f(v_4) = 12, f(v_5) = 18, f(v_6) = 24, f(v_7) = 30, f(v_8) = 36, f(v_9) = 8, f(v_{10}) = 10, f(v_{11}) = 14, f(v_{12}) = 16, f(v_{13}) = 20, f(v_{14}) = 22, f(v_{15}) = 26, f(v_{16}) = 28, f(v_{17}) = 32, f(v_{18}) = 34, f(v_{19}) = 38$  and  $f(u_1) = 3, f(u_2) = 7, f(u_3) = 9, f(u_4) = 15, f(u_5) = 21, f(u_6) = 27, f(u_7) = 33, f(u_8) = 39, f(u_9) = 11, f(u_{10}) = 13, f(u_{11}) = 17, f(u_{12}) = 19, f(u_{13}) = 23, f(u_{14}) = 25, f(u_{15}) = 29, f(u_{16}) = 31, f(u_{17}) = 35, f(u_{18}) = 37, f(u_{19}) = 1$ . Then  $e_f(0) = 29, e_f(1) = 28$ .

Now for the remaining two cases let,

$$s = \left\lfloor \frac{n}{3} \right\rfloor, k = \left\lfloor \frac{2n+1}{3} \right\rfloor - \left\lfloor \frac{n}{3} \right\rfloor, t = \left( n + \left\lfloor \frac{2n+1}{3} \right\rfloor - 2 \right) - \left\lfloor \frac{3n}{2} \right\rfloor,$$

$$m = \left\lfloor \frac{2n+1}{2} \right\rfloor - (2 + s + t), h_e = \text{largest even number not divisible by } 3 \leq 2n,$$

$$h_o = \text{largest odd number not divisible by } 3 \leq 2n + 1.$$

$$f(v) = 2,$$

$$f(v_1) = 4, \quad f(v_2) = 5,$$

$$f(v_{2+i}) = 6i; \quad 1 \leq i \leq s$$

$$f(u_1) = 7,$$

$$f(u_{1+i}) = 3 + 6(i-1); \quad 1 \leq i \leq k$$

If  $k = s$ , then  $f(u_{k+2}) = 1$  or  $f(u_n) = 1$ .

**Case 3:**  $t = 0$  ( $n = 10, 12, 13, 15, 17$ )

For  $m$  odd, consider  $x_1 = \left\lfloor \frac{m}{2} \right\rfloor, x_2 = x_3 = x_4 = \left\lfloor \frac{m}{2} \right\rfloor$  and for  $m$  even consider,  $x_1 = x_2 = x_3 = x_4 = \frac{m}{2}$

$$f(v_{s+1+2i}) = 8 + 6(i-1); \quad 1 \leq i \leq x_1$$

$$f(v_{s+2+2i}) = 10 + 6(i-1), \quad 1 \leq i \leq x_3$$

$$f(u_{s+1+2i}) = 11 + 6(i-1); \quad 1 \leq i \leq x_2$$

$$f(u_{s+2+2i}) = 13 + 6(i-1); \quad 1 \leq i \leq x_4$$

which assigns all the vertex labels for case 3.

**Case 4:**  $t \geq 1$  ( $n = 16, 18, n \geq 20$ )

For  $m$  odd, consider  $x_1 = x_2 = x_3 = \left\lfloor \frac{m}{2} \right\rfloor, x_4 = \frac{m-3}{2}$  and for  $m$  even consider,  $x_1 = \frac{m}{2}, x_2 = x_3 = x_4 =$

$$\frac{m-2}{2}.$$

$$\begin{aligned} f(v_{s+1+2i}) &= 8 + 6(i-1); & 1 \leq i \leq x_1 \\ f(v_{s+2+2i}) &= 10 + 6(i-1), & 1 \leq i \leq x_3 \\ f(u_{s+1+2i}) &= 11 + 6(i-1); & 1 \leq i \leq x_2 \\ f(u_{s+2+2i}) &= 13 + 6(i-1); & 1 \leq i \leq x_4 \end{aligned}$$

For the vertices  $u_{n-1}, u_{n-2}, \dots, u_{n-(t+1)}$  we assign even numbers (not congruent 0 mod 3) in descending order starting from  $h_e$  respectively while for  $u_n, v_n, v_{n-1}, v_{n-2}, \dots, v_{n-t}$  we assign odd numbers(not congruent 0 mod 3) in descending order starting from  $h_0$  respectively such that  $f^*(v_j u_{j-i})$  or  $f^*(v_j u_{j+i})$  do not generate edge label 0. Which assigns all the vertex labels for case 4.

In view of the above defined labeling pattern for cases 3 and 4, we have

$$e_f(0) = \left\lfloor \frac{3n}{2} \right\rfloor \text{ and } e_f(1) = \left\lceil \frac{3n}{2} \right\rceil.$$

Thus, we have  $|e_f(0) - e_f(1)| \leq 1$ .

Hence,  $G_n$  is a prime cordial graph for  $n \geq 4$ . □

**Example 2.1.** For the graph  $G_{20}$ ,  $|V(G_{20})| = 41$  and  $|E(G_{20})| = 60$ . In accordance with Theorem 2.1 we have  $s = 6, k = 7, t = 1, m = 11, x_1 = x_2 = x_3 = 5, x_4 = 4$  and using the labeling pattern described in case 4. The corresponding prime cordial labeling is shown in Fig. 1. It is easy to visualise that  $e_f(0) = 30 = e_f(1)$ .

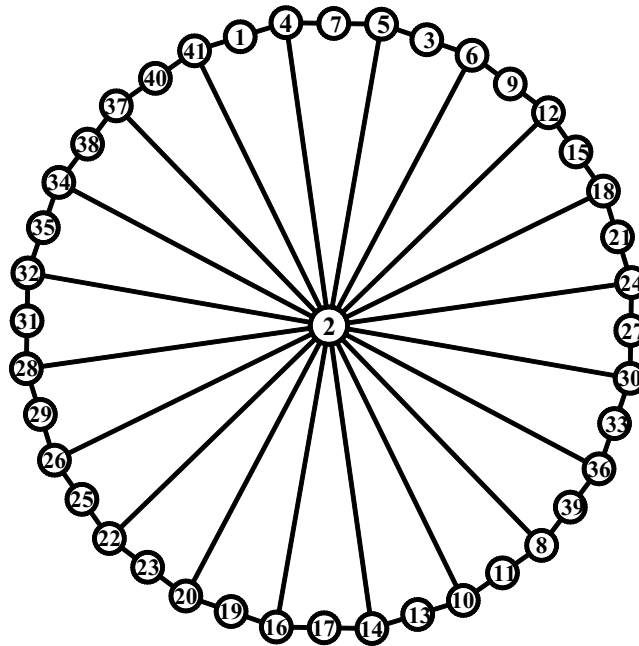


Fig. 1

**Theorem 2.2.** Helm graph  $H_n$  is a prime cordial graph for every  $n$ .

*Proof.* Let  $v$  be the apex,  $v_1, v_2, \dots, v_n$  be the vertices of degree 4 and  $u_1, u_2, \dots, u_n$  be the pendant vertices of  $H_n$ . Then  $|V(H_n)| = 2n + 1$  and  $|E(H_n)| = 3n$ . To define  $f : V(G) \rightarrow \{1, 2, 3, \dots, 2n + 1\}$ , we consider following three cases.

**Case 1:**  $n = 3$  to 9

For  $n = 3$ ,  $f(v) = 6, f(v_1) = 2, f(v_2) = 4, f(v_3) = 3$  and  $f(u_1) = 1, f(u_2) = 7, f(u_3) = 5$ . Then  $e_f(0) = 4, e_f(1) = 5$ .

For  $n = 4$ ,  $f(v) = 6, f(v_1) = 3, f(v_2) = 2, f(v_3) = 4, f(v_4) = 8$  and  $f(u_1) = 1, f(u_2) = 9, f(u_3) = 5, f(u_4) = 7$ . Then  $e_f(0) = 6 = e_f(1)$ .

For  $n = 5$ ,  $f(v) = 6, f(v_1) = 3, f(v_2) = 2, f(v_3) = 5, f(v_4) = 8, f(v_5) = 10$  and  $f(u_1) = 11, f(u_2) = 1, f(u_3) = 5, f(u_4) = 7, f(u_5) = 9$ . Then  $e_f(0) = 8, e_f(1) = 7$ .

For  $n = 6$ ,  $f(v) = 2, f(v_1) = 3, f(v_2) = 6, f(v_3) = 8, f(v_4) = 10, f(v_5) = 7, f(v_6) = 11$  and  $f(u_1) = 1, f(u_2) = 4, f(u_3) = 12, f(u_4) = 5, f(u_5) = 9, f(u_6) = 13$ . Then  $e_f(0) = 9 = e_f(1)$ .

For  $n = 7$ ,  $f(v) = 2, f(v_1) = 3, f(v_2) = 6, f(v_3) = 4, f(v_4) = 10, f(v_5) = 5, f(v_6) = 11, f(v_7) = 1$  and

$f(u_1) = 9, f(u_2) = 8, f(u_3) = 12, f(u_4) = 14, f(u_5) = 7, f(u_6) = 13, f(u_7) = 15$ . Then  $e_f(0) = 11, e_f(1) = 10$ .  
 For  $n = 8, f(v) = 2, f(v_1) = 6, f(v_2) = 8, f(v_3) = 4, f(v_4) = 10, f(v_5) = 5, f(v_6) = 9, f(v_7) = 13, f(v_8) = 17$   
 and  $f(u_1) = 3, f(u_2) = 12, f(u_3) = 14, f(u_4) = 16, f(u_5) = 7, f(u_6) = 11, f(u_7) = 15, f(u_8) = 1$ . Then  $e_f(0) = 12 = e_f(1)$ .

For  $n = 9, f(v) = 2, f(v_1) = 3, f(v_2) = 6, f(v_3) = 8, f(v_4) = 4, f(v_5) = 10, f(v_6) = 5, f(v_7) = 9, f(v_8) = 13, f(v_9) = 17$  and  $f(u_1) = 1, f(u_2) = 12, f(u_3) = 14, f(u_4) = 16, f(u_5) = 18, f(u_6) = 7, f(u_7) = 11, f(u_8) = 15, f(u_9) = 19$ . Then  $e_f(0) = 17, e_f(1) = 16$ .

**Case 2:**  $n$  is even,  $n \geq 10$

$$\begin{aligned} f(v) &= 2, & f(v_1) &= 10, \\ f(v_2) &= 4, & f(v_3) &= 8, \\ f(v_{3+i}) &= 12 + 2(i - 1); & 1 \leq i \leq \frac{n}{2} - 4 \\ f\left(v_{\frac{n}{2}}\right) &= 6, & f\left(v_{\frac{n}{2}+1}\right) &= 1, \\ f\left(u_{\frac{n}{2}}\right) &= 3, & f\left(u_{\frac{n}{2}+1}\right) &= 2n + 1, \\ f(u_i) &= 2n - 2(i - 1); & 1 \leq i \leq \frac{n}{2} - 1 \\ f(v_{n-i}) &= 5 + 4i; & 0 \leq i \leq \frac{n}{2} - 1 \\ f(u_{n-i}) &= 7 + 4i; & 0 \leq i \leq \frac{n}{2} - 1 \end{aligned}$$

**Case 3:**  $n$  is odd,  $n \geq 11$

$$\begin{aligned} f(v) &= 2, & f(v_1) &= 10, \\ f(v_2) &= 4, & f(v_3) &= 8, \\ f(v_{3+i}) &= 12 + 2(i - 1); & 1 \leq i \leq \frac{n-1}{2} - 4 \\ f\left(v_{\frac{n-1}{2}}\right) &= 6, & f\left(v_{\frac{n+1}{2}}\right) &= 3, \\ f\left(u_{\frac{n+1}{2}}\right) &= 1, \\ f(u_i) &= 2n - 2(i - 1); & 1 \leq i \leq \frac{n-1}{2} \\ f(v_{n-i}) &= 5 + 4i; & 0 \leq i < \frac{n-1}{2} \\ f(u_{n-i}) &= 7 + 4i; & 0 \leq i < \frac{n-1}{2} \end{aligned}$$

In view of the above defined labeling pattern for cases 2 and 3,

If  $2n - 1 \equiv 0 \pmod{3}$  then  $e_f(0) = \left\lfloor \frac{3n}{2} \right\rfloor$  and  $e_f(1) = \left\lfloor \frac{3n}{2} \right\rfloor$ ,

otherwise  $e_f(0) = \left\lfloor \frac{3n}{2} \right\rfloor$  and  $e_f(1) = \left\lfloor \frac{3n}{2} \right\rfloor$ .

Thus, we have  $|e_f(0) - e_f(1)| \leq 1$ .

Hence,  $H_n$  is a prime cordial graph for every  $n$ . □

**Example 2.2.** The graph  $H_{13}$  and its prime cordial labeling is shown in Fig. 2.

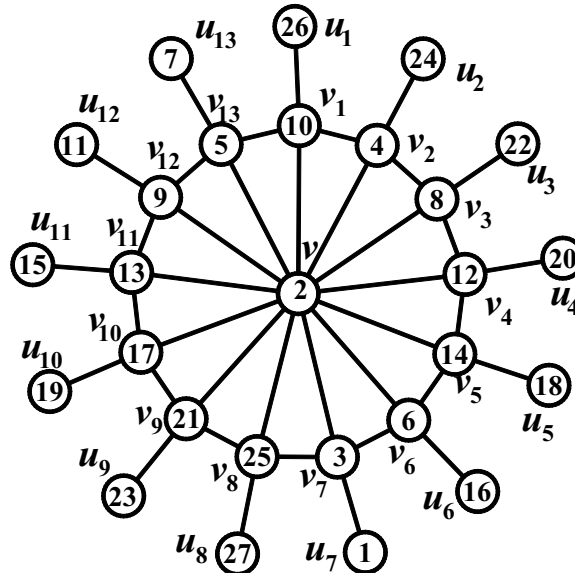


Fig. 2

**Theorem 2.3.** Closed helm  $CH_n$  is a prime cordial graph for  $n \geq 5$ .

*Proof.* Let  $v$  be the apex,  $v_1, v_2, \dots, v_n$  be the vertices of degree 4 and  $u_1, u_2, \dots, u_n$  be the vertices of degree 3 of  $CH_n$ . Then  $|V(CH_n)| = 2n + 1$  and  $|E(CH_n)| = 4n$ . To define  $f : V(G) \rightarrow \{1, 2, 3, \dots, 2n + 1\}$ , we consider following three cases.

**Case 1:**  $n = 3, 4$

In  $CH_3$  to satisfy the edge condition for prime cordial labeling it is essential to label six edges with label 0 and six edges with label 1 out of twelve edges. But all the possible assignments of vertex labels will give rise to 0 labels for at most four edges and 1 labels for at least eight edges. That is,  $|e_f(0) - e_f(1)| = 4 > 1$ . Hence,  $CH_3$  is not prime cordial graph.

In  $CH_4$  to satisfy the edge condition for prime cordial labeling it is essential to label eight edges with label 0 and eight edges with label 1 out of sixteen edges. But all the possible assignments of vertex labels will give rise to 0 labels for at most seven edges and 1 labels for at least nine edges. That is,  $|e_f(0) - e_f(1)| = 2 > 1$ . Hence,  $CH_4$  is not prime cordial graph.

**Case 2:**  $n = 5, 6$

For  $n = 5$ ,  $f(v) = 6, f(v_1) = 2, f(v_2) = 4, f(v_3) = 3, f(v_4) = 9, f(v_5) = 11$  and  $f(u_1) = 10, f(u_2) = 8, f(u_3) = 1, f(u_4) = 7, f(u_5) = 5$ . Then  $e_f(0) = 10 = e_f(1)$ .

For  $n = 6$ ,  $f(v) = 6, f(v_1) = 1, f(v_2) = 5, f(v_3) = 10, f(v_4) = 4, f(v_5) = 12, f(v_6) = 3$  and  $f(u_1) = 11, f(u_2) = 13, f(u_3) = 8, f(u_4) = 2, f(u_5) = 9, f(u_6) = 7$ . Then  $e_f(0) = 12 = e_f(1)$ .

**Case 3:**  $n \geq 7$

- $f(v) = 2, \quad f(v_1) = 4,$
- $f(v_2) = 6, \quad f(v_3) = 3,$
- $f(v_4) = 12, \quad f(v_5) = 8,$
- $f(v_6) = 10,$
- $f(v_{6+i}) = 14 + 2(i - 1); \quad 1 \leq i \leq n - 6$
- $f(u_1) = 1, \quad f(u_2) = 5,$
- $f(u_3) = 7, \quad f(u_4) = 9,$
- $f(u_5) = 13, \quad f(u_6) = 11,$
- $f(u_{6+i}) = 15 + 2(i - 1); \quad 1 \leq i \leq n - 6$

In view of the above defined labeling pattern we have  $e_f(0) = 2n = e_f(1)$ .

Thus, we have  $|e_f(0) - e_f(1)| \leq 1$ .

Hence,  $CH_n$  is a prime cordial graph for  $n \geq 5$ . □

**Example 2.3.** The graph  $CH_{10}$  and its prime cordial labeling is shown in Fig. 3.

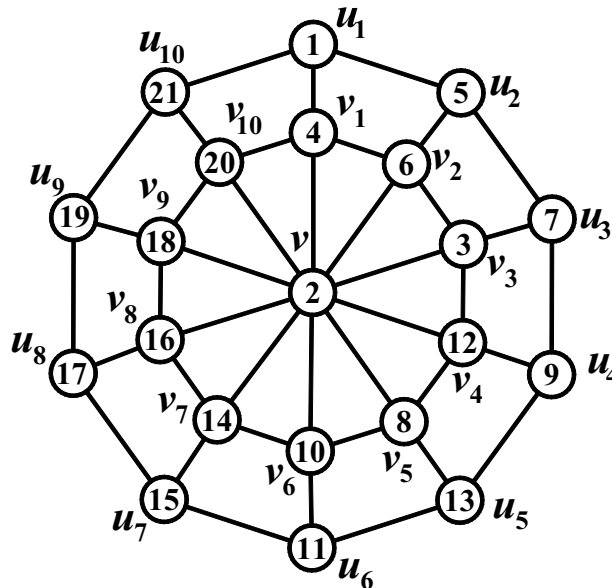


Fig. 3

**Theorem 2.4.** Flower graph  $Fl_n$  is a prime cordial graph for  $n \geq 4$ .

*Proof.* Let  $v$  be the apex,  $v_1, v_2, \dots, v_n$  be the vertices of degree 4 and  $u_1, u_2, \dots, u_n$  be the vertices of degree 2 of  $Fl_n$ . Then  $|V(Fl_n)| = 2n + 1$  and  $|E(Fl_n)| = 4n$ . To define  $f : V(G) \rightarrow \{1, 2, 3, \dots, 2n + 1\}$ , we consider following four cases.

**Case 1:**  $n = 3$

In  $Fl_3$  to satisfy the edge condition for prime cordial labeling it is essential to label six edges with label 0 and six edges with label 1 out of twelve edges. But all the possible assignments of vertex labels will give rise to 0 labels for at most four edges and 1 labels for at least eight edges. That is,  $|e_f(0) - e_f(1)| = 4 > 1$ . Hence,  $Fl_3$  is not prime cordial graph.

**Case 2:**  $n = 4$  to 9.

For  $Fl_4$ ,  $f(v) = 6, f(v_1) = 4, f(v_2) = 2, f(v_3) = 9, f(v_4) = 3$  and  $f(u_1) = 7, f(u_2) = 8, f(u_3) = 5, f(u_4) = 1$ . Then  $e_f(0) = 8 = e_f(1)$ .

For  $Fl_5$ ,  $f(v) = 6, f(v_1) = 2, f(v_2) = 4, f(v_3) = 8, f(v_4) = 10, f(v_5) = 3$  and  $f(u_1) = 11, f(u_2) = 7, f(u_3) = 5, f(u_4) = 1, f(u_5) = 9$ . Then  $e_f(0) = 10 = e_f(1)$ .

For  $Fl_6$ ,  $f(v) = 6, f(v_1) = 2, f(v_2) = 4, f(v_3) = 8, f(v_4) = 10, f(v_5) = 12, f(v_6) = 3$  and  $f(u_1) = 5, f(u_2) = 7, f(u_3) = 9, f(u_4) = 11, f(u_5) = 13, f(u_6) = 1$ . Then  $e_f(0) = 12 = e_f(1)$ .

For  $Fl_7$ ,  $f(v) = 2, f(v_1) = 3, f(v_2) = 12, f(v_3) = 10, f(v_4) = 8, f(v_5) = 14, f(v_6) = 4, f(v_7) = 6$  and  $f(u_1) = 1, f(u_2) = 5, f(u_3) = 11, f(u_4) = 7, f(u_5) = 13, f(u_6) = 15, f(u_7) = 9$ . Then  $e_f(0) = 14 = e_f(1)$ .

For  $Fl_8$ ,  $f(v) = 2, f(v_1) = 3, f(v_2) = 6, f(v_3) = 4, f(v_4) = 8, f(v_5) = 10, f(v_6) = 14, f(v_7) = 16, f(v_8) = 12$  and  $f(u_1) = 1, f(u_2) = 9, f(u_3) = 7, f(u_4) = 5, f(u_5) = 11, f(u_6) = 13, f(u_7) = 15, f(u_8) = 17$ . Then  $e_f(0) = 16 = e_f(1)$ .

For  $Fl_9$ ,  $f(v) = 2, f(v_1) = 3, f(v_2) = 12, f(v_3) = 4, f(v_4) = 8, f(v_5) = 10, f(v_6) = 14, f(v_7) = 16, f(v_8) = 18, f(v_9) = 6$  and  $f(u_1) = 17, f(u_2) = 1, f(u_3) = 5, f(u_4) = 7, f(u_5) = 11, f(u_6) = 13, f(u_7) = 15, f(u_8) = 19, f(u_9) = 9$ . Then  $e_f(0) = 18 = e_f(1)$ .

**Case 3:**  $n$  is even,  $n \geq 10$

$$\begin{aligned} f(v) &= 2, & f(v_1) &= 10, \\ f(v_2) &= 4, & f(v_3) &= 8, \\ f(v_{3+i}) &= 12 + 2(i-1); & 1 \leq i \leq \frac{n}{2} - 4 \\ f(v_{\frac{n}{2}}) &= 6, \\ f(v_{n-i}) &= 5 + 4i; & 0 \leq i < \frac{n}{2} - 2 \\ f(u_i) &= 2n - 2(i-1); & 1 \leq i \leq \frac{n}{2} - 1 \\ f(u_{\frac{n}{2}}) &= 3, \\ f(u_{n-i}) &= 7 + 4i; & 0 \leq i < \frac{n}{2} - 2 \end{aligned}$$

For  $2n + 1 \equiv 0 \pmod{3}$

$$\begin{aligned} f(v_{\frac{n}{2}+i}) &= 2n + 1 - 4(i-1); & 1 \leq i \leq 2 \\ f(u_{\frac{n}{2}+1}) &= 1, & f(u_{\frac{n}{2}+2}) &= 2n - 1 \end{aligned}$$

For  $2n + 1 \equiv 1 \pmod{3}$

$$\begin{aligned} f(v_{\frac{n}{2}+1}) &= 2n - 3, & f(v_{\frac{n}{2}+2}) &= 1, \\ f(u_{\frac{n}{2}+1}) &= 2n + 1, & f(u_{\frac{n}{2}+2}) &= 2n - 1 \end{aligned}$$

For  $2n + 1 \equiv 2 \pmod{3}$

$$\begin{aligned} f(v_{\frac{n}{2}+1}) &= 2n - 1, & f(v_{\frac{n}{2}+2}) &= 2n - 3, \\ f(u_{\frac{n}{2}+1}) &= 2n + 1, & f(u_{\frac{n}{2}+2}) &= 1 \end{aligned}$$

**Case 4:**  $n$  is odd,  $n \geq 11$

$$\begin{aligned} f(v) &= 2, & f(v_1) &= 10, \\ f(v_2) &= 4, & f(v_3) &= 8, \\ f(v_{3+i}) &= 12 + 2(i-1); & 1 \leq i \leq \frac{n-1}{2} - 4 \\ f(v_{\frac{n-1}{2}}) &= 6, & f(v_{\frac{n+1}{2}}) &= 3, \\ f(v_{n-i}) &= 5 + 4i; & 0 \leq i < \frac{n-1}{2} \\ f(u_i) &= 2n - 2(i-1); & 1 \leq i \leq \frac{n-1}{2} \\ f(u_{n-i}) &= 7 + 4i; & 0 \leq i < \frac{n-5}{2} \end{aligned}$$

For  $2n + 1 \equiv 0 \pmod{3}$

$$\begin{aligned} f(u_{\frac{n+1}{2}}) &= 2n + 1, & f(u_{\frac{n+1}{2}+1}) &= 1, \\ f(u_{\frac{n+1}{2}+2}) &= 2n - 3, \end{aligned}$$

For  $2n + 1 \equiv 1 \pmod{3}$

$$f\left(u_{\frac{n+1}{2}}\right) = 2n - 3, \quad f\left(u_{\frac{n+1}{2}+1}\right) = 2n + 1,$$

$$f\left(u_{\frac{n+1}{2}+2}\right) = 1,$$

For  $2n + 1 \equiv 2 \pmod{3}$

$$f\left(u_{\frac{n+1}{2}}\right) = 1,$$

$$f\left(u_{\frac{n+1}{2}+i}\right) = 2n + 1 - 4(i - 1); \quad 1 \leq i \leq 2$$

In view of the above defined labeling pattern we have  $e_f(0) = 2n = e_f(1)$ .

Thus, we have  $|e_f(0) - e_f(1)| \leq 1$ .

Hence,  $Fl_n$  is a prime cordial graph for  $n \geq 4$ . □

**Example 2.4.** The graph  $Fl_{11}$  and its prime cordial labeling is shown in Fig. 4.

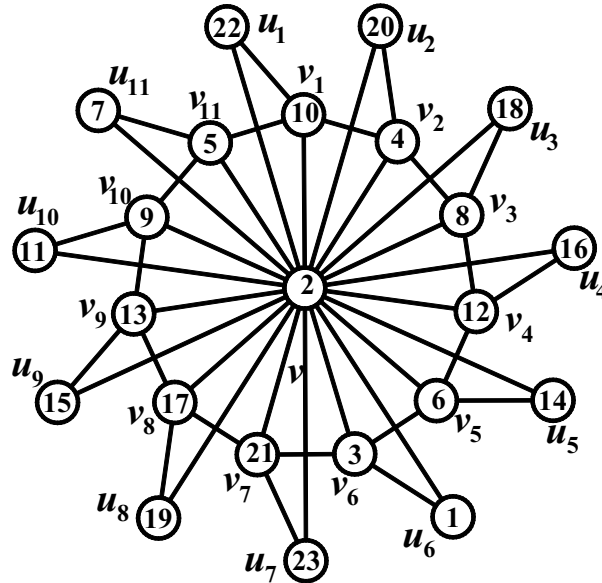


Fig. 4

### 3 Open problems

- To investigate necessary and sufficient conditions for a graph to admit a prime cordial labeling.
- To investigate some new graph or graph families which admit prime cordial labeling.
- To obtain forbidden subgraph(s) characterisation for prime cordial labeling.

### 4 Conclusion

As all the graphs are not prime cordial graphs it is very interesting and challenging as well to investigate prime cordial labeling for the graph or graph families which admit prime cordial labeling. Here we have contributed some new results by investigating prime cordial labeling for some wheel related graphs.

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