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Note on nonparametric M-estimation for spatial regression

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Abstract

In this paper, we investigate a nonparametric robust estimation for spatial regression. More precisely, given a strictly stationary random field $Z_{\mathbf{i}} = (X_{\mathbf{i}}, Y_{\mathbf{i}})$, $\mathbf{i} \in \mathbb{N}^N$, we consider a family of robust nonparametric estimators for a regression function based on the kernel method. We establish a p-mean consistency results of the kernel estimator under some conditions.

Keywords: Quadratic error, p-mean consistency, Nonparametric regression, Spatial process, Robust estimation.

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1 Introduction

In the last years spatial statistics has been widely applied in diverse areas such as climatology, epidemiology, agronomy, meteorology, econometrics, image processing, etc. There is a vast literature on spatial models, see, for example, the books by Cressie (1991), Guyon (1995), Anselin and Florax (1995), Banerjee, Carlin and Gelfand (2004), Gelfand et al. (2010) and Cressie and Wikle (2011) for broad discussion and applications. However, the nonparametric treatment of such data has so for being limited. The first results were obtained by Tran (1990). For relevant works on the nonparametric modelization of spatial data, see Biau and Cadre (2004), Carbon et al. (2007), Li et al. (2009). In this paper, we consider the problem of the estimation of the regression function as the analysis tool of such kind of data. Noting that, this model is very interesting in practice. It is used as an alternative approach to classical methods, in particular when the data are affected by the presence of outliers. There is an extensive literature on robust estimation (see, for instance Huber (1964), Robinson (1984), Collomb and Härdle (1986), Fan et al. (1994) for previous results and Boente et al. (2009) for recent advances and references). The first results concerning the nonparametric robust estimation in functional statistic were obtained by Azzedine et al. (2008). They studied the almost complete convergence of robust estimators based on a kernel method, considering independent observations. Crambes et al. (2008) stated the convergence in L_p norm in both cases (i.i.d and strong mixing). While the asymptotic normality of these estimators is proved by Attouch et al. (2010). The main goal of this paper is to study the robust nonparametric, we study L_p mean consistency results of a nonparametric estimation of the spatial regression by using the robust approach.

The paper is organized as follows. We present our model and estimator in Section 2. Section 3 is devoted to assumptions. The p-mean consistency of the robust nonparametric estimators is stated in Section 4. Proofs are provided in the appendix.

2 The model

Consider $Z_{\mathbf{i}} = (X_{\mathbf{i}}, Y_{\mathbf{i}})$, $\mathbf{i} \in \mathbb{N}^N$ be a $\mathbb{R}^d \times \mathbb{R}$-valued measurable and strictly stationary spatial process, defined on a probability space $(\Omega, \mathcal{A},)$. We assume that the process under study $(Z_{\mathbf{i}})$ is observed over a

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rectangular domain $I_{\mathbf{n}} = \{\mathbf{i} = (i_1, ..., i_N) \in \mathbb{N}^N,\ 1 \le i_k \le n_k, k = 1, ..., N\}$, $\mathbf{n} = (n_1, ..., n_N) \in \mathbb{N}^N$. A point $\mathbf{i}$ will be referred to as a *site*. We will write $\mathbf{n} \to \infty$ if $\min\{n_k\} \to \infty$ and $|\frac{n_j}{n_k}| < C$ for a constant C such that $0 < C < \infty$ for all j, k such that $1 \le j, k \le N$. For $\mathbf{n} = (n_1, ..., n_N) \in \mathbb{N}^N$, we set $\hat{\mathbf{n}} = n_1 \times ... \times n_N$. The nonparametric model studied in this paper, denoted by θ_x, is implicitly defined, for all vectors $x \in \mathbb{R}^d$, as a zero with respect to (w.r.t.) $t \in \mathbb{R}$ of the equation

$$\Psi(x,t) = [\psi_x(Y_{\mathbf{i}},t)|X_{\mathbf{i}} = x] = 0$$

where ψ_x is a real-valued integrable function satisfying some regularity conditions to be stated below. In what follows, we suppose that, for all $x \in \mathbb{R}^d$, θ_x exists and is unique (see, for instance, Boente and Fraiman (1989)).

For all $(x,t) \in \mathbb{R}^{d+1}$, we propose a nonparametric estimator of $\Psi(x,t)$ given by

$$\hat{\Psi}(x,t) = \frac{\sum_{\mathbf{i}\in\mathcal{I}_{\mathbf{n}}} K(h^{-1}(x - X_{\mathbf{i}}))\psi_x(Y_{\mathbf{i}},t)}{\sum_{\mathbf{i}\in\mathcal{I}_{\mathbf{n}}} K(h^{-1}(x - X_{\mathbf{i}}))},$$

with the convention $\frac{0}{0} = 0$, where K is a kernel and $h = h_n$ is a sequence of positive real numbers. A natural estimator $\hat{\theta}_x$ of θ_x is a zero w.r.t. t of the equation

$$\hat{\Psi}(x,t) = 0.$$

In this work, we will assume that the random filed $(Z_{\mathbf{i}}, \mathbf{i} \in \mathbb{N}^N)$ satisfies the following mixing condition:

$$\begin{cases} \text{There exists a function } \varphi(t) \downarrow 0 \text{ quand } t \to \infty, \text{ such that} \\ \quad \forall E,\ E' \text{ subsets of } \mathbb{N}^N \text{with finite cardinals} \\ \alpha\left(\mathcal{B}(E), \mathcal{B}\left(E'\right)\right) = \sup_{B\in\mathcal{B}(E), C\in\mathcal{B}\left(E'\right)} |(B\cap C) - (B)\ (C)| \\ \quad \le s\left(\text{Card}(E), \text{Card}\left(E'\right)\right)\varphi\left(\text{dist}\left(E,E'\right)\right), \end{cases} \tag{2.1}$$

where $\mathcal{B}(E)$(*resp.* $\mathcal{B}\left(E'\right)$) denotes the Borel σ-field generated by $(Z_{\mathbf{i}},\ \mathbf{i} \in E)$ (*resp.* $\left(Z_{\mathbf{i}},\ \mathbf{i} \in E'\right)$), Card$(E)$ (*resp.* Card$\left(E'\right)$) the cardinality of E (*resp.* E'), dist$\left(E,E'\right)$ the Euclidean distance between E and E' and $s : \mathbb{N}^2 \to \mathbb{R}^+$ is a symmetric positive function nondecreasing in each variable such that either

$$s(n,m) \le C \min(n,m), \quad \forall n,m \in \mathbb{N} \tag{2.2}$$

or

$$s(n,m) \le C(n+m+1)^{\widetilde{\beta}}, \quad \forall n,m \in \mathbb{N} \tag{2.3}$$

for some $\widetilde{\beta} \ge 1$ and some $C > 0$.We assume also that the process satisfies a polynomial mixing condition:

$$\varphi(t) \le Ct^{-\theta}, \quad \theta > 0, \quad t \in \mathbb{R}. \tag{2.4}$$

3 Assumptions

From now on, let x stand for a fixed point in $\mathbb{R}^d$ and we assume that the $Z_{\mathbf{i}}$'s have the same distribution with (X,Y). Moreover, we set $f(\cdot)$ to be the density of X and $h(\cdot|x)$ the conditional density of Y given $X = x$. Consider the following hypotheses.

(H1) The functions f and h such that:

(i) The density $f(.)$ has continuous derivative in the neighborhood of x with $f(x) > 0$

(ii) For all $t \in \mathbb{R}$, the function $h(t/.)$ has continuous derivative in the neighborhood of x.

(iii) The following functions, defined for $(u,t) \in \mathbb{R}^{d+1}$ by

$$\begin{cases} g(u,t) &= (\psi_x(Y,t)/X=u)f(u) \\ \lambda(u,t) &= (\psi_x^2(Y,t)/X=u)f(u) \quad \text{and} \\ \Gamma(u,t) &= (\frac{\partial \psi_x}{\partial t}(Y,t)/X=u)f(u) \end{cases} \tag{3.5}$$

have, also, continuous derivative w.r.t. the first component.

(H2) The function ψ_x is continuous, differentiable, strictly monotone bounded w.r.t. the second component and its derivative $\dfrac{\partial \psi_x(y,t)}{\partial t}$ is bounded and continuous at θ_x uniformly in y.

(H3) The joint probability density $f_{\mathbf{i},\mathbf{j}}$ of $X_{\mathbf{i}}$ and $X_{\mathbf{j}}$ exists and satisfies

$$|f_{\mathbf{i},\mathbf{j}}(u,v) - f(u)f(v)| \leq C$$

for some constant C and for all u, v,$\mathbf{i}$ and $\mathbf{j}$.

(H4) The mixing coefficient defined in (2.2) satisfies, for some $q > 2$ and some integer $r \geq 1$

$$\lim_{T\to\infty} T^a \sum_{i=T}^{\infty} t^{Nr-1}(\varphi(t))^{qr-2/qr} = 0,$$

for some $a \geq (rq-2)Nr/(2+rq-4r)$ with $q > (4r-2)/r$.

(H5) The probability kernel function K is a symmetric and bounded density function on $\mathbb{R}^d$ with compact support, C_K, and finite variance such that
$|K(x) - K(y)| \leq M\|x-y\|$ for $x, y \in C_K$ and $0 < M < \infty$.

(H6) The individual h_n satisfy

$$\lim_{n\to\infty} h = 0 \quad \text{and} \liminf_{n\to\infty} \hat{\mathbf{n}} h^{(2(r-1)a+N(qr-2))/(a+N)q} > 0.$$

4 Main results

Let $p \in [1, +\infty[$. In this section we state a pointwise p-mean consistency result for the estimator $\hat{\theta}_x$. We start with the case where $p = 2$, we give precise asymptotic evaluations of the quadratic error of this estimator.

4.1 Mean square error

Theorem 4.1. *If the assumptions (H1)-(H6) are satisfied and if* $\Gamma(x,\theta_x) \neq 0$ *then*

$$(\hat{\theta}_x - \theta_x)^2 = B^2(x,\theta_x)h^4 + \frac{A(x,\theta_x)}{\hat{\mathbf{n}}h^d} + O\left(\frac{1}{\hat{\mathbf{n}}h^d}\right)$$

where $A(x,\theta_x) = \dfrac{\lambda(x,\theta_x)}{\Gamma(x,\theta_x)} \displaystyle\int K^2(t)\mathrm{d}t$ *and* $B(x,\theta_x) = \dfrac{g^{(2)}(x,\theta_x)}{\Gamma(x,\theta_x)} \displaystyle\int t^2 K^2(t)\mathrm{d}t$

Before giving the proof, let us introduce some notation. For $y \in \mathbb{R}$, let

$$\hat{g}(x,t) = \frac{1}{\hat{\mathbf{n}}h^d} \sum_{\mathbf{i}\in\mathcal{I}_\mathbf{n}} K(h^{-1}(x - X_\mathbf{i}))\psi_x(Y_\mathbf{i},t) \quad \text{and} \quad \hat{f}(x) = \frac{1}{\hat{\mathbf{n}}h^d} \sum_{\mathbf{i}\in\mathcal{I}_\mathbf{n}} K(h^{-1}(x - X_\mathbf{i})).$$

So that if $\hat{f}(x) \neq 0$ we have

$$\hat{\Psi}(x,t) = \frac{\hat{g}(x,t)}{\hat{f}(x)}$$

Proof.
A Taylor expansion of the function $\hat{g}(x,.)$ in a neighborhood of θ_x gives:

$$\hat{g}(x,\hat{\theta}_x) = \hat{g}(x,\theta_x) + (\hat{\theta}_x - \theta_x)\frac{\partial\hat{g}}{\partial t}(x,\theta^*_{x\mathbf{n}})$$

where $\theta^*_{x\mathbf{n}}$ is between $\hat{\theta}_x$ and θ_x such that

$$\hat{g}(x,\hat{\theta}_x) = g(x,\theta_x) = 0.$$

Thus, we have under the case where $\hat{f}(x) \neq 0$

$$\hat{\theta}_x - \theta_x = \frac{-\hat{g}(x,\theta_x)}{\frac{\partial\hat{g}}{\partial t}(x,\theta^*_{x\mathbf{n}})}.$$

We have by lemma 6 of Gheriballah et al (2010),

$$\frac{\partial\hat{g}}{\partial t}(x,\theta^*_{x\mathbf{n}}) - \Gamma(x,\theta_x) \to 0 \quad \text{almost completely (a.co.)}$$

It follows that

$$(\hat{\theta}_x - \theta_x)^2 = \frac{1}{\Gamma(x,\theta_x)}[(\hat{g}(x,\theta_x))^2] + o([(\hat{g}(x,\theta_x))^2]) + (\hat{f}(x) = 0)$$

Now, Theorem 4.1 is a consequence of the following intermediate results, whose proofs are given in the Appendix.

Lemma 4.1. *Under Hypotheses (H1) and (H2)-(H4), we have,*

$$Var\left[\left(\hat{g}(x,\theta_x)\right)\right] = \frac{A(x,\theta_x)}{\hat{\mathbf{n}}h^d} + O\left(\frac{1}{\hat{\mathbf{n}}h^d}\right).$$

Lemma 4.2. *Under Hypotheses (H1) and (H2)-(H4), we have,*

$$\left[\left(\hat{g}(x,\theta_x)\right)\right] = B(x,\theta_x)h^2 + o(h^2).$$

Lemma 4.3. *Under the conditions of Theorem 4.1, we have*

$$(\hat{f}(x) = 0) = O\left(\frac{1}{\hat{\mathbf{n}}h^d}\right)^p.$$

4.2 Convergence in L_p norm

Theorem 4.2. *Under conditions (H1)-(H6) and if* $\Gamma(x,\theta_x) \neq 0$ *we get*

$$\|\hat{\theta}_x - \theta_x\|_p = O\left(h^2\right) + O\left(\frac{1}{\hat{\mathbf{n}}h^d}\right)^{1/2}$$

where $\|.\|_p$ *is the norm* L_p

Proof. We prove the case where $p = 2r$ (for all $r \in \mathbb{N}^*$) and we use the Holder inequality for lower values of p. Moreover, we use the same analytical arguments as those used in previous theorem, we have

$$\|\hat{\theta}_x - \theta_x\|_{2r} \leq C\|\hat{g}(x,\theta_x)\|_{2r} + \|o(\,[\hat{g}(x,\theta_x)])\|_{2r}. \tag{4.6}$$

Furthermore, we write

$$\|\hat{g}(x,\theta_x)\|_{2r} = \frac{1}{\hat{\mathbf{n}}h^d}\left(\left[\left(\sum_{\mathbf{i}\in\mathcal{I}_\mathbf{n}}\xi_\mathbf{i}\right)^{2r}\right]\right)^{1/2r}.$$

where $\xi_\mathbf{i} = K_\mathbf{i}\psi_x(Y_\mathbf{i},\theta_x) = K_\mathbf{i}[\psi_x(Y_\mathbf{i},\theta_x) - [\psi_x(Y_\mathbf{i},\theta_x)/X_\mathbf{i} = x]]$ with $K_\mathbf{i} = K(h^{-1}(x - X_\mathbf{i}))$.
Therefore, the first term of (4.6) is a consequence of the application of Theorem 2.2 of (Gao et al. 2008, P. 689) on $\xi_\mathbf{i}$ while the second one is given in Lemma 4.2.

5 Appendix

Proof of Lemma 4.1. Let $\Delta_{\mathbf{i}}(x) = K(h^{-1}(x - X_{\mathbf{i}}))\psi_x(Y_{\mathbf{i}}, \theta_x)$, then

$$\begin{aligned}
\operatorname{Var}\Big(\hat{g}(x,\theta_x)\Big) &= \operatorname{Var}\left(\frac{1}{\hat{\mathbf{n}}h^d}\sum_{\mathbf{i}\in\mathcal{I}_{\mathbf{n}}}\Big(K_{\mathbf{i}}\psi_x(Y_{\mathbf{i}},\theta_x)\Big)\right) \\
&= \frac{1}{\hat{\mathbf{n}}^2 h^{2d}}\operatorname{Var}\left(\sum_{\mathbf{i}\in\mathcal{I}_{\mathbf{n}}}\Delta_{\mathbf{i}}\right) \\
&= \frac{1}{\hat{\mathbf{n}}h^{2d}}\operatorname{Var}(\Delta_{\mathbf{i}}) + \frac{1}{\hat{\mathbf{n}}^2 h^{2d}}\sum_{\mathbf{i}\neq\mathbf{j}\in\mathcal{I}_{\mathbf{n}}}\left|\operatorname{Cov}(\Delta_{\mathbf{i}},\Delta_{\mathbf{j}})\right|
\end{aligned}$$

Concerning the variance term, we have

$$\operatorname{Var}(\Delta_{\mathbf{i}}) = [\Delta_{\mathbf{i}}]^2 - {}^2[\Delta_{\mathbf{i}}]$$

By the stationarity of the observations $(X_{\mathbf{i}}, Y_{\mathbf{i}})$ we get, firstly,

$$\begin{aligned}
[\Delta_{\mathbf{i}}] &= [K(h^{-1}(x - X_{\mathbf{1}}))\psi_x(Y_{\mathbf{1}},\theta_x)] \\
&= [K(h^{-1}(x - X_{\mathbf{1}}))(\psi_x(Y_{\mathbf{1}},\theta_x)|X = X_{\mathbf{1}})] \\
&= \int_{\mathbb{R}^d} K(h^{-1}(x - z))(\psi_x(Y_{\mathbf{1}},\theta_x)|X = X_{\mathbf{1}})f(z)\mathrm{d}z.
\end{aligned}$$

Next, by a classical change of variables, $u = h^{-1}(x - z)$ we write

$$[\Delta_{\mathbf{i}}] = h^d \int_{\mathbb{R}^d} K(u)g(x - hu, \theta_x)du$$

and by the Taylor expansion, under (H1), we obtain

$$[\Delta_{\mathbf{i}}] = h^d g(x,\theta_x)\int_{\mathbb{R}^d} K(u)du + o(h^d) = o(h^d) \quad \text{since} \quad g(x,\theta_x) = 0$$

Secondly, by a similar arguments, we have

$$\begin{aligned}
[\Delta_{\mathbf{i}}]^2 &= [K^2(h^{-1}(x - X_{\mathbf{1}}))\psi_x^2(Y_{\mathbf{1}},\theta_x)] \\
&= [K^2(h^{-1}(x - X_{\mathbf{1}}))(\psi_x^2(Y_{\mathbf{1}},\theta_x)|X = X_{\mathbf{1}})] \\
&= \int_{\mathbb{R}^d} K^2(h^{-1}(x - z))(\psi_x^2(Y_{\mathbf{1}},\theta_x)|X_{\mathbf{1}} = z)f(z)\mathrm{d}z \\
&= h^d\lambda(x,\theta_x)\int_{\mathbb{R}^d} K^2(u)\mathrm{d}u + o(h^d).
\end{aligned}$$

Hence,

$$\operatorname{Var}(\Delta_{\mathbf{i}}) = A(x,\theta_x)h^d + o(h^d). \tag{5.7}$$

Now, to evaluate the second part, denoted by $R_{\mathbf{n}} = \sum_{\mathbf{i}\neq\mathbf{j}\in\mathcal{I}_{\mathbf{n}}}\left|\operatorname{Cov}(\Delta_{\mathbf{i}}(x),\Delta_{\mathbf{j}}(x))\right|$, we divide the rectangular region $\mathcal{I}_{\mathbf{n}}$ into two sets.

$$S_1 = \{\mathbf{i},\mathbf{j} \in \mathcal{I}_{\mathbf{n}} : 0 < \|\mathbf{i} - \mathbf{j}\| \le c_{\mathbf{n}}\},\ S_2 = \{\mathbf{i},\mathbf{j} \in \mathcal{I}_{\mathbf{n}} : \|\mathbf{i} - \mathbf{j}\| > c_{\mathbf{n}}\},$$

where $c_{\mathbf{n}}$ is a real sequence that converges to infinity and will be made precise later:

$$\begin{aligned}
R_{\mathbf{n}} &= \sum_{(\mathbf{i},\mathbf{j})\in S_1}\left|\operatorname{Cov}\left(\Delta_{\mathbf{i}}(x),\Delta_{\mathbf{j}}(x)\right)\right| + \sum_{(\mathbf{i},\mathbf{j})\in S_2}\left|\operatorname{Cov}\left(\Delta_{\mathbf{i}}(x),\Delta_{\mathbf{j}}(x)\right)\right| \\
&= R_{\mathbf{n}}^1 + R_{\mathbf{n}}^2.
\end{aligned}$$

On the one hand, on S_1 we have, under (H2):

$$\begin{aligned} \left|Cov\left(\Delta_{\mathbf{i}}(x),\Delta_{\mathbf{j}}(x)\right)\right| &\leq C\left|\left[K_{\mathbf{i}}K_{\mathbf{j}}\right]-\left[K_{\mathbf{i}}\right]\left[K_{\mathbf{j}}\right]\right| \\ &\leq C\int_{\mathbb{R}^d}\int_{\mathbb{R}^d}K(h^{-1}(x-v))|f_{\mathbf{ij}}(u,v)-f(u,v)|dudv \\ &\leq C\left(\int_{\mathbb{R}^d}K(h^{-1}(x-v))du\right)^2 \\ &\leq Ch^{2d}. \end{aligned}$$

$$\begin{aligned} R_{\mathbf{n}}^1 &= \sum_{(\mathbf{i},\mathbf{j})\in S_1}\left|\left[K_{\mathbf{i}}K_{\mathbf{j}}\right]-\left[K_{\mathbf{i}}\right]\left[K_{\mathbf{j}}\right]\right| \\ &\leq C\hat{\mathbf{n}}c_{\mathbf{n}}^N h^{2d}. \end{aligned}$$

On the other hand, on S_2 we apply Lemma 2.1(ii) of Tran(1990) and we deduce that

$$\left|Cov\left(\Delta_{\mathbf{i}},\Delta_{\mathbf{j}}\right)\right|\leq C\varphi\left(\|\mathbf{i}-\mathbf{j}\|\right). \tag{5.8}$$

By (5.8) we have for some $v>N$

$$\begin{aligned} R_{\mathbf{n}}^2=\sum_{(\mathbf{i},\mathbf{j})\in S_2}\left|Cov\left(\Delta_{\mathbf{i}},\Delta_{\mathbf{j}}\right)\right| &\leq C\sum_{(\mathbf{i},\mathbf{j})\in S_2}\varphi\left(\|\mathbf{i}-\mathbf{j}\|\right)\leq C\hat{\mathbf{n}}\sum_{\mathbf{i}:\|\mathbf{i}\|\geq c_{\mathbf{n}}}\varphi\left(\|\mathbf{i}\|\right) \\ &\leq C\hat{\mathbf{n}}c_{\mathbf{n}}^{-v}\sum_{\mathbf{i}:\|\mathbf{i}\|\geq c_{\mathbf{n}}}\|\mathbf{i}\|^v\varphi\left(\|\mathbf{i}\|\right). \end{aligned}$$

Taking $c_{\mathbf{n}}=h^{-d/v}$, we see

$$\begin{aligned} R_{\mathbf{n}}^2 &\leq C\hat{\mathbf{n}}h^d\sum_{\mathbf{i}:\|\mathbf{i}\|\geq c_{\mathbf{n}}}\|\mathbf{i}\|^v\varphi\left(\|\mathbf{i}\|\right) \\ &\leq C\hat{\mathbf{n}}h^d\sum_{\mathbf{i}:\|\mathbf{i}\|\geq c_{\mathbf{n}}}\|\mathbf{i}\|^v\varphi\left(\|\mathbf{i}\|\right). \end{aligned}$$

Employing (H4) and the fact that $a^{-1}>2$ choose v positive numbers such that

$$\sum_{\mathbf{i}}\|\mathbf{i}\|^v\varphi\left(\|\mathbf{i}\|\right)<\infty \quad \text{and} \quad v>N.$$

Which allows us to write

$$R_{\mathbf{n}}^2=o(\hat{\mathbf{n}}h^d) \quad \text{and} \quad R_{\mathbf{n}}^1=o(\hat{\mathbf{n}}h^d)$$

Finally

$$R_{\mathbf{n}}=o\left(\frac{1}{\hat{\mathbf{n}}h^d}\right).$$

Combining the last result together with equation (5.7) we derive

$$Var\Big(\hat{g}(x,\theta_x)\Big)=\frac{A(x,\theta_x)}{\hat{\mathbf{n}}h^d}+o\left(\frac{1}{\hat{\mathbf{n}}h^d}\right)$$

Proof of Lemma 4.2. Keeping the notation of previous lemma, we write

$$\begin{aligned} \Big[\hat{g}(x,\theta_x)\Big] &= \left[\frac{1}{\hat{\mathbf{n}}h^d}\sum_{\mathbf{i}\in\mathcal{I}_{\mathbf{n}}}\Big(K_{\mathbf{i}}\psi_x(Y_{\mathbf{i}},\theta_x)\Big)\right] \\ &= \frac{1}{h^d}\Big[\Delta_{\mathbf{i}}\Big] \\ &= \frac{1}{h^d}\int_{\mathbb{R}^d}K(h^{-1}(x-z))(\psi_x(Y_{\mathbf{1}},\theta_x)/X_{\mathbf{1}}=z)f(z)dz. \end{aligned}$$

Next, by the classical change of variables, $u = h^{-1}(x - z)$ we have

$$\frac{1}{h^d}[\Delta_{\mathbf{i}}] = \int_{\mathbb{R}^d} K(u)g(x - hu, \theta_x)du$$

Using a Taylor expansion of order two , under (H1), we obtain

$$\left[\hat{g}(x, \theta_x)\right] = B(x, \theta_x)h^2 + o(h^d) = o(h^d).$$

Proof of Lemma 4.3. We have

$$\begin{array}{rcl}(\hat{f}(x) = 0) & = & (\hat{f}(x) \leq f(x) - \epsilon) \\ & \leq & (|\hat{f}(x) - f(x)| \geq \epsilon)\end{array}$$

The Markov's inequality allows to get, for any $p > 0$,

$$(\hat{f}(x) = 0) \leq \frac{(|\hat{f}(x) - f(x)|)^p}{\epsilon^p}$$

This yields the proof.

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