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A New Legendre Wavelets Decomposition Method for Solving PDEs

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Abstract

In this paper, we present a novel technique based on the Legendre wavelets decomposition. The properties of Legendre wavelets are used to reduces the PDEs problem into the solution of ODEs system. To illustrate our results, two examples are studied using a special software package which implements the proposed algorithms.

Keywords: Legendre Wavelets, Legendre Polynomials, Operational Matrix of integration, Telegraph Equation.

2000 MSC: 49M37, 30E25.

1 Introduction

Since the introduction of Legendre wavelets method (LWM), for the resolution of variational problems, by Rezzaghi and Yousefi in 2000 and 2001 [5, 7], several works applying this method were born. To mention a few, we give : the resolution of differential equations [3, 11, 12], the study of optimal control problem with constraints [6], the resolution of linear integro-differential equations [8], the numerical resolution of Abel equation [10], the resolution of fractional differential equations [2, 4].

The LWM transforms a boundary value problem (BVP) into a system of algebraic equations [5]. The unknown parameter of this system is the vector whose components are the decomposition coefficients of the BVP solution into Legendre wavelets basis.

In this paper, we apply the Legendre wavelets method to solve a partial differential equation, whose unknown function depends on spatial and temporal variables. But the decomposition of this unknown function into Legendre wavelets basis will be done only on the spatial variable. Obviously, the coefficients of this decomposition will depend on the temporal variable. Hence, via this technique, the solution of a partial differential equation is reduced to the solution of a time-dependent differential equation. As an application of this procedure, we present a numerical simulation of the telegraph equation. This equation modelled several phenomenas in electronics and electricity. It appears in particular during the description of the propagation of the electric signals along a transmission line.

This paper is organized as follows: In section 2, we give a detailed description of the Legendre wavelets decomposition of a function dependent on the temporal variable t and the space variable x. Section 3 is devoted to the operational matrix of integration. In section 4, we apply our technique on the telegraph equation. In section 5, we present formulas of errors calculation. For the last section, the performance of the new method is illustrated with two numerical examples.

2 Decomposition in Legendre wavelets basis

We start this section by recalling that the Legendre wavelets [5] are defined on the interval $[0,1]$ as follows : for all $j \in \mathbb{N}^*$, $n = 1, 2, 3, ..., 2^{j-1}$ (the number of levels), $m = 0, 1, ..., nc - 1$ (the order of the Legendre

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polynomials) and nc is the number of collocation points ;

$$\psi_{n,m}(x) = \begin{cases} \sqrt{m+\frac{1}{2}}2^{j/2}L_m\left(2^j x - 2n + 1\right) & \frac{n-1}{2^{j-1}} \leq x < \frac{n}{2^{j-1}} \\ 0, \quad otherwise, & \end{cases} \tag{2.1}$$

where $L_m(x)$ are the Legendre polynomials of order m defined on the interval $[-1,1]$ and satisfy the following recursive formula:

$$L_{k+1}(x) = \left(\frac{2k+1}{k+1}\right) x L_k(x) - \left(\frac{k}{k+1}\right) L_{k-1}(x) \tag{2.2}$$

with $L_0(x) = 1$, $L_1(x) = x$ and $k = 1, 2, 3, ...nc - 2$.

It is established [3, 5–7] that the family $\{\psi_{n,m} : n \geq 1 \ \ , \ \ m \geq 0\}$ forms an orthonormal basis of the Hilbert space $L^2([0,1])$, i.e. any element $h \in L^2([0,1])$ may be expanded as

$$h(x) = \sum_{n=1}^{+\infty}\sum_{m=0}^{+\infty} C_{n,m}\psi_{n,m}(x), \tag{2.3}$$

where the approximation coefficients are entirely determined by $C_{n,m} = \langle h, \psi_{n,m}\rangle$, in which $\langle ., . \rangle$ denotes the inner product of $L^2([0,1])$.

Since the series (2.3) converges on $[0,1]$, the function h can be approximated as

$$h(x) \simeq \sum_{n=1}^{2^{j-1}}\sum_{m=0}^{nc-1} C_{n,m}\psi_{n,m}(x) = C^T\Psi(x), \tag{2.4}$$

where C and $\Psi(x)$ are $2^{j-1}nc$ vectors given by

$$C = \left[C_{1,0}, C_{1,1}, ..., C_{1,nc-1}, C_{2,0}, ..., C_{2,nc-1}, ..., C_{2^{j-1},0}, ..., C_{2^{j-1},nc-1}\right]^T, \tag{2.5}$$

and

$$\Psi(x) = \left[\psi_{1,0}(x), ..., \psi_{1,nc-1}(x), ..., \psi_{2^{j-1},0}(x), ..., \psi_{2^{j-1},nc-1}(x)\right]^T. \tag{2.6}$$

Decomposition in an other L^2 -space

Let us consider the space

$$H = L^2\left([0,T]; L^2([0,1])\right), \tag{2.7}$$

see [1]. We want to give a Legendre wavelets decomposition of an element h in H:

$$\begin{array}{cccc} h: & [0,T] & \rightarrow & L^2([0,1]) \\ & t & \rightarrow & h(t,.). \end{array} \tag{2.8}$$

As the function $h(t,.)$ belongs to $L^2([0,1])$, then by (2.3) we have

$$h(t,x) = \sum_{n=1}^{+\infty}\sum_{m=0}^{+\infty} C_{n,m}(t)\,\psi_{n,m}(x), \tag{2.9}$$

where the coeficients $C_{n,m}(t)$ depending on the variable t are defined by

$$C_{n,m}(t) = \langle h(t,.), \psi_{n,m}\rangle = \int_0^1 h(t,x)\psi_{n,m}(x)\,dx. \tag{2.10}$$

The functions $h(t,.)$ and $\psi_{n,m}$, being both in $L^2([0,1])$, their product is in $L^1([0,1])$ (according to Cauchy-Schwartz inequality), which allows us to conclude that the coefficients $C_{n,m}(t)$ are well defined for all $t \in [0,T]$. Consequently, the relations (2.9) and (2.10) are justified.

For the sequel, we need the following lemmas.

Lemma 2.1. *If $h \in C\left([0,T], L^2([0,1])\right)$ then the function coefficients $C_{n,m}(t)$ are continuous in $[0,T]$.*

Proof. It arises from the fact that the inner product is a continuous function of its both arguments. □

Lemma 2.2. *If $h \in C^1\left(]0,T[\,, L^2\left([0,1]\right)\right)$, then the function coefficients $C_{n,m}(t)$ belong to $C^1(]0,T[)$.*
Forthermore, if $\frac{\partial h}{\partial t} \in L^2\left([0,T]\,, L^2\left([0,1]\right)\right)$, then

$$\frac{dC_{n,m}(t)}{dt} = \int_0^1 \frac{\partial h(t,x)}{\partial t} \psi_{n,m}(x)\, dx. \tag{2.11}$$

Proof. Lemma 2.2 is based on

$$\frac{C_{n,m}(t+\Delta t) - C_{n,m}(t)}{\Delta t} = \int_0^1 \frac{h(t+\Delta t, x) - h(t,x)}{\Delta t} \psi_{n,m}(x)\, dx,$$

and

$$\frac{h(t+\Delta t, x) - h(t,x)}{\Delta t} = \frac{\partial h}{\partial t}(t,x) + \varepsilon(t, \Delta t, x),$$

with

$$\lim_{\Delta t \to 0} \varepsilon(t, \Delta t, x) = 0.$$

□

Lemma 2.3. *(Generalization) If $h \in C^k\left(]0,T[\,, L^2\left([0,1]\right)\right)$, then the function coefficients $C_{n,m}(t)$ belong to $C^k(]0,T[)$ $(k \geq 2)$.*

For all $t \in [0,T]$, the series (2.9) is convergent, we can thus approach any function h in $L^2\left([0,T]\,; L^2\left([0,1]\right)\right)$ as

$$h(t,x) \simeq \sum_{n=1}^{2^{j-1}} \sum_{m=0}^{nc-1} C_{n,m}(t)\, \psi_{n,m}(x) = C^T(t)\, \Psi(x), \tag{2.12}$$

where $C(t)$ and $\Psi(x)$ are $\left(2^{j-1}nc\right)$ functions vectors given by

$$C(t) = \left[C_{1,0}(t), C_{1,1}(t), ..., C_{1,nc-1}(t), ..., C_{2^{j-1},0}(t), ..., C_{2^{j-1},nc-1}(t)\right]^T, \tag{2.13}$$

and

$$\Psi(x) = \left[\psi_{1,0}(x), ..., \psi_{1,nc-1}(x), ..., \psi_{2^{j-1},0}(x), ..., \psi_{2^{j-1},nc-1}(x)\right]^T. \tag{2.14}$$

3 Operational matrix of integration

In this section, the operational matrix of integration [7] will be obtained. The integration into $[0,x]$, where $x \in]0,1]$ of the vector $\Psi(x)$ defined in Eq. (2.14) can be written as

$$\int_0^x \Psi(t)dt = P\Psi(x), \tag{3.1}$$

where

$$P = \frac{1}{2^j} \begin{bmatrix} L & F & F & \cdots & F \\ 0 & L & F & \cdots & F \\ \vdots & 0 & \ddots & \ddots & \vdots \\ & & & & F \\ 0 & 0 & \cdots & 0 & L \end{bmatrix}, \tag{3.2}$$

is the $(2^{j-1}nc) \times (2^{j-1}nc)$ operational matrix of integration, F and L are $nc \times nc$ matrices given by

$$F = \begin{bmatrix} 2 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & 0 & & 0 \end{bmatrix} \tag{3.3}$$

and

$$L=\begin{bmatrix} 1 & \frac{1}{\sqrt{3}} & 0 & 0 & \cdots & 0 & 0 & 0 \\ \frac{-\sqrt{3}}{3} & 0 & \frac{\sqrt{3}}{3\sqrt{5}} & 0 & \cdots & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{5}}{5\sqrt{3}} & 0 & \frac{\sqrt{5}}{5\sqrt{7}} & \ddots & 0 & 0 & 0 \\ 0 & 0 & -\frac{\sqrt{7}}{7\sqrt{5}} & 0 & \ddots & 0 & 0 & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \ddots & -\frac{\sqrt{2nc-3}}{(2nc-3)\sqrt{2nc-5}} & 0 & \frac{\sqrt{2nc-3}}{(2nc-3)\sqrt{2nc-1}} \\ 0 & 0 & 0 & 0 & \cdots & 0 & \frac{\sqrt{2nc-1}}{(2nc-1)\sqrt{2nc-3}} & 0 \end{bmatrix}. \tag{3.4}$$

4 Application to telegraph equation

The standard form of the telegraph equation is given by

$$\frac{\partial^2 u(x,t)}{\partial x^2}=a\frac{\partial^2 u(x,t)}{\partial t^2}+b\frac{\partial u(x,t)}{\partial t}+cu(x,t), \tag{4.1}$$

with boundary conditions

$$u(0,t)=\alpha(t) \quad and \quad \frac{\partial u(0,t)}{\partial x}=\beta(t) \tag{4.2}$$

and initial conditions

$$u(x,0)=f(x) \quad and \quad \frac{\partial u(x,0)}{\partial t}=g(x), \tag{4.3}$$

where

a, b, c are constants related respectively to resistance, induction, capacity and conductibility of the cable.
$\alpha(t), \beta(t)$ are continuous functions in $[0,T]$ and $f(x), g(x)$ are continuous in $[0,1]$
We are interested in the evolution of the tension $u(x,t)$ in a coaxial cable.
Let

$$\frac{\partial^2 u(x,t)}{\partial x^2}=C^T(t)\,\Psi(x). \tag{4.4}$$

Integrating (4.4) with respect to second variable over $[0,x]$, we get

$$\frac{\partial u(x,t)}{\partial x}=C^T(t)\,P\Psi(x)+\beta(t). \tag{4.5}$$

By a second integration with respect to x into $[0,x]$, we have

$$u(x,t)=C^T(t)\,P^2\Psi(x)+\beta(t)\,x+\alpha(t). \tag{4.6}$$

As well as

$$\frac{\partial u(x,t)}{\partial t}=\frac{dC^T(t)}{dt}P^2\Psi(x)+\frac{d\beta(t)}{dt}x+\frac{d\alpha(t)}{dt} \tag{4.7}$$

$$\frac{\partial^2 u(x,t)}{\partial t^2}=\frac{d^2C^T(t)}{dt^2}P^2\Psi(x)+\frac{d^2\beta(t)}{dt^2}x+\frac{d^2\alpha(t)}{dt^2} \tag{4.8}$$

We have also

$$\begin{cases} 1=d^T\Psi(x) \\ x=e^T\Psi(x). \end{cases} \tag{4.9}$$

Substituting (4.4) to (4.9) in (4.1), we obtain

$$\begin{aligned} C^T(t)\Psi(x) \quad &= \quad a\Big(\frac{d^2C^T(t)}{dt^2}P^2\Psi(x)+\frac{d^2\beta(t)}{dt^2}e^T\Psi(x)+\frac{d^2\alpha(t)}{dt^2}d^T\Psi(x)\Big) \\ &+b\left(\frac{dC^T(t)}{dt}P^2\Psi(x)+\frac{d\beta(t)}{dt}e^T\Psi(x)+\frac{d\alpha(t)}{dt}d^T\Psi(x)\right) \\ &+c\left(C^T(t)\,P^2\Psi(x)+\beta(t)\,e^T\Psi(x)+\alpha(t)d^T\Psi(x)\right). \end{aligned}$$

Hence, we have

$$\begin{aligned}&-a\left(P^2\right)^T C''(t) - b\left(P^2\right)^T C'(t) + \left(I - c\,\left(P^2\right)^T\right) C(t)\\ =\;&\left(a\beta''(t) + b\beta'(t) + c\beta(t)\right) e + \left(a\alpha''(t) + b\alpha'(t) + c\alpha(t)\right) d\end{aligned}$$

This system can be solved for unknown coefficients of the vector $C(t)$. Consequently, the solution $u(t,x)$ given in (4.6) can be calculated.

5 Error calculation

A reasonable scalar index for the closeness of two functions is the Euclidian norm.

The absolute error for each solution is produced by cumulates of truncation, Legendre Wavelets method and Finite Difference errors. This error is estimated when we know the exact solution by

$$E_A = \|u - u_e\|_2, \tag{5.1}$$

where u_e is the analytic solution and u is the approximate solution. Also, we consider the pointwise error :

$$E_{A,i} = |u_i - u_e(x_i)|. \tag{5.2}$$

6 Numerical results

In this section, we consider two examples to show the efficiency and the accuracy of our method.

6.1 Example 1

Consider in $[0,1]$ the following boundary values problem

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial^2 u(x,t)}{\partial t^2}, \tag{6.1}$$

with

$$u(0,t) = \cos(t) \quad \text{and} \quad \frac{\partial u(0,t)}{\partial x} = 0, \tag{6.2}$$

and

$$u(x,0) = \cos(x) \quad \text{and} \quad \frac{\partial u(x,0)}{\partial t} = 0. \tag{6.3}$$

The exact solution of the problem (6.1)-(6.2) and (6.3) is given by

$$u_{ex}(t,x) = \cos(x)\cos(t). \tag{6.4}$$

Suppose that the derivatives on x can be expressed as

$$\frac{\partial^2 u(x,t)}{\partial x^2} = C^T(t)\,\Psi(x), \tag{6.5}$$

$$\frac{\partial u(x,t)}{\partial x} = C^T(t)\,P\Psi(x), \tag{6.6}$$

$$u(x,t) = \left(C^T(t)\,P^2 + \cos(t)d^T\right)\Psi(x), \tag{6.7}$$

then their derivates on t are given by

$$\frac{\partial u(x,t)}{\partial t} = \left(\frac{\partial C^T(t)}{\partial t}P^2 - \sin(t)d^T\right)\Psi(x), \tag{6.8}$$

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \left(\frac{\partial^2 C^T(t)}{\partial t^2}P^2 - \cos(t)d^T\right)\Psi(x). \tag{6.9}$$

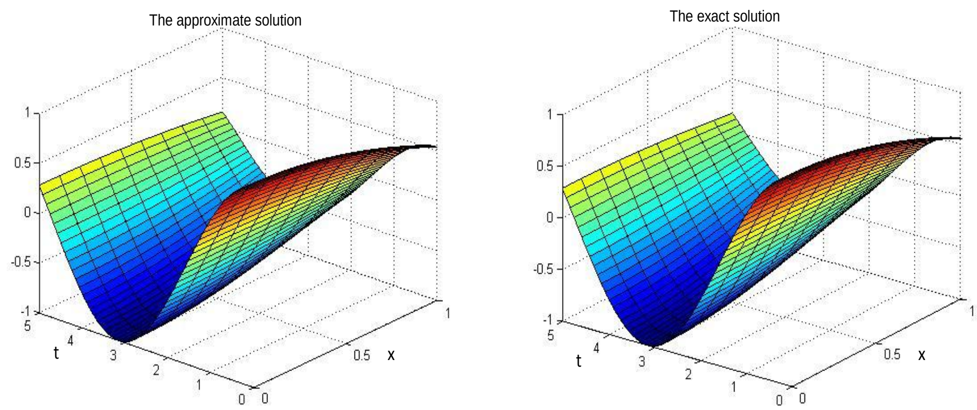

Figure 1: The analytical and approximate solutions.

Substituting (6.5) to (6.9) in (6.1), we obtain

$$C^T(t)\,\Psi(x) = \left(\frac{\partial^2 C^T(t)}{\partial t^2} P^2 - \cos(t) d^T\right)\Psi(x), \tag{6.10}$$

and

$$\frac{\partial^2 C(t)}{\partial t^2} - ((P^2)^T)^{-1} C(t) = ((P^2)^T)^{-1}\cos(t) d, \tag{6.11}$$

with

$$C^T(0) = \left(e^T - d^T\right)\left(P^2\right)^{-1} \quad and \quad \frac{\partial C(0)}{\partial t} = \overrightarrow{0}, \tag{6.12}$$

where

$$\begin{cases} 1 = d^T \Psi(x) \\ \cos(x) = e^T \Psi(x). \end{cases} \tag{6.13}$$

For the resolution of this problem, we can use a second time the wavelets Legendre method or a finite difference schemes.

We observe a good agreement between the analytical and approximate solutions (see Figure 1). However, the obtained result shows that this technique can provide good performance even when the mesh discretization has low resolution.

We also calculate the absolute error by using formula (5.1), for $j = 2$ and $nc = 5$

$$E_A = 1.29e - 2. \tag{6.14}$$

6.2 Example 2

We consider the telegraph equation

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial^2 u(x,t)}{\partial t^2} + 4\frac{\partial u(x,t)}{\partial t} + 4u(x,t), \tag{6.15}$$

with the conditions

$$u(0,t) = \left(1 + e^{-2t}\right) \quad and \quad \frac{\partial u(0,t)}{\partial x} = 2, \tag{6.16}$$

and

$$u(x,0) = 1 + e^{2x} \quad and \quad \frac{\partial u(x,0)}{\partial t} = -2. \tag{6.17}$$

Suppose that the second derivative of $u(x,t)$ can be expressed approximately as

$$\frac{\partial^2 u(x,t)}{\partial x^2} = C^T(t)\,\Psi(x). \tag{6.18}$$

Using boundary condition (6.16), we get

$$\frac{\partial u(x,t)}{\partial x} = C^T(t)\, P\Psi(x) + 2d^T\Psi(x), \tag{6.19}$$

$$u(x,t) = \left(C^T(t)\, P^2 + 2l^T + \left(1+e^{-2t}\right)d^T\right)\Psi(x). \tag{6.20}$$

We can also express the functions of the right-hand sides of (6.19) and (6.20) as

$$\begin{cases} 1 = d^T\Psi(x) \\ x = l^T\Psi(x). \end{cases} \tag{6.21}$$

The derivates on t are given by

$$\frac{\partial u(x,t)}{\partial t} = \left(\frac{\partial C^T(t)}{\partial t}P^2 - 2e^{-2t}d^T\right)\Psi(x), \tag{6.22}$$

and

$$\frac{\partial^2 u(x,t)}{\partial t^2} = \left(\frac{\partial^2 C^T(t)}{\partial t^2}P^2 + 4e^{-2t}d^T\right)\Psi(x). \tag{6.23}$$

Now by inserting Eqs. (6.18) to (6.23) into equation (6.15), we get

$$\begin{aligned} C^T(t)\,\Psi(x) \quad = \quad & \left(\frac{\partial^2 C^T(t)}{\partial t^2}P^2 + 4e^{-2t}d^T\right)\Psi(x) \\ & +4\left(\frac{\partial C^T(t)}{\partial t}P^2 - 2e^{-2t}d^T\right)\Psi(x) + \\ & 4\left(C^T(t)\,P^2 + 2l^T + \left(1+e^{-2t}\right)d^T\right)\Psi(x). \end{aligned}$$

Thus

$$\frac{\partial^2 C(t)}{\partial t^2} + 4\frac{\partial C(t)}{\partial t} + (4I - ((P^2)^T)^{-1})C(t) = -((P^2)^T)^{-1}(8l+4d), \tag{6.24}$$

where

$$C(0) = ((P^2)^T)^{-1}(ex - 2l - 2d) \quad \textit{and} \quad \frac{\partial C(0)}{\partial t} = 0, \tag{6.25}$$

and

$$1 + e^{2x} = ex^T\Psi(x) \tag{6.26}$$

The exact solution of the problem (6.15)-(6.16) and (6.17) is

$$u_{ex}(t,x) = e^{2x} + e^{-2t}, \tag{6.27}$$

Applying the same technique as the preceding example, we observe a good agreement between the analytical and approximate solutions with an absolute errors of order $1.0e-013$ (see figure 2). However, the results obtained show that this technique can provide good performance even when the mesh discretization has low resolution.

We also calculate the absolute error by using formula (5.1), for $j=3$ and $nc=10$

$$E_A = 1.7705e - 013. \tag{6.28}$$

We calculated the absolute error for different values of j and nc (see figure 3), we observe that :

The effect of the levels number on the solution is shown in Table 1. For $j = 1, 2, ..., 6$ (2^{j-1} levels), the absolute errors with respect to analytic solutions are presented. As j increase from 1 to 6 for $nc = 3$, the errors decrease to 10^{-4}.

The considerable effect of collocation points on the solution is shown in Table 2. For $nc = 2, 3, ..., 10$, the absolute errors with respect to analytic solutions are presented. As nc changes from 2 to 4 for $j = 3$, then from 6 to 10 for $j = 3$, the errors decrease by a factor of 10^2 and 10^6 respectively.

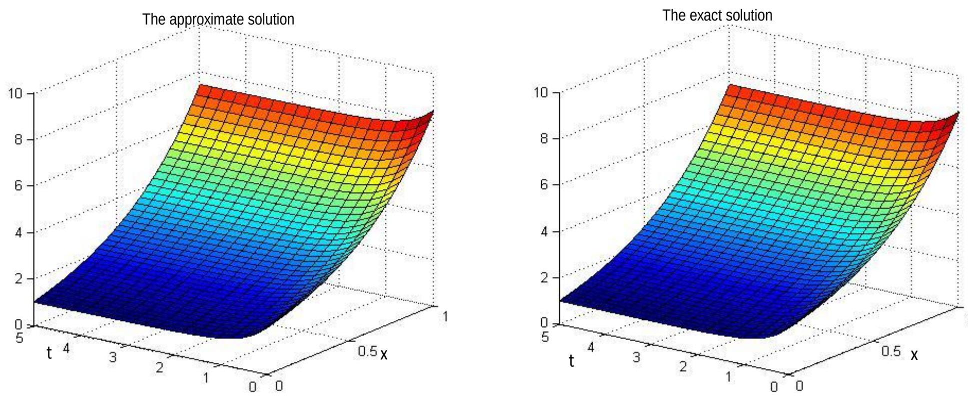

Figure 2: The analytical and approximate solutions.

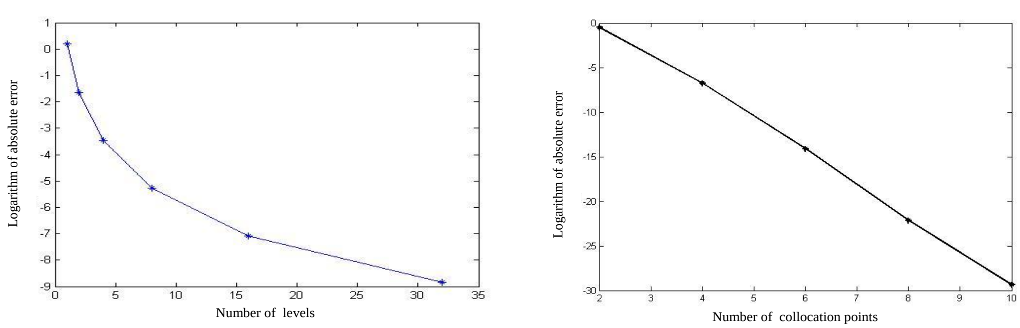

Figure 3: Evolution of absolute errors.

Table 1: Evolution of absolute errors for $nc = 3$

	j = 1	j = 2	j = 3	j = 4	j = 5	j = 6
Number of levels 2^{j-1}	1	2	4	8	16	32
Absolute errors	1.2048	1.910E-001	3.1442E-002	5.0610E-003	8.4109E-004	1.43699E-004

Table 2: Evolution of absolute errors for $j = 3$.

nc	2	4	6	8	10
Absolute errors	6.1105e-001	1.1544e-003	7.3398e-007	2.3729e-010	1.7700e-013

7 Conclusion

This work shows that the Legendre wavelets method is a very effective technique for reducing partial differential equations into a set of ordinary differential equations. This numerical method has been tested under different examples. Satisfactory results have been obtained even for a small number of collocation points. The convergence, stability of the solution and accuracy of the results prove the high quality of this method.

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