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(λ, μ)-Fuzzy quasi-ideals and (λ, μ)-fuzzy bi-ideals in ternary semirings

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Abstract

In this paper we introduce the notion of (λ, μ)-Fuzzy quasi ideals and (λ, μ)-Fuzzy bi-ideals in ternary semirings which can be regarded as the generalization of fuzzy quasi ideals and fuzzy bi-ideals in ternary semirings.

Keywords: (λ, μ)-Fuzzy ternary subsemirings, (λ, μ)-Fuzzy ideal, (λ, μ)-Fuzzy quasi-ideal, (λ, μ)-Fuzzy bi-ideal.

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1 Introduction

The notion of ternary algebraic system was introduced by Lehmer [12] in 1932. He investigated certain ternary algebraic systems called triplexes. In 1971, Lister [13] characterized additive semigroups of rings which are closed under the triple ring product and he called this algebraic system a ternary ring. Dutta and Kar [1] introduced a notion of ternary semirings which is a generalization of ternary rings and semirings, and they studied some properties of ternary semirings [1, 2, 3, 4, 5, 6, 7, 8]. The theory of fuzzy sets was first studied by Zadeh [16] in 1965. Many papers on fuzzy sets appeared showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology, measure theory, etc. Kavikumar et al.[9] and [10] studied fuzzy ideals, fuzzy bi-ideals and fuzzy quasi-ideals in ternary semirings. Ronnason Chinram et al. [14] studied L-fuzzy ideals in ternary semirings. In [11], we introduce the notion of (λ, μ)-Fuzzy ideals in ternary semirings. In this paper we introduce the notion of (λ, μ)-Fuzzy quasi ideals and (λ, μ)-Fuzzy bi-ideals in ternary semirings which can be regarded as the generalization of fuzzy quasi ideals and fuzzy bi-ideals in ternary semirings.

2 Preliminaries

Definition 2.1. *A nonempty set S together with two associative binary operations called addition and multiplication (denoted by $+$ and $\cdot$ respectively) is called a semiring if $(S, +)$ is a commutative semigroup, $(S, \cdot)$ is a semigroup and multiplicative distributes over addition both from the left and the right, i.e., $a(b + c) = ab + ac$ and $(a + b)c = ac + bc$ for all $a, b, c \in S$.*

Definition 2.2. *A nonempty set S together with a binary operation called, addition $+$ and a ternary multiplication, denoted by juxtaposition, is said to be a ternary semiring if $(S, +)$ is a commutative semigroup satisfying the following conditions:*
(i) $(abc)de = a(bcd)e = ab(cde)$,

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(ii) $(a+b)cd = acd + bcd$,
(iii) $a(b+c)d = abd + acd$
and (iv) $ab(c+d) = abc + abd$ *for all* $a, b, c, d, e \in S$.

We can see that any semiring can be reduced to a ternary semiring. However, a ternary semiring does not necessarily reduce to a semiring by this example. We consider Z_0^-, the set of all non-positive integers under usual addition and multiplication, we see that Z_0^- is an additive semigroup which is closed under the triple multiplication but is not closed under the binary multiplication. Moreover, Z_0^- is a ternary semiring but is not a semiring under usual addition and multiplication.
Throughout this paper S denotes a ternary semiring with zero.

Definition 2.3. *Let* S *be a ternary semiring. If there exists an element* $0 \in S$ *such that* $0 + x = x = x + 0$ *and* $0xy = x0y = xy0 = 0$ *for all* $x, y \in S$*, then* 0 *is called the zero element or simply the zero of the ternary semiring* S*. In this case we say that* S *is a ternary semiring with zero.*

Definition 2.4. *An additive subsemigroup* T *of* S *is called a ternary subsemiring of* S *if* $t_1t_2t_3 \in T$ *for all* $t_1, t_2, t_3 \in T$.

Definition 2.5. *An additive subsemigroup* I *of* S *is called a left [resp. right, lateral] ideal of* S *if* $s_1s_2i \in I$ *[resp.* $is_1s_2 \in I, s_1is_2 \in I$*] for all* $s_1, s_2 \in S$ *and* $i \in I$*. If* I *is a left, right and lateral ideal of* S*, then* I *is called an ideal of* S*.*

Definition 2.6. *An additive subsemigroup* $(Q,+)$ *of a ternary semiring* S *is called a quasi-ideal of* S *if* $QSS \cap (SQS + SSQSS) \cap SSQ \subseteq Q$.

Definition 2.7. *An additive subsemigroup* $(Q,+)$ *of a ternary semiring* S *is called a bi-ideal of* S *if* $QSQSQ \subseteq Q$.

It is obvious that every ideal of a ternary semiring with zero contains the zero element.

Definition 2.8. *Let* X *be a non-empty set. A map* $A : X \to [0,1]$ *is called a fuzzy set in* X*.*

Definition 2.9. *Let* A*,* B *and* C *be any three fuzzy subsets of a ternary semiring* S*. Then* $A\cap B$*,* $A\cup B$*,* $A+B$*,* $A\cdot B\cdot C$ *are fuzzy subsets of* S *defined by*

$$(A\cap B)(x) = min\{A(x),\ B(x)\}$$
$$(A\cup B)(x) = max\{A(x),\ B(x)\}$$

$$(A+B)(x) = \begin{cases} sup\{min\{A(y), B(z)\}\} & if \quad x = y+z \\ 0 & otherwise \end{cases}$$

$$(A\cdot B\cdot C)(x) = \begin{cases} sup\{min\{A(u), B(v), C(w)\}\} & if \quad x = uvw, \\ 0 & otherwise \end{cases}$$

Definition 2.10. *Let* X *be a nonempty set and let* A *be a fuzzy subset of* X*. Let* $0 \le t \le 1$*. Then the set* $A_t = \{x \in X \ / \ A(x) \ge t\}$ *is called a level set of* X *with respect to* A*.*

Definition 2.11. *Let* A *be a fuzzy set of a ternary semiring* S*. Then* A *is called a fuzzy ternary subsemiring of* S *if*
1. $A(x+y) \ge min\{A(x), A(y)\}$
2. $A(xyz) \ge min\{A(x), A(y), A(z)\}$ *for all* $x, y, z \in S$.

Definition 2.12. *A fuzzy set* A *of a ternary semiring* S *is called a fuzzy ideal of* S *if*
(i) $A(x+y) \ge min\{A(x), A(y)\}$
(ii) $A(xyz) \ge A(x)$
(iii) $A(xyz) \ge A(z)$ *and*
(iv) $A(xyz) \ge A(y)$ *for all* $x, y, z \in S$.
A fuzzy subset A *with conditions (i) and (ii) is called a fuzzy right ideal of* S*. If* A *satisfies (i)and(iii), then it is called a fuzzy left ideal of* S*. Also if* A *satisfies (i)and(iv), then it is called a fuzzy lateral ideal of* S*. It is clear that* A *is a fuzzy ideal of a ternary semiring* S *if and only if* $A(xyz) \ge max\{A(x), A(y), A(z)\}$ *for all* $x, y, z \in S$*.*

Definition 2.13. *A fuzzy ternary subsemiring A of a ternary semiring S is called a fuzzy quasi-ideal of S if for all $x \in S$,*
$A(x) \geq \{(A \cdot \boldsymbol{S} \cdot \boldsymbol{S}) \cap (\boldsymbol{S} \cdot A \cdot \boldsymbol{S} + \boldsymbol{S} \cdot \boldsymbol{S} \cdot A \cdot \boldsymbol{S} \cdot \boldsymbol{S}) \cap (\boldsymbol{S} \cdot \boldsymbol{S} \cdot A)\}(x)$.

Definition 2.14. *A fuzzy ternary subsemiring A of S is called a fuzzy bi ideal of S if for all $x \in S$, $A(x) \geq (A \cdot \boldsymbol{S} \cdot A \cdot \boldsymbol{S} \cdot A)(x)$.*

Lemma 2.1. *For any non-empty subsets A, B and C of S,*
(1) $f_A \cdot f_B \cdot f_C = f_{ABC}$
(2) $f_A \cap f_B \cap f_C = f_{A \cap B \cap C}$
(3) $f_A + f_B = f_{A+B}$.

Lemma 2.2. *[10] Let Q be an additive subsemigroup of S.*
(1) Q is a quasi-ideal of S if and only if f_Q is a fuzzy quasi-ideal of S.
(2) Q is a bi-ideal of S if and only if f_Q is a fuzzy bi-ideal of S.

Theorem 2.1. *[9] Let A be a fuzzy subset of S. Then A is a fuzzy quasi-ideal of S, if and only if A_t is a quasi-ideal of S, for all $t \in Im(A)$.*

Theorem 2.2. *[10] Let A be a fuzzy subset of S. Then A is a fuzzy bi-ideal of S, if and only if A_t is a bi-ideal of S, for all $t \in Im(A)$.*

3 (λ, μ)- Fuzzy quasi ideals

Based on the concept of (λ, μ)-fuzzy subrings and (λ, μ)-fuzzy ideals introduced by B.Yao [15], we introduce the following concepts which are the generalization of fuzzy sets. Throughout this paper λ and μ $(0 \leq \lambda < \mu \leq 1)$, are arbitrary, but fixed. In this section we introduce the notion of (λ, μ)-fuzzy quasi ideals in ternary semirings.

Definition 3.1. *[11] Let A be a fuzzy set of S. Then A is called a (λ, μ)-fuzzy ternary subsemiring of S if*
1. $A(x+y) \vee \lambda \geq min\{A(x), A(y), \mu\}$
2. $A(xyz) \vee \lambda \geq min\{A(x), A(y), A(z), \mu\}$ for all $x, y, z \in S$.

Definition 3.2. *[11] Let A be a fuzzy set of a ternary semiring S. A is called a (λ, μ)-fuzzy right(resp. left, lateral) ideal of S if*
1. $A(x+y) \vee \lambda \geq min\{A(x), A(y), \mu\}$
2. $A(xyz) \vee \lambda \geq min\{A(x), \mu\}$ [resp. $A(xyz) \vee \lambda \geq min\{A(z), \mu\}$, $A(xyz) \vee \lambda \geq min\{A(y), \mu\}$] for all $x, y, z \in S$.

Definition 3.3. *A (λ, μ)-fuzzy ternary subsemiring A of S is called a (λ, μ)-fuzzy quasi-ideal of S if for all $x \in S$,*
$A(x) \vee \lambda \geq min\{[(A \cdot \boldsymbol{S} \cdot \boldsymbol{S}) \cap (\boldsymbol{S} \cdot A \cdot \boldsymbol{S} + \boldsymbol{S} \cdot \boldsymbol{S} \cdot A \cdot \boldsymbol{S} \cdot \boldsymbol{S}) \cap (\boldsymbol{S} \cdot \boldsymbol{S} \cdot A)](x), \mu\}$.

Remark 3.1. *Every fuzzy quasi ideal is a (λ, μ)-fuzzy quasi ideal of S by taking $\lambda = 0$ and $\mu = 1$. But the converse need not be true.*

Example 3.1. *Consider the set of integer modulo 6, non-positive integer $Z_6^- = \{0, -1, -2, -3, -4, -5\}$ with the usual addition and ternary multiplication,*
we have

+	0	-1	-2	-3	-4	-5
0	0	-1	-2	-3	-4	-5
-1	-1	-2	-3	-4	-5	0
-2	-2	-3	-4	-5	0	-1
-3	-3	-4	-5	0	-1	-2
-4	-4	-5	0	-1	-2	-3
-5	-5	0	-1	-2	-3	-4

$\cdot$	0	-1	-2	-3	-4	-5
0	0	0	0	0	0	0
-1	0	1	2	3	4	5
-2	0	2	4	0	2	4
-3	0	3	0	3	0	3
-4	0	4	2	0	4	2
-5	0	5	4	3	2	1

$\cdot$	0	1	2	3	4	5
0	0	0	0	0	0	0
-1	0	-1	-2	-3	-4	-5
-2	0	-2	-4	0	-2	-4
-3	0	-3	0	-3	0	-3
-4	0	-4	-2	0	-4	-2
-5	0	-5	-4	-3	-2	-1

Then $(Z_6^-,+,\cdot)$ is a ternary semiring. Let a fuzzy subset $A : Z_6^- \to [0,1]$ be defined by

$$A(x) = \begin{cases} 0.8 & if \quad x = 0 \\ 0.9 & if \quad x = -3 \\ 0.2 & \quad otherwise \end{cases}$$

Clearly A is a $(0.3, 0.8)$- fuzzy quasi ideal of S. But A is not a fuzzy quasi ideal of S, since $A_{0.9}$ is not a quasi ideal.

Lemma 3.3. *Let A and B be any two (λ,μ)-fuzzy ternary subsemirings of S. Then $A\cap B$ is a (λ,μ)-fuzzy ternary subsemiring of S.*

Proof. Let $x, y, z \in S$.
i) $(A\cap B)(x+y)\vee\lambda = min\{A(x+y), B(x+y)\}\vee\lambda$
$\geq min\{A(x+y)\vee\lambda, B(x+y)\vee\lambda\}$
$\geq min\{min\{A(x),A(y),\mu\}, min\{B(x),B(y),\mu\}\}$
$= min\{min\{A(x),B(x)\}, min\{A(y),B(y)\},\mu\}$
$= min\{(A\cap B)(x),(A\cap B)(y),\mu\}$.
ii)$(A\cap B)(xyz)\vee\lambda = min\{A(xyz),B(xyz)\}\vee\lambda$
$\geq min\{A(xyz)\vee\lambda, B(xyz)\vee\lambda\}$
$\geq min\{min\{A(x),A(y),A(z),\mu\}, min\{B(x),B(y),B(z),\mu\}\}$
$\geq min\{min\{A(x),B(x)\}, min\{A(y),B(y)\}, min\{A(z),B(z)\},\mu\}$
$= min\{(A\cap B)(x),(A\cap B)(y),(A\cap B)(z),\mu\}$.
Thus $A\cap B$ is a (λ,μ)-fuzzy ternary subsemiring of S. □

Lemma 3.4. *Let Q be any nonempty subset of S. Then Q is a quasi-ideal in S if and only if f_Q is a (λ,μ)-fuzzy quasi-ideal of S.*

Proof. Let Q be a quasi-ideal in S. By Lemma 2.2, f_Q is a fuzzy quasi-ideal of S and by Remark 3.1, f_Q is a (λ,μ)-fuzzy quasi-ideal of S. Conversely, let f_Q be a (λ,μ)-fuzzy quasi-ideal of S with $0\leq\lambda<\mu\leq 1$. Let $x\in QSS\cap(SQS+SSQSS)\cap SSQ$. Then we have
$f_Q(x)\vee\lambda \geq min\{(f_Q\cdot f_S\cdot f_S)(x),(f_S\cdot f_Q\cdot f_S+f_S\cdot f_S\cdot f_Q\cdot f_S\cdot f_S)(x),(f_S\cdot f_S\cdot f_Q)(x),\mu\}$
$= min\{f_{(QSS\cap(SQS+SSQSS)\cap SSQ)}(x),\mu\} = min\{1,\mu\} = \mu$.
Thus $f_Q(x)\vee\lambda\geq\mu>\lambda$ and hence $f_Q(x)\geq\mu$.
This implies that $x\in Q$ and so $QSS\cap(SQS+SSQSS)\cap SSQ\subseteq Q$. This means that Q is a quasi-ideal of S. □

Theorem 3.3. *Let A be a fuzzy set of a ternary semiring S. If A is a (λ,μ)-fuzzy lateral(right, left) ideal of S then A is a (λ,μ)-fuzzy quasi-ideal of S.*

Proof. Let A be a (λ,μ)-fuzzy lateral ideal of S. Let $x\in S$.
Suppose $x = as_1s_2 = s_3(b+s_4cs_5)s_6 = s_7s_8d$ where $a,b,c,d,s_1,s_2,s_3,s_4,s_5,s_6,s_7,s_8\in S$.
We have $\{A\cdot\mathbf{S}\cdot\mathbf{S}\cap(\mathbf{S}\cdot A\cdot\mathbf{S}+\mathbf{S}\cdot\mathbf{S}\cdot A\cdot\mathbf{S}\cdot\mathbf{S})\cap\mathbf{S}\cdot\mathbf{S}\cdot A\}(x)$
$= min\{A\cdot\mathbf{S}\cdot\mathbf{S}(x),(\mathbf{S}\cdot A\cdot\mathbf{S}+\mathbf{S}\cdot\mathbf{S}\cdot A\cdot\mathbf{S}\cdot\mathbf{S})(x),\mathbf{S}\cdot\mathbf{S}\cdot A(x)\}$
$= min\{\sup_{x=as_1s_2}\{A(a)\}, \sup_{x=s_3(b+s_4cs_5)s_6}\{min\{A(b),A(c)\}\}, \sup_{x=s_7s_8d}\{A(d)\}\}$
Now since A is a (λ,μ)-fuzzy lateral ideal,
$A(s_3(b+s_4cs_5)s_6)\vee\lambda\geq min\{A(b+s_4cs_5),\mu\}\geq min\{A(b),A(c),\mu\}$.
We have $min\{(A\cdot\mathbf{S}\cdot\mathbf{S}\cap(\mathbf{S}\cdot A\cdot\mathbf{S}+\mathbf{S}\cdot\mathbf{S}\cdot A\cdot\mathbf{S}\cdot\mathbf{S})\cap\mathbf{S}\cdot\mathbf{S}\cdot A)(x),\mu\}$

$= min\{min\{supA(a), sup\{min\{A(b), A(c)\}\}, supA(d)\}, \mu\}$
$\leq min\{min\{1, sup\{min\{A(b), A(c)\}\}, 1\}, \mu\}$
$= min\{sup\{min\{A(b), A(c)\}\}, \mu\}$
$\leq \sup_{x=s_3(b+s_4cs_5)s_6} \{min\{A(b), A(c), \mu\}\}$
$\leq \sup A(s_3(b + s_4cs_5)s_6) \vee \lambda = A(x) \vee \lambda.$
We remark that if x is not expressed as $x = as_1s_2 = s_3(b + s_4cs_5)s_6 = s_7s_8d$ then $(A \cdot \mathbf{S} \cdot \mathbf{S} \cap (\mathbf{S} \cdot A \cdot \mathbf{S} + \mathbf{S} \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot \mathbf{S}) \cap \mathbf{S} \cdot \mathbf{S} \cdot A)(x) = 0 \leq A(x) \vee \lambda$.
Thus $A(x) \vee \lambda \geq min\{(A \cdot \mathbf{S} \cdot \mathbf{S} \cap (\mathbf{S} \cdot A \cdot \mathbf{S} + \mathbf{S} \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot \mathbf{S}) \cap \mathbf{S} \cdot \mathbf{S} \cdot A)(x), \mu\}$ for all $x \in S$.
Hence A is a (λ, μ)-fuzzy quasi-ideal of S. □

Theorem 3.4. *Let A be a fuzzy set of a ternary semiring S. A is a (λ, μ)-fuzzy quasi-ideal of S if and only if A_t is a quasi-ideal in S for all $t \in (\lambda, \mu]$ whenever nonempty.*

Proof. Let A be a (λ, μ)-fuzzy quasi-ideal of S. Let $x, y \in S$. Suppose $x, y \in A_t$, $t \in (\lambda, \mu]$ then $A(x) \geq t, A(y) \geq t$ and $\mu \geq t$. This implies that $min\{A(x), A(y), \mu\} \geq t$. Since A is (λ, μ)-fuzzy quasi-ideal, $A(x + y) \vee \lambda \geq min\{A(x), A(y), \mu\} \geq t > \lambda$. This implies that $A(x + y) \geq t$. Hence $x + y \in A_t$. Next, let $x \in A_tSS \cap (SA_tS + SSA_tSS) \cap SSA_t$. Then there exist $a, b, c, d \in A_t$ and $s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8 \in S$ such that $x = as_1s_2 = s_3(b + s_4cs_5)s_6 = s_7s_8d$. Thus $A(a) \geq t, A(b) \geq t, A(c) \geq t, A(d) \geq t$.
Then $(A \cdot \mathbf{S} \cdot \mathbf{S} \cap (\mathbf{S} \cdot A \cdot \mathbf{S} + \mathbf{S} \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot \mathbf{S}) \cap \mathbf{S} \cdot \mathbf{S} \cdot A)(x)$
$= min\{A \cdot \mathbf{S} \cdot \mathbf{S}(x), (\mathbf{S} \cdot A \cdot \mathbf{S} + \mathbf{S} \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot \mathbf{S})(x), \mathbf{S} \cdot \mathbf{S} \cdot A(x)\}$
$= min\{\sup_{x=as_1s_2} \{A(a)\}, \sup_{x=s_3(b+s_4cs_5)s_6} \{min\{A(b), A(c)\}\}, \sup_{x=s_7s_8d} \{A(d)\}\} \geq t.$
Now $min\{(A \cdot \mathbf{S} \cdot \mathbf{S} \cap (\mathbf{S} \cdot A \cdot \mathbf{S} + \mathbf{S} \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot \mathbf{S}) \cap \mathbf{S} \cdot \mathbf{S} \cdot A)(x), \mu\} \geq min\{t, \mu\} = t$.
Since A is a (λ, μ)-fuzzy quasi-ideal of S, $A(x) \vee \lambda \geq t > \lambda$. Then $A(x) \geq t$ and $x \in A_t$ and hence A_t is a quasi-ideal in S.
Conversely let us assume that A_t, $t \in (\lambda, \mu]$ is a quasi-ideal in S. By Theorem:2.1, A is a fuzzy quasi-ideal of S. By remark:3.1, A is a (λ, μ)-fuzzy quasi-ideal of S. □

Theorem 3.5. *Let A and B be any two (λ, μ)-fuzzy quasi-ideals of S. Then $A \cap B$ is also a (λ, μ)-fuzzy quasi-ideal of S.*

Proof. Let A and B be (λ, μ)-fuzzy quasi-ideals of S. By Lemma:3.3, $A \cap B$ is a (λ, μ)-fuzzy ternary subsemiring of S. Let $x \in S$.
Suppose $x = as_1s_2 = s_3(b + s_4cs_5)s_6 = s_7s_8d$ where $a, b, c, d, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8 \in S$. Since A and B are (λ, μ)-fuzzy quasi-ideals of S, we have
$A(x) \vee \lambda \geq min\{\sup\{A(a), min\{A(b), A(c)\}, A(d)\}, \mu\}$ and
$B(x) \vee \lambda \geq min\{\sup\{B(a), min\{B(b), B(c)\}, B(d)\}, \mu\}$.
Consider $min\{[((A \cap B) \cdot \mathbf{S} \cdot \mathbf{S}) \cap (\mathbf{S} \cdot (A \cap B) \cdot \mathbf{S} + \mathbf{S} \cdot \mathbf{S} \cdot (A \cap B) \cdot \mathbf{S} \cdot \mathbf{S}) \cap (\mathbf{S} \cdot \mathbf{S} \cdot (A \cap B))](x), \mu\}$
$= min\{min\{((A \cap B) \cdot \mathbf{S} \cdot \mathbf{S})(x), (\mathbf{S} \cdot (A \cap B) \cdot \mathbf{S} + \mathbf{S} \cdot \mathbf{S} \cdot (A \cap B) \cdot \mathbf{S} \cdot \mathbf{S})(x), (\mathbf{S} \cdot \mathbf{S} \cdot (A \cap B))(x)\}, \mu\}$
$= min\{min\{\sup_{x=as_1s_2} \{(A \cap B)(a)\},$
$\sup_{x=s_3(b+s_4cs_5)s_6} \{min\{(A \cap B)(b), (A \cap B)(c)\}\}, \sup_{x=s_7s_8d} \{(A \cap B)(d)\}\}, \mu\}$
$= min\{min\{\sup_{x=as_1s_2} \{min\{A(a), B(a)\}\},$
$\sup_{x=s_3(b+s_4cs_5)s_6} \{min\{min\{A(b), B(b)\}, min\{A(c), B(c)\}\}\}, \sup_{x=s_7s_8d} \{min\{A(d), B(d)\}\}\}, \mu\}$
$= min\{min\{\sup\{min\{A(a), B(a)\}\},$
$\sup\{min\{min\{A(b), A(c)\}, min\{B(b), B(c)\}\}\}, \sup\{min\{A(d), B(d)\}\}\}, \mu\}$
$\leq min\{min\{min\{\sup\{A(a), B(a)\}\},$
$min\{\sup\{min\{A(b), A(c)\}, min\{B(b), B(c)\}\}\}, min\{\sup\{A(d), B(d)\}\}\}, \mu\}$
$= min\{min\{min\{\sup\{A(a), min\{A(b), A(c)\}, A(d)\}\},$
$min\{\sup\{B(a), min\{B(b), B(c)\}, B(d)\}\}\}, \mu\}$
$= min\{min\{\sup\{A(a), min\{A(b), A(c)\}, A(d)\}, \mu\},$
$min\{\sup\{B(a), min\{B(b), B(c)\}, B(d)\}, \mu\}\}$
$\leq min\{[A(x) \vee \lambda], [B(x) \vee \lambda]\} \leq min\{A(x), B(x)\} \vee \lambda = (A \cap B)(x) \vee \lambda$. Thus $A \cap B$ is a (λ, μ)-fuzzy quasi-ideal of S. □

4 (λ,μ)- Fuzzy Bi ideals

Definition 4.1. *A (λ,μ)-fuzzy ternary subsemiring A of S is called a (λ,μ)-fuzzy bi ideal of S if for all $x \in S$, $A(x) \vee \lambda \geq min\{(A \cdot \boldsymbol{S} \cdot A \cdot \boldsymbol{S} \cdot A)(x), \mu\}$.*

Remark 4.1. *Every fuzzy bi ideal is a (λ,μ)-fuzzy bi ideal of S by taking $\lambda = 0$ and $\mu = 1$. But the converse need not be true.*

Example 4.1. *Consider the ternary semiring $(Z_6^-, +, \cdot)$ as defined in Example 3.1. Let a fuzzy subset $A : Z_6^- \to [0,1]$ be defined by $A(0) = 0.9$, $A(-1) = 0$, $A(-2) = 0.9$, $A(-3) = 0$, $A(-4) = 0.8$ and $A(-5) = 0$. Clearly A is a $(0.3, 0.8)$- fuzzy bi-ideal. But A is not a fuzzy bi-ideal, since $A_{0.9}$ is not a fuzzy bi-ideal.*

Lemma 4.1. *Let Q be any nonempty subset of S. Then Q is a bi-ideal in S if and only if f_Q is a (λ,μ)-fuzzy bi-ideal of S.*

Proof. Let Q be a bi-ideal of S. By Lemma 2.2, f_Q is a fuzzy bi-ideal of S and by Remark 4.1, f_Q is a (λ,μ)-fuzzy bi-ideal of S. Conversely, let f_Q be a (λ,μ)-fuzzy bi-ideal of S. Let $x \in QSQSQ$. Then we have $f_Q(x) \vee \lambda \geq min\{(f_Q \cdot f_S \cdot f_Q \cdot f_S \cdot f_Q)(x), \mu\} = min\{f_{(QSQSQ)}(x), \mu\} = min\{1, \mu\} = \mu$. Thus $f_Q(x) \vee \lambda \geq \mu$. Hence $x \in Q$ and so $QSQSQ \subseteq Q$. This means that Q is a bi-ideal of S. □

Lemma 4.2. *Any (λ,μ)-fuzzy quasi-ideal of S is a (λ,μ)-fuzzy bi-ideal of S.*

Proof. Let A be a (λ,μ)-fuzzy quasi-ideal of S. Then we have
$A \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot A \subseteq A \cdot (\mathbf{S} \cdot \mathbf{S} \cdot \mathbf{S}) \cdot \mathbf{S} \subseteq A \cdot \mathbf{S} \cdot \mathbf{S}$,
$A \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot A \subseteq \mathbf{S} \cdot (\mathbf{S} \cdot \mathbf{S} \cdot \mathbf{S}) \cdot A \subseteq \mathbf{S} \cdot \mathbf{S} \cdot A$,
$A \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot A \subseteq \mathbf{S} \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot \mathbf{S}$ and taking $\{0\} \subseteq \mathbf{S} \cdot A \cdot \mathbf{S}$.
So $A \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot A \subseteq (\mathbf{S} \cdot A \cdot \mathbf{S} + \mathbf{S} \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot \mathbf{S})$.
Hence we have $A \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot A \subseteq A \cdot \mathbf{S} \cdot \mathbf{S} \cap (\mathbf{S} \cdot A \cdot \mathbf{S} + \mathbf{S} \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot \mathbf{S}) \cap \mathbf{S} \cdot \mathbf{S} \cdot A \subseteq A$.
Let $x \in S$. Now $min\{(A \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot A)(x), \mu\} \leq min\{(A \cdot \mathbf{S} \cdot \mathbf{S} \cap (\mathbf{S} \cdot A \cdot \mathbf{S} + \mathbf{S} \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot \mathbf{S}) \cap \mathbf{S} \cdot \mathbf{S} \cdot A)(x), \mu\} \leq A(x) \vee \lambda$.
It follows that A is a (λ,μ)-fuzzy bi-ideal of S. □

Theorem 4.1. *Let A be a fuzzy set of a ternary semiring S. If A is a (λ,μ)-fuzzy lateral(right, left) ideal of S then A is a (λ,μ)-fuzzy bi-ideal of S.*

Proof. By Theorem:3.3 , every (λ,μ)-fuzzy lateral ideal of S is a (λ,μ)-fuzzy quasi ideal of S and by Lemma:4.2 , it is a (λ,μ)-fuzzy bi-ideal of S. □

Theorem 4.2. *Let A be a fuzzy set of a ternary semiring S. A is a (λ,μ)-fuzzy bi-ideal of S if and only if A_t is a bi-ideal in S for all $t \in (\lambda,\mu]$ whenever nonempty.*

Proof. Let A be a (λ,μ)-fuzzy bi-ideal of S. Let $x, y \in S$. Suppose $x, y \in A_t$, $t \in (\lambda,\mu]$. Then $A(x) \geq t, A(y) \geq t$ and $\mu \geq t$ and hence $min\{A(x), A(y), \mu\} \geq t$. Since A is a (λ,μ)-fuzzy bi-ideal, $A(x+y) \vee \lambda \geq min\{A(x), A(y), \mu\} \geq t$. This implies that $A(x+y) \geq t$. Thus $x + y \in A_t$. Let $u \in S$. Suppose $u \in A_tSA_tSA_t$ then there exist $x, y, z \in A_t$ and $s_1, s_2 \in S$ such that $u = xs_1ys_2z$. Thus $A(x) \geq t, A(y) \geq t, A(z) \geq t$. Now $(A \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot A)(u) = sup\{min\{A(x), A(y), A(z)\}\} \geq t$. Since A is a (λ,μ)-fuzzy bi-ideal of S, $A(u) \vee \lambda \geq min\{(A \cdot \mathbf{S} \cdot A \cdot \mathbf{S} \cdot A)(u), \mu\} \geq min\{t, \mu\} = t$. Thus $A(u) \geq t$ and $u \in A_t$. Hence A_t is a bi-ideal in S.
Conversely let us assume that A_t, $t \in (\lambda,\mu]$ is a bi-ideal in S. By Theorem:2.2, A is a fuzzy bi-ideal of S. By Remark:4.1, A is a (λ,μ)-fuzzy bi-ideal of S. □

Theorem 4.3. *Let A and B be any two (λ,μ)-fuzzy bi-ideals of S. Then $A \cap B$ is also a (λ,μ)-fuzzy bi-ideal of S.*

Proof. Let A and B be (λ,μ)-fuzzy bi-ideals of S. By Lemma:3.3, $A \cap B$ is a (λ,μ)-fuzzy ternary subsemiring of S. Let $a \in S$ and $s_1, s_2, x, y, z \in S$ such that $a = xs_1ys_2z$. Since A and B are (λ,μ)-fuzzy bi-ideals of S, we have $A(a) \vee \lambda \geq min\{\sup\limits_{a=xs_1ys_2z}\{A(x), A(y), A(z)\}, \mu\}$ and
$B(a) \vee \lambda \geq min\{\sup\limits_{a=xs_1ys_2z}\{B(x), B(y), B(z)\}, \mu\}$.
Consider $min\{((A \cap B) \cdot \mathbf{S} \cdot (A \cap B) \cdot \mathbf{S} \cdot (A \cap B))(a), \mu\}$

$$= min\{\sup_{a=xs_1ys_2z}\{min\{(A\cap B)(x), \mathbf{S}(s_1), (A\cap B)(y), \mathbf{S}(s_2), (A\cap B)(z)\}\}, \mu\}$$
$$= min\{\sup_{a=xs_1ys_2z}\{min\{(A\cap B)(x), (A\cap B)(y), (A\cap B)(z)\}\}, \mu\}$$
$$\leq min\{min\{\sup_{a=xs_1ys_2z}\{min\{A(x), B(x)\}, min\{A(y), B(y)\}, min\{A(z), B(z)\}\}\}, \mu\}$$
$$\leq min\{min\{min\{\sup_{a=xs_1ys_2z}\{A(x), A(y), A(z)\}, \sup_{a=xs_1ys_2z}\{B(x), B(y), B(z)\}\}\}, \mu\}$$
$$= min\{min\{\sup_{a=xs_1ys_2z}\{A(x), A(y), A(z)\}\}, \mu\}, min\{\sup_{a=xs_1ys_2z}\{B(x), B(y), B(z)\}\}, \mu\}\}$$
$$\leq min\{\{A(a)\vee\lambda\}, \{B(a)\vee\lambda\}\}$$
$\leq min\{A(a), B(a)\}\vee\lambda = (A\cap B)(a)\vee\lambda$. Thus $A\cap B$ is a (λ, μ)-fuzzy bi-ideal of S. □

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