

## Lattice for covering rough approximations

Dipankar Rana<sup>a,\*</sup> and Sankar Kumar Roy<sup>b</sup>

<sup>a,b</sup> Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore-721102, West Bengal, India.

### Abstract

Covering is a common type of data structure and covering-based rough set theory is an efficient tool to process this type of data. Lattice is an important algebraic structure and used extensively in investigating some types of generalized rough sets. This paper presents the lattice based on covering rough approximations and lattice for covering numbers. An important result is investigated to illustrate the paper.

*Keywords:* Covering, Rough Set, Lattice, Covering approximation.

2010 MSC: 03G10, 14E20, 18B35.

©2012 MJM. All rights reserved.

### 1 Introduction

Theory of rough sets was introduced by Z. Pawlak [7], assumed that sets are chosen from a universe  $U$ , but that elements of  $U$  can be specified only upto an indiscernibility equivalence relation  $E$  on  $U$ . If a subset  $X \subseteq U$  contains an element indiscernible from some elements not in  $X$ , then  $X$  is rough. Also a rough set  $X$  is described by two approximations. Basically, in rough set theory, it is assume that our knowledge is restricted by an indiscernibility relation. An indiscernibility relation is an equivalence relation  $E$  such that two elements of an universe of discourse  $U$  are  $E$ -equivalent if we can not distinguish these two elements by their properties known by us. By the means of an indiscernibility relation  $E$ , we can partition the elements of  $U$  into three disjoint classes respect to any set  $X \subseteq U$ , defined as follows:

- The elements which are certainly in  $X$ . These are elements  $x \in U$  whose  $E$ -class  $x/E$  is included in  $X$ .
- The elements which certainly are not in  $X$ . These are elements  $x \in U$  such that their  $E$ -class  $x/E$  is included in  $X^{co}$ , which is the complement of  $X$
- The elements which are possibly belongs to  $X$ . These are elements whose  $E$ -class intersects with both  $X$  and  $X^{co}$ . In other words,  $x/E$  is not included in  $X$  nor in  $X^{co}$ .

From this observation, Pawlak [7] defined lower approximation set  $X \downarrow$  of  $X$  to be the set of those elements  $x \in U$  whose  $E$ -class is included in  $X$ , i.e,  $X \downarrow = \{x \in U : x/E \subseteq X\}$  and for the upper approximation set  $X \uparrow$  of  $X$  consists of elements  $x \in U$  whose  $E$ -class intersect with  $X$ , i.e,  $X \uparrow = \{x \in U : x/E \cap X \neq \emptyset\}$ . The difference between  $X \downarrow$  and  $X \uparrow$  is treated as the actual area of uncertainty.

Covering-based rough set theory ([17], [19]) is a generalization of rough set theory. The structure of covering-based rough sets ([18],[19],[20]) have been a interested field of study. The classical rough set theory is based on equivalence relations. An equivalence relation corresponds to a partition, while a covering is an extension of a partition. This paper focuses on establishing algebraic structure of covering-based rough sets through down-sets and up-sets. Firstly, we connect posets with covering-based rough sets, then covering-based rough sets can be investigated in posets. Down-sets and up-sets are defined in the poset environment. In order to

\*Corresponding author.

E-mail address: [sankroy2006@gmail.com](mailto:sankroy2006@gmail.com) (Sankar Kumar Roy).

achieve this goal, many theories and methods have been proposed, for example, fuzzy set theory ([4], [16]), computing with words ([9], [15]), rough set theory ([5], [14]) and granular computing ([1], [3], [11], [13]). From the structures of these theories, two structures are mainly used, that is, algebraic structure ([2], [10],[12]) and topological structure [17]. This paper focuses on establish the algebraic structures of covering-based rough lattice through down sets, up sets and lattice for covering numbers.

## 2 Preliminaries

In this section, we present some definition and fundamental concept on covering lattice.

**Definition 2.1.** Let  $U$  be a domain of discourse, and  $C$  be a family of subsets of  $U$ . If none of subsets in  $C$  is empty and  $\cup C = U$ , then  $C$  is called a covering of  $U$ . We call  $(U, C)$  the covering approximation space and the covering  $C$  is called the family of approximation sets. It is clear that a partition of  $U$  is certainly a covering of  $U$ , so the concept of a covering is an extension of a partition.

Let  $(U, C)$  be an approximation space and  $x$  be any element of  $U$ . then the family.  $Mind(x) = \{K \in C : x \in K \wedge (\forall S \in C \wedge x \in S \wedge S \subseteq K \Rightarrow K = S)\}$  is called the minimal description of the object  $x$ . In order to describe an object we need only the essential characteristics related to this object. This is the purpose of the minimal description concept.

**Definition 2.2.** A relation  $R$  on a set  $P$  is called a partial order if  $R$  is reflexive, antisymmetric, and transitive. If  $R$  is a partial order on  $P$ , then  $(P, R)$  is called a poset.

**Definition 2.3.** An upper semi-lattice is a poset  $(P, R)$  in which every subset  $\{a, b\}$  has a least upper bound  $a \vee b$ . A lower semi-lattice is a poset  $(P, R)$  in which every subset  $\{a, b\}$  has a greatest lower bound  $a \wedge b$ . The upper semi-lattice and the lower semi-lattice are also called semi-lattices.

**Definition 2.4.** [8] The lattice as a poset will be denoted by  $(L, \leq)$ , and the lattice as an algebra by  $(L, \wedge, \vee)$ . We write simply  $L$  to denote the lattice in both senses. A poset  $(L, \leq)$  is a lattice if  $\sup\{a, b\}$  and  $\inf\{a, b\}$  exist for all  $a, b \in L$ .

**Definition 2.5.** Let  $C$  be a covering of domain  $U$  and  $K \in C$ . If  $K$  is a union of some sets in  $C - \{K\}$ , we say  $K$  is reducible in  $C$ , otherwise  $K$  is irreducible.

**Definition 2.6.** (Down-set and Up-set) Let  $(P, \prec)$  be a poset. For all  $A \subseteq P$ , one can define  $\downarrow A = \{x \in P : \exists a \in A, x \prec a\}$ ,

$\uparrow A = \{x \in P : \exists a \in A, a \prec x\}$ .  $\downarrow A$  is called a down-set of  $A$  on the poset  $(P, \prec)$ ;  $\uparrow A$  is called an up-set  $A$  on the poset  $(P, \prec)$ . When there is no confusion, we say  $\downarrow A$  is a down-set of  $A$ , and  $\uparrow A$  an up-set of  $A$ .

Let  $(U, C)$  be a covering approximation space and  $N(x) = \{K \in C : x \in K\}$  neighborhood of point  $x$  for each  $x \in U$ . There are six types of covering approximation operations that are defined as follows: for  $X \subseteq U$ ,

- $X \downarrow_{C_1} = \cup\{K : K \in C \wedge K \subseteq X\}$ ;  $X \uparrow_{C_1} = \cup\{K : K \in C \wedge K \cap X \neq \emptyset\}$ ;
- $X \downarrow_{C_2} = \cup\{K : K \in C \wedge K \subseteq X\}$ ;  $X \uparrow_{C_2} = U - (U - X) \uparrow_{C_2}$ ;
- $X \downarrow_{C_3} = \{x \in U : N(x) \subseteq X\}$ ;  $X \uparrow_{C_3} = \{x \in U : N(x) \cap X \neq \emptyset\}$ ;
- $X \downarrow_{C_4} = \{x \in U : \exists a(a \in N(x) \wedge N(a) \subseteq X)\}$ ;  $X \uparrow_{C_4} = \{x \in U : \forall a(a \in N(x) \rightarrow N(a) \cap X \neq \emptyset)\}$ ;
- $X \downarrow_{C_5} = \{x \in U : \forall a(a \in N(a) \rightarrow N(a) \subseteq X)\}$ ;  $X \uparrow_{C_5} = \cup\{N(x) : x \in U \wedge N(x) \cap X \neq \emptyset\}$ ;
- $X \downarrow_{C_6} = \{x \in U : \forall a(a \in N(a) \rightarrow a \in X)\}$ ;  $X \uparrow_{C_6} = \cup\{N(x) : x \in X\}$ . We call  $X \downarrow_{C_n}$  the covering lower approximation operation and  $X \uparrow_{C_n}$  the covering upper approximation operation ( $n = 1, 2, 3, 4, 5, 6$ ).

## 3 Rough set approximations based on covering

Let  $X^{co}$  be the complement of  $X$  in  $U$ ,  $X^{co} = U - X$ . Let  $(U, C)$  be a covering approximation space. For any subset,  $X \subseteq U$ , the covering lower approximation of  $X$  be defined by  $X \downarrow = \cup C \downarrow (X)$  and the covering upper approximation of  $X$  be defined by  $X \uparrow = \cap\{K : K \subseteq X^{co} \text{ and } K \in C\}$ . The set  $X$  is called new type covering based rough when  $X \downarrow \neq X \uparrow$ , otherwise  $X$  is called an exact set. The boundary of  $X$  denoted by  $BN_C(X) = X \uparrow - X \downarrow$  is called as the boundary region of  $X$  of the new type covering  $C$ . With this concept, we construct the following proposition as:

**Proposition 3.1.**  $X \downarrow = X$  if and only if  $X$  is the union of some elements of  $C$  and also  $X \uparrow = X$  if and only if  $X^{co}$  is the union of some elements of  $C$ .

**Proposition 3.2.** Let  $C$  be a covering of a universe  $U$ . If  $K$  is a reducible element of  $C$ ,  $C - \{K\}$  is still a covering of  $U$ .

**Proposition 3.3.** Let  $C$  be a covering of a universe  $U$ ,  $K \in C$ ,  $K$  is a reducible element of  $C$ , and  $K_1 \in C - \{K\}$ , then  $K_1$  is a reducible element of  $C$  if and only if it is a reducible element of  $C - \{K\}$ .

### 3.1 Lattice based on covering rough approximation

**Definition 3.7.** Let  $C$  be a covering of  $U$ . We define  $L_C = \{X \subseteq U : C_6 \downarrow (X) = X\}$ .  $L_C$  is called the fixed point set of neighborhoods induced by  $C$ . We omit the subscript  $C$  when there is no confusion.

**Theorem 3.1.**  $(L, \subseteq)$  is a lattice, where  $X \vee Y = X \cup Y$  and  $X \wedge Y = X \cap Y$  for any  $X, Y \in L$ .

*Proof.* For any  $X, Y \in L$ , if  $X \cup Y \notin L$ , then there exists  $x \in X \cup Y$  such that  $N(x) \not\subseteq X \cup Y$ . Since  $x \in X \cup Y, \Rightarrow x \in X$  or  $x \in Y$ . Hence  $N(x) \not\subseteq X$  or  $N(x) \not\subseteq Y$ , which is contradictory with  $X, Y \in L$ . Therefore,  $X \cup Y \in L$ . For any  $X, Y \in L$ , if  $X \cap Y \notin L$ , then there exists  $y \in X \cap Y$  such that  $N(y) \not\subseteq X \cap Y$ . Since  $y \in X \cap Y, y \in X$  and  $y \in Y$ . Hence there exist three cases as follows:

(1)  $N(y) \not\subseteq X$  and  $N(y) \not\subseteq Y$ ,

(2)  $N(y) \not\subseteq X$  and  $N(y) \subseteq Y$ ,

(3)  $N(y) \subseteq X$  and  $N(y) \not\subseteq Y$ . But these three cases are all contradictory with  $X, Y \in L$ . Therefore,  $X \cap Y \in L$ .

Thus  $(L, \subseteq)$  is a lattice.  $\emptyset$  and  $U$  are the least and greatest elements of  $(L, \subseteq)$ . In fact,  $(L, \cap, \cup)$  is defined from the viewpoint of algebra and  $(L, \subseteq)$  is defined from the viewpoint of partially ordered set. Both of them are lattices. Therefore, we no longer differentiate  $(L, \cap, \cup)$  and  $(L, \subseteq)$ , and both of them are called lattice  $L$ .  $\square$

**Proposition 3.4.** Let  $C$  be a covering of  $U$ . For all  $a \in U, N(a) \in L$ .

*Proof.* For any  $b \in N(a), N(b) \subseteq N(a)$ , which implies  $b \in c : N(c) \subseteq N(a) = C_6 \downarrow (N(a))$ . Hence  $N(a) \subseteq C_6 \downarrow (N(a))$ . According to last definition of approximation  $C_6 \downarrow (N(a)) \subseteq N(a)$ . Thus  $C_6 \downarrow (N(a)) = N(a)$ , i.e.,  $N(a) \in L$ .  $\square$

**Theorem 3.2.** Let  $C$  be a covering of  $U$ , then  $L$  is a complete distributive lattice.

*Proof.* For any  $D \subseteq L$ , we need to prove that  $\cap D \in L$  and  $\cup D \in L$ . If  $\cap D \notin L$ , then there exists  $y \in \cap D$  such that  $N(y) \not\subseteq \cap D$ , i.e., there are two index sets  $I, J \subseteq \{1, 2, \dots, |D|\}$  with  $I \cap J = \emptyset$  and  $|I \cup J| = |D|$  such that  $N(y) \not\subseteq X_i$  and  $N(y) \subseteq X_j$  for any  $i \in I, j \in J$ , where  $X_i, X_j \in D$ . This is contradictory with  $X_i (i \in I), X_j (j \in J) \in L$ . Hence  $\cap D \in L$ . If  $\cup D \notin L$ , then there exists  $x \in \cup D$  such that  $N(x) \not\subseteq \cup D$ , i.e., there exists  $X \in D$  such that  $x \in X$  and  $N(x) \not\subseteq X$ , which is contradictory with  $X \in L$ . Hence  $\cup D \in L$ . Again for any  $X, Y, Z \in L, X, Y, Z \subseteq U$ . It is straightforward that  $X \cup (Y \cap Z) = (X \cup Y) \cap (X \cup Z)$ ,  $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$ . Hence  $L$  is a distributive lattice.  $\square$

**Theorem 3.3.** If  $\{N(x) : x \in U\}$  is a partition of  $U$ , then  $L$  is a Boolean lattice.

*Proof.* According to Theorem 3.6,  $L$  is a distributive lattice. Moreover,  $L$  is a bounded lattice. Therefore, we need to prove only that  $L$  is a complemented lattice. In other words, we need to prove that  $X^{co} \in L$  for any  $X \in L$ . If  $X^{co} \notin L$ , i.e.,  $\cup_{x \in X^{co}} N(x) \neq X^{co}$ , then there exists  $y \in \cup_{x \in X^{co}} N(x)$  such that  $y \notin X^{co}$ . Since  $y \in \cup_{x \in X^{co}} N(x)$ , then there exists  $z \in X^{co}$  such that  $y \in N(z)$ . Since  $\{N(x) : x \in U\}$  is a partition of  $U, z \in N(y)$ . Therefore,  $N(y) \not\subseteq X$ , i.e.,  $\cup_{x \in X} N(x) \neq X$ , which is contradictory with  $X \in L$ . Hence,  $X^{co} \in L$  for any  $X \in L$ , i.e.,  $L$  is a complemented lattice. Consequently,  $L$  is a Boolean lattice.  $\square$

## 4 Covering numbers

Various techniques have been proposed to characterize rough sets ([2], [3], [5]). Similarly, we establish some measurements to describe covering-based rough sets quantitatively.

## 4.1 Definitions and Properties of Covering Numbers

The upper covering number of a subset of a domain is the minimal number of some elements in a covering which can cover the subset. The lower covering number of a subset is the maximal number of some elements in a covering which can be included in the subset.

**Definition 4.1.** Let  $C$  be a covering of  $U$ . For all  $X \subseteq U$ , we define

- $N \uparrow_C (X) = \min\{|B| : (X \subseteq \cup B) \wedge (B \subseteq C)\}$ .
- $N \downarrow_C (X) = |\{K \in C | K \subseteq X\}|$ .

$N \uparrow_C (X)$  and  $N \downarrow_C (X)$  are called the upper and lower covering numbers of  $X$  with respect to  $C$ . When there is no confusion,  $N \uparrow_C (X)$  is denoted simply by  $N \uparrow (X)$ , and  $N \downarrow_C (X)$  by  $N \downarrow (X)$ .

**Example-1:** Let  $U = \{a, b, c, d\}$ ,  $D_1 = \{a, b\}$ ,  $D_2 = \{a, c\}$ ,  $D_3 = \{b, c\}$ ,  $D_4 = \{d\}$ ,  $C = \{D_1, D_2, D_3, D_4\}$ ,  $X = \{a, d\}$ ,  $Y = \{a, b, c\}$ . Then  $B_1 = \{D_1, D_4\}$ ,  $B_2 = \{D_2, D_4\}$ ,  $B_3 = \{D_1, D_2, D_4\}$ ,  $B_4 = \{D_1, D_3, D_4\}$ ,  $B_5 = \{D_2, D_3, D_4\}$ , and  $B_6 = \{D_1, D_2, D_3, D_4\}$  are also coverings of  $X$ ; in other words,  $X \subseteq \cup B_i$  for  $i \in \{1, 2, 3, 4, 5, 6\}$ . So  $N \uparrow (X) = \min\{|B_i| : 1 \leq i \leq 6\} = 2$ .  $N \downarrow (X) = |\{K \in C | K \subseteq X\}| = |\{K_4\}| = 1$ . Similarly,  $N \uparrow (Y) = 2$  and  $N \downarrow (Y) = 3$ . In particular,  $N \uparrow \emptyset = 0$  since,  $\{\emptyset\} \subseteq \cup\{\emptyset\}$  and  $\{\emptyset\} \subseteq C$ . The result makes the concept of the covering numbers more reasonable.

**Lemma 4.1.** Let  $C$  be a covering of  $U$ . For all  $K \in C$ ,  $N \uparrow (K) = 1$ .

**Lemma 4.2.** Let  $C$  be a covering of  $U$ . For all  $x \in U$ ,  $N \uparrow (\{x\}) = 1$ .

## 5 Lattice for covering numbers

Lattices are important algebraical structures, and have a variety of applications in the real world. This subsection establishes a lattice structure and two semi-lattices in covering-based rough sets with covering numbers.

**Definition 5.1.** Let  $C$  be a covering of  $U$ . For all  $X, Y \subseteq U$ , if  $X \subseteq Y$  and  $N \uparrow (X) = N \uparrow (Y)$ , we call  $Y$  an upper-set of  $X$ , and  $X$  a lower-set of  $Y$ . The family of all upper-sets and the family of all lower-sets are semi-lattices.

**Proposition 5.1.** Let  $C$  be a covering of  $U$ . For all  $X \subseteq U$ , we call  $D_X, D'_X$  the family of all upper-sets, lower-sets of  $X$ , respectively, i.e.,  $D_X = \{Y \subseteq U : (X \subseteq Y) \wedge (N \uparrow (X) = N \uparrow (Y))\}$ ,  $D'_X = \{Y \subseteq U : (Y \subseteq X) \wedge (N \uparrow (X) = N \uparrow (Y))\}$ . Then  $(D_X, \cap)$ , and  $(D'_X, \cup)$  are semi-lattices.

*Proof.* In fact, we only need to prove  $Y_1 \cap Y_2 \in D_X$  for all  $Y_1, Y_2 \in D_X$ , and  $Y_1 \cup Y_2 \in D'_X$  for all  $Y_1, Y_2 \in D'_X$ . For all  $Y_1, Y_2 \in D_X$ ,  $N \uparrow (Y_1) = N \uparrow (X)$ ,  $X \subseteq Y_1$  and  $N \uparrow (Y_2) = N \uparrow (X)$ ,  $X \subseteq Y_2$ . So  $X \subseteq Y_1 \cap Y_2 \subseteq Y_1$ . Thus  $N \uparrow (X) \leq N \uparrow (Y_1 \cap Y_2) \leq N \uparrow (Y_1) = N \uparrow (X)$ , that is,  $N \uparrow (Y_1 \cap Y_2) = N \uparrow (X)$ . Therefore,  $Y_1 \cap Y_2 \in D_X$ . Similarly, we can prove  $Y_1 \cup Y_2 \in D'_X$  for all  $Y_1, Y_2 \in D'_X$ .  $\square$

**Definition 5.2.** Let  $C$  be a covering of  $U$  and  $|C| = n$ . For  $X \subseteq U$ , if  $N \downarrow (X) + N \downarrow (X^{co}) = n$ , we call  $X$  a detached-set of  $U$  with respect to  $C$ . With the detached-set, a covering is divided into two smaller coverings of two smaller domains. Moreover, the concept of the detached-set leads to a lattice structure.

**Proposition 5.2.** Let  $C$  be a covering of  $U$  and  $|C| = n$ .  $\mathbf{D}$  is denoted as the family of all detached-sets of  $U$ , i.e.,  $\mathbf{D} = \{X \subseteq U : N \downarrow (X) + N \downarrow (X^{co}) = n\}$ . Then  $(\mathbf{D}, \cup, \cap)$  is a lattice.

*Proof.* For all  $X, Y \in \mathbf{D}$ ,  $N \downarrow (X) + N \downarrow (X^{co}) = n$ ,  $N \downarrow (Y) + N \downarrow (Y^{co}) = n$ .  $2n = (N \downarrow (X) + N \downarrow (X^{co})) + (N \downarrow (Y) + N \downarrow (Y^{co})) \leq (N \downarrow (X \cup Y) + N \downarrow (X \cap Y)) + (N \downarrow (X^{co} \cup Y^{co}) + N \downarrow (X^{co} \cap Y^{co})) = [N \downarrow (X \cup Y) + N \downarrow ((X \cup Y)^{co})] + [N \downarrow (X \cap Y) + N \downarrow ((X \cap Y)^{co})]$ .  $N \downarrow (X \cup Y) + N \downarrow ((X \cup Y)^{co}) = n$  and  $N \downarrow (X \cap Y) + N \downarrow ((X \cap Y)^{co})$  since  $N \downarrow (X) + N \downarrow (X^{co}) \leq n$  for all  $X \subseteq U$ . Thus  $X \cup Y \in \mathbf{D}$  and  $X \cap Y \in \mathbf{D}$ .  $\square$

**Proposition 5.3.** *The covering lower and upper approximations have the following properties:*

- (1)  $X \downarrow_C \subseteq X \uparrow_C$
- (2)  $\emptyset \downarrow_C = \emptyset \uparrow_C = \emptyset$  and  $U \downarrow_C = U \uparrow_C = U$
- (3)  $(X \cap Y) \downarrow_C = X \downarrow_C \cap Y \downarrow_C$  and  $(X \cup Y) \uparrow_C = X \uparrow_C \cup Y \uparrow_C$
- (4)  $(X \downarrow_C) \downarrow_C = X \downarrow_C$  and  $(X \uparrow_C) \uparrow_C = X \uparrow_C$
- (5) If  $X \subseteq Y$  then  $X \downarrow_C \subseteq Y \downarrow_C$  and  $X \uparrow_C \subseteq Y \uparrow_C$
- (6)  $X \uparrow_C = \sim (\sim X) \downarrow_C$ .

## 6 Conclusion

In this paper, we investigated some fundamental issues of approximation in the context of rough set theory based on covering based rough set approximation. Lattice based on covering rough approximation and lattice for covering numbers are also introduced. Our discussion is based on the notion of lattice that represents the relationships between elements of a universe with neighborhood system. Furthermore one can find the lattice for successive rough approximation and stratified rough approximation based on covering system.

## References

- [1] Bargiela, A., Pedrycz, W. : Granular Computing: An Introduction, *Kluwer Academic Publishers, Boston*, 180, 2002.
- [2] Comer, S. : An algebraic approach to the approximation of information, *Fundamenta Informaticae*, 14(1991), 492-502.
- [3] Lin, T.Y. : Granular computing - structures, representations, and applications, *In: LNAI*, 2639(2003), 16 - 24.
- [4] Liu, G., Sai, Y. : Invertible approximation operators of generalized rough sets and fuzzy rough sets, *Information Sciences*, 180(2010), 2221-2229.
- [5] Pawlak, Z.: Rough sets: A new approach to vagueness. In: L.A. Zadeh and J. Kacprzyk, Eds. *Fuzzy Logic for the Management of Uncertainty*, New York, John Wiley and Sons, 105-118, 1992.
- [6] Pawlak, Z. : Rough sets : Theoretical aspects of reasoning about data, *Kluwer Academic Publishers, Boston*, 1991.
- [7] Pawlak, Z., Some issues on rough sets, *Transactions on Rough Set, I, Journal Subline, Lecture Notes in Computer Science*, 3100(2004), 1-58.
- [8] Rana, D., Roy, S. K. : Rough Set Approach on Lattice, *Journal of Uncertain Systems*, 5(1)(2011), 72-80.
- [9] Wang, F.Y. : Outline of a computational theory for linguistic dynamical systems: Toward computing with words, *International Journal of Intelligent Control and Systems*, 2(1998), 211-224.
- [10] Yao, Y. : Algebraic approach to rough sets, *Bull. Polish Acad. Sci. Math*, 35(1987), 673 - 683.
- [11] Yao, Y. : A partition model of granular computing, *LNCS*, 3100(2004), 232 - 253.
- [12] Yao, Y. : Constructive and algebraic methods of theory of rough sets, *Information Sciences*, 109, 21-47, 1998.
- [13] Yao, Y. : Granular computing: basic issues and possible solutions, *In: Proceedings of the 5th Joint Conference on Information Sciences*, 1(2000), 186 - 189.
- [14] Yao, Y. : Three-way decisions with probabilistic rough sets, *Information Science*, 180(2010), 341-353.
- [15] Zadeh, L. A. : Fuzzy Logic = computing with words, *IEEE Transactions on Fuzzy Systems*, 4(1996), 103-111.
- [16] Zadeh, L. A. : Fuzzy Sets, *Information and Control*, 8(1965), 338-353.

- [17] Zhu, W.: Topological Approaches to Covering Rough Sets, *Information Sciences*, 177(2007), 1499-1508.
- [18] Zhu, W.: Relationship among Basic Concepts in Covering-based Rough Sets, *Information Sciences*, 179(14)(2007), 2478-2486.
- [19] Zhu, W.: Basic Concepts in Covering-Based Rough Sets, in ICNC'07, *Haikou, China, 24-27 August, 2007*, 5, 283-286, 2007.
- [20] Zhu, W., Wang, F.Y.: On Three Types of Covering Rough Sets, *IEEE Transactions On Knowledge and Data Engineering*, 19(8)(2007), 1131-1144.

*Received:* October 19, 2012; *Accepted:* April 14, 2014

**UNIVERSITY PRESS**

Website: <http://www.malayajournal.org/>