Malaya J. Mat. 2(3)(2014) 236–242

Malaya Journal of Matematik
MJM
an international journal of mathematical sciences with computer applications...
www.malayajournal.org
ISSN : 2319-3786

Generating relations involving 2-variable Hermite matrix polynomials

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Abstract

In the present paper, some generating relations involving the 2-variable Hermite matrix polynomials are derived by using operational techniques. Further, some new and known generating relations for the scalar Hermite polynomials are obtained as applications of the main results.

Keywords: Hermite matrix polynomials, Generating relations, Operational techniques.

2010 MSC: 15A60, 33C05, 33C25, 33C45, 33C50.

1 Introduction

An important generalization of special functions is special matrix functions. The study of special matrix polynomials is important due to their applications in certain areas of statistics, physics and engineering. The Hermite matrix polynomials are introduced by Jódar and Company in [12]. Some properties of the Hermite matrix polynomials are given in [9, 10, 12, 13, 14]. The extensions and generalizations of Hermite matrix polynomials have been introduced and studied in [2, 3, 15, 16, 19] for matrices in $\mathbb{C}^{N\times N}$ whose eigenvalues are all situated in the right open half-plane.

Throughout this paper, for a matrix A in $\mathbb{C}^{N\times N}$, its spectrum $\sigma(A)$ denotes the set of all the eigenvalues of A. If $f(z)$ and $g(z)$ are holomorphic functions of the complex variable z, which are defined in an open set Ω of the complex plane and if A is a matrix in $\mathbb{C}^{N\times N}$ such that $\sigma(A)\subset\Omega$, then the matrix functional calculus [11] yields that

$$f(A)g(A) = g(A)f(A).$$

If D_0 is the complex plane cut along the negative real axis and $\log(z)$ denotes the principal logarithm of z, then $z^{\frac{1}{2}}$ represents $\exp(\frac{1}{2}\log(z))$. If A is a matrix with $\sigma(A)\subset D_0$, then $A^{\frac{1}{2}}=\sqrt{A}=\exp(\frac{1}{2}\log(A))$ denotes the image by $z^{\frac{1}{2}}=\sqrt{z}=\exp(\frac{1}{2}\log(z))$ of the matrix functional calculus acting on the matrix A. We say that A is a positive stable matrix [10] if

$$Re(z) > 0, \quad \text{for all } z\in\sigma(A). \tag{1.1}$$

We recall that the 2-variable Hermite matrix polynomials (2VHMaP) $H_n(x,y,A)$ are defined by the series [2; p.84]

$$H_n(x,y,A) = n!\sum_{k=0}^{[\frac{n}{2}]}\frac{(-1)^k y^k (x\sqrt{2A})^{n-2k}}{(n-2k)!k!} \quad (n\geq 0) \tag{1.2}$$

and specified by the generating function

$$\exp(xt\sqrt{2A}-yt^2I) = \sum_{n=0}^{\infty} H_n(x,y,A)\frac{t^n}{n!}. \tag{1.3}$$

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It is worth to mention that these matrix polynomials are linked to 2-variable Hermite-Kampé de Fériet polynomials (2VHKdFP) $H_n(x,y)$ [1] by the following relation:

$$H_n(x,y,A) = H_n(x\sqrt{2A}, -y), \tag{1.4a}$$

or, equivalently

$$H_n\left((\sqrt{2A})^{-1}x, -y, A\right) = H_n(x,y), \tag{1.4b}$$

where $H_n(x,y)$ are defined by the series [1]

$$H_n(x,y) = n!\sum_{k=0}^{[\frac{n}{2}]} \frac{y^k x^{n-2k}}{k!(n-2k)!} \tag{1.5}$$

and specified by the generating function

$$\exp(xt+yt^2) = \sum_{n=0}^{\infty} H_n(x,y)\frac{t^n}{n!}. \tag{1.6}$$

Also, for $A = \frac{1}{2} \in \mathbb{C}^{1\times 1}$ in equation (1.3) and in view of generating function (1.6), we have

$$H_n\left(x, -y, \frac{1}{2}\right) = H_n(x,y). \tag{1.7}$$

In particular, we note that

$$H_n(x,y,A) = y^{\frac{n}{2}} H_n\left(\frac{x}{\sqrt{y}}, A\right), \tag{1.8}$$

$$H_n(x,1,A) = H_n(x,A), \tag{1.9}$$

where $H_n(x,A)$ denotes the Hermite matrix polynomials (HMaP) defined by [12]

$$\exp(xt\sqrt{2A} - t^2 I) = \sum_{n=0}^{\infty} H_n(x,A)\frac{t^n}{n!} \tag{1.10}$$

and linked to the classical Hermite polynomials $H_n(x)$ [18] by the following relation:

$$H_n(x,A) = H_n\left(x\sqrt{\frac{A}{2}}\right), \tag{1.11}$$

where $H_n(x)$ are defined by [18]

$$H_n(x) = n!\sum_{k=0}^{[\frac{n}{2}]} \frac{(-1)^k (2x)^{n-2k}}{k!(n-2k)!}. \tag{1.12}$$

The 2VHMaP $H_n(x,y,A)$ are also defined by the following operational rule [2; p.90]:

$$H_n(x,y,A) = \exp\left(-y(2A)^{-1}\frac{\partial^2}{\partial x^2}\right)\left\{(x\sqrt{2A})^n\right\} \tag{1.13}$$

and have the following representation [3; p.99]:

$$H_n(x,y,A) = \left(x\sqrt{2A} - 2y(\sqrt{2A})^{-1}\frac{\partial}{\partial x}\right)^n \{I\}. \tag{1.14}$$

Recently, Dattoli and his co-workers have shown that operational methods can be used to simplify the derivations of many properties of ordinary and generalized special functions and also provide a unique tool to treat various polynomials from a general and unified point of view, see for example [4-8]. In this paper, we derive some generating relations involving the 2VHMaP $H_n(x,y,A)$ which further prove the usefulness of the methods of operational nature.

2 Generating relations

We prove the following results by using operational techniques:

Theorem 2.1. *For a matrix A in $\mathbb{C}^{N\times N}$ satisfying condition (1.1), the following generating relation involving the 2VHMaP $H_n(x,y,A)$ holds true:*

$$\sum_{n=0}^{\infty} H_{2n}(x,y,A)\frac{t^n}{n!} = \frac{1}{\sqrt{1+4yt}} \exp\left(\frac{2Ax^2t}{1+4yt}\right). \tag{2.1}$$

Proof. By making use of equation (1.13) in the l.h.s. of equation (2.1), we find

$$\sum_{n=0}^{\infty} H_{2n}(x,y,A)\frac{t^n}{n!} = \exp\left(-y(2A)^{-1}\frac{\partial^2}{\partial x^2}\right)\sum_{n=0}^{\infty}(x\sqrt{2A})^{2n}\frac{t^n}{n!}, \tag{2.2}$$

which on using the exponential function becomes

$$\sum_{n=0}^{\infty} H_{2n}(x,y,A)\frac{t^n}{n!} = \exp\left(-y(2A)^{-1}\frac{\partial^2}{\partial x^2}\right)\ \exp(2Ax^2t). \tag{2.3}$$

Using the generalized Glaisher identity [6]

$$\exp\left(\lambda\frac{d^2}{dx^2}\right)\left\{\exp(-ax^2+bx)\right\} = \frac{1}{\sqrt{1+4a\lambda}}\exp\left(-\frac{ax^2-bx-b^2\lambda}{1+4a\lambda}\right), \tag{2.4}$$

with $b=0$ in the r.h.s. of equation (2.3), we get assertion (2.1) of Theorem 2.1. □

Remark 2.1. *Taking $y=1$ and replacing t by $-\left(\frac{t}{2}\right)^2$ in assertion (2.1) of Theorem 2.1 and using equation (1.9), we get the result [9; p.122]*

$$\sum_{n=0}^{\infty}(-1)^n\ H_{2n}(x,A)\frac{t^n}{n!\,2^{2n}} = (1-t^2)^{-\frac{1}{2}}\exp\left(\frac{A}{2}\ \frac{-x^2t^2}{(1-t^2)}\right). \tag{2.5}$$

Theorem 2.2. *For a matrix A in $\mathbb{C}^{N\times N}$ satisfying condition (1.1), the following generating relation involving the 2VHMaP $H_n(x,y,A)$ holds true:*

$$\sum_{n=0}^{\infty} H_{n+k}(x,y,A)\frac{t^n}{n!} = \exp(xt\sqrt{2A}-yt^2I)\ H_k\left(xI-yt\left(\sqrt{\frac{A}{2}}\right)^{-1},y,A\right). \tag{2.6}$$

Proof. By making use of equation (1.14) in the l.h.s. of equation (2.6), we find

$$\sum_{n=0}^{\infty} H_{n+k}(x,y,A)\frac{t^n}{n!} = \sum_{n=0}^{\infty}\left(x\sqrt{2A}-2y(\sqrt{2A})^{-1}\frac{\partial}{\partial x}\right)^{n+k}\frac{t^n}{n!}, \tag{2.7}$$

which on simplifying the r.h.s. and again using equation (1.14) becomes

$$\sum_{n=0}^{\infty} H_{n+k}(x,y,A)\frac{t^n}{n!} = \exp\left(xt\sqrt{2A}-2yt(\sqrt{2A})^{-1}\frac{\partial}{\partial x}\right)\ H_k(x,y,A). \tag{2.8}$$

Now, decoupling the exponential operator in the r.h.s. of the above equation by using the Weyl identity [7]

$$e^{\hat{A}+\hat{B}} = e^{\hat{A}}e^{\hat{B}}e^{-k/2} \qquad \left([\hat{A},\hat{B}]=k,\ k\in\mathbb{C}\right), \tag{2.9}$$

we get

$$\sum_{n=0}^{\infty} H_{n+k}(x,y,A)\frac{t^n}{n!} = \exp(xt\sqrt{2A}-yt^2I)\exp\left(-2yt(\sqrt{2A})^{-1}\frac{\partial}{\partial x}\right)H_k(x,y,A). \tag{2.10}$$

Using the shift operator [7]

$$\exp\left(\lambda\frac{\partial}{\partial x}\right)f(x) = f(x+\lambda), \tag{2.11}$$

in the r.h.s. of equation (2.10), we get assertion (2.6) of Theorem 2.2. □

Remark 2.2. *Taking $y = 1$ in assertion (2.6) of Theorem 2.2 and using equation (1.9), we get the result [15, p. 170] with $b = 1$*

$$\sum_{n=0}^{\infty} H_{n+k}(x,A)\frac{t^n}{n!} = \exp(xt\sqrt{2A} - t^2 I)\ H_k\left(xI - t\left(\sqrt{\frac{A}{2}}\right)^{-1}, A\right). \tag{2.12}$$

Theorem 2.3. *For a matrix A in $\mathbb{C}^{N\times N}$ satisfying condition (1.1), the following bilinear generating relation of the 2VHMaP $H_n(x,y,A)$ holds true:*

$$\sum_{n=0}^{\infty} H_n(x,y,A)H_n(z,w,A)\frac{t^n}{n!} = \frac{1}{\sqrt{1-4ywt^2}}\ \exp\left(\frac{2A(xzt-(x^2w+z^2y)t^2)}{1-4ywt^2}\right). \tag{2.13}$$

Proof. By making use of equation (1.13) in the l.h.s. of equation (2.13), we find

$$\sum_{n=0}^{\infty} H_n(x,y,A)H_n(z,w,A)\frac{t^n}{n!} = \exp\left(-y(2A)^{-1}\frac{\partial^2}{\partial x^2}\right)\sum_{n=0}^{\infty} H_n(z,w,A)\ \frac{(xt\sqrt{2A})^n}{n!}, \tag{2.14}$$

which on using generating function (1.3) becomes

$$\sum_{n=0}^{\infty} H_n(x,y,A)H_n(z,w,A)\frac{t^n}{n!} = \exp\left(-y(2A)^{-1}\frac{\partial^2}{\partial x^2}\right)\ \exp(2Axzt - 2Aw(xt)^2). \tag{2.15}$$

Using the generalized Glaisher identity (2.4) in the r.h.s. of equation (2.15), we get assertion (2.13) of Theorem 2.3. □

Remark 2.3. *Taking $w = 1$ in assertion (2.13) of Theorem 2.3 and using equation (1.9), we deduce the following consequence of Theorem 2.3.*

Corollary 2.1. *For a matrix A in $\mathbb{C}^{N\times N}$ satisfying condition (1.1), the following generating relation involving the 2VHMaP $H_n(x,y,A)$ and HMaP $H_n(z,A)$ holds true:*

$$\sum_{n=0}^{\infty} H_n(x,y,A)H_n(z,A)\frac{t^n}{n!} = \frac{1}{\sqrt{1-4yt^2}}\ \exp\left(\frac{2A(xzt-(x^2+z^2y)t^2)}{1-4yt^2}\right). \tag{2.16}$$

Remark 2.4. *Taking $y = w = 1$ and replacing t by $\frac{t}{2}$ in assertion (2.13) of Theorem 2.3 and using equation (1.9), we get the result [13] (see [9])*

$$\sum_{n=0}^{\infty} H_n(x,A)H_n(z,A)\frac{t^n}{n!\,2^n} = (1-t^2)^{-\frac{1}{2}}\ \exp\left(\frac{A}{2}\ \frac{2xzt-(x^2+z^2)t^2}{(1-t^2)}\right). \tag{2.17}$$

It is worthy to mention that all the above main results can be proved alternately by using the series rearrangement techniques.

3 Special cases

In this section, we derive some new generating relations for Hermite polynomials in terms of matrix argument as applications of the results derived in Section 2.

I. Replacing y by $-y$ in equation (2.1) and making use of equation (1.4a) in the resultant equation, we get

$$\sum_{n=0}^{\infty} H_{2n}(x\sqrt{2A},y)\frac{t^n}{n!} = \frac{1}{\sqrt{1-4yt}}\exp\left(\frac{2Ax^2t}{1-4yt}\right), \tag{3.1}$$

which is new generating relation for the 2VHKdFP $H_n(x,y)$ in terms of matrix argument and is a generalization of the generating relation [8, p. 412]

$$\sum_{n=0}^{\infty} H_{2n}(x,y)\frac{t^n}{n!} = \frac{1}{\sqrt{1-4yt}}\exp\left(\frac{x^2t}{1-4yt}\right). \tag{3.2}$$

Again, making use of equation (1.11)in equation (2.5), we get

$$\sum_{n=0}^{\infty} H_{2n}\left(x\sqrt{\frac{A}{2}}\right)\frac{t^n}{n!\,2^{2n}} = (1-t^2)^{-\frac{1}{2}}\exp\left(\frac{A}{2}\,\frac{-x^2t^2}{(1-t^2)}\right), \tag{3.3}$$

which is new generating relation for the classical Hermite polynomials $H_n(x)$ in terms of matrix argument.

II. Replacing y by $-y$ in equation (2.6) and making use of equation (1.4a) in the resultant equation, we get

$$\sum_{n=0}^{\infty} H_{n+k}(x\sqrt{2A}, y)\frac{t^n}{n!} = \exp(xt\sqrt{2A} + yt^2 I)\; H_k(x\sqrt{2A} + 2yt, y), \tag{3.4}$$

which is new generating relation for the 2VHKdFP $H_n(x,y)$ in terms of matrix argument and is a generalization of the generating relation [17, p. 452]

$$\sum_{n=0}^{\infty} H_{n+k}(x,y)\frac{t^n}{n!} = \exp(xt + yt^2)\; H_k(x + 2yt, y). \tag{3.5}$$

Again, making use of equation (1.11) in equation (2.12), we get

$$\sum_{n=0}^{\infty} H_{n+k}\left(x\sqrt{\frac{A}{2}}\right)\frac{t^n}{n!} = \exp(xt\sqrt{2A} - t^2 I)\; H_k\left(x\sqrt{\frac{A}{2}} - tI\right), \tag{3.6}$$

which is new generating relation for the classical Hermite polynomials $H_n(x)$ in terms of matrix argument and is a generalization of the generating relation [18, p. 197]

$$\sum_{n=0}^{\infty} H_{n+k}(x)\frac{t^n}{n!} = \exp(2xt - t^2)\; H_k(x - t). \tag{3.7}$$

Next, replacing t by $t\sqrt{2A}$ in equation (2.12), we obtain the generating relation [20, p. 191]

$$\sum_{n=0}^{\infty} H_{n+k}(x, A)\frac{(t\sqrt{2A})^n}{n!} = \exp(2xtA - 2t^2A)\; H_k\left(x - 2t, A\right). \tag{3.8}$$

III. Replacing y by $-y$ and w by $-w$ in equation (2.13) and making use of equation (1.4a) in the resultant equation, we get

$$\sum_{n=0}^{\infty} H_n(x\sqrt{2A}, y)H_n(z\sqrt{2A}, w)\frac{t^n}{n!} = \frac{1}{\sqrt{1-4ywt^2}}\;\exp\left(\frac{2A(xzt + (x^2w + z^2y)t^2)}{1-4ywt^2}\right), \tag{3.9}$$

which is new bilinear generating relation for the 2VHKdFP $H_n(x,y)$ in terms of matrix argument and is a generalization of the generating relation [5, p. 116] (see also [17, p. 453])

$$\sum_{n=0}^{\infty} H_n(x,y)H_n(z,w)\frac{t^n}{n!} = \frac{1}{\sqrt{1-4ywt^2}}\;\exp\left(\frac{xzt + (x^2w + z^2y)t^2}{1-4ywt^2}\right). \tag{3.10}$$

Again, replacing y by $-y$ in equation (2.16) and making use of equations (1.4a) and (1.11) in the resultant equation, we get

$$\sum_{n=0}^{\infty} H_n(x\sqrt{2A}, y)H_n\left(z\sqrt{\frac{A}{2}}\right)\frac{t^n}{n!} = \frac{1}{\sqrt{1+4yt^2}}\;\exp\left(\frac{2A(xzt - (x^2 - z^2y)t^2)}{1+4yt^2}\right), \tag{3.11}$$

which is new generating relation for the 2VHKdFP $H_n(x,y)$ and the classical Hermite polynomials $H_n(x)$ in terms of matrix argument.

Further, making use of equation (1.11) in equation (2.17), we get

$$\sum_{n=0}^{\infty} H_n\left(x\sqrt{\frac{A}{2}}\right)H_n\left(z\sqrt{\frac{A}{2}}\right)\frac{t^n}{n!\,2^n} = (1-t^2)^{-\frac{1}{2}}\;\exp\left(\frac{A}{2}\,\frac{2xzt - (x^2+z^2)t^2}{(1-t^2)}\right), \tag{3.12}$$

which is new bilinear generating relation for the classical Hermite polynomials $H_n(x)$ in terms of matrix argument and is a generalization of the generating relation [18, p. 198]

$$\sum_{n=0}^{\infty} H_n(x)H_n(z)\frac{t^n}{n!} = \frac{1}{\sqrt{1-4t^2}}\;\exp\left(\frac{4(xzt - (x^2+z^2)t^2)}{1-4t^2}\right). \tag{3.13}$$

4 Concluding remarks

Recently, Subuhi Khan and Raza [15] introduced the 2-variable Hermite matrix polynomials of the second form $\mathcal{H}_n(x,y;A)$, defined by the series [15, p. 162]

$$\mathcal{H}_n(x,y;A) = n! \sum_{k=0}^{[\frac{n}{2}]} \frac{y^k \left(x\sqrt{\frac{A}{2}}\right)^{n-2k}}{k!(n-2k)!} \tag{4.1}$$

and specified by the generating function

$$\exp\left(xt\sqrt{\frac{A}{2}} + yt^2 I\right) = \sum_{n=0}^{\infty} \mathcal{H}_n(x,y;A)\frac{t^n}{n!}. \tag{4.2}$$

From generating functions (1.3) and (4.2), we note that the 2VHMaP of the second form $\mathcal{H}_n(x,y;A)$ are linked to the 2VHMaP $H_n(x,y;A)$ by the following relation:

$$\mathcal{H}_n(x,y;A) = H_n\left(\frac{x}{2}, -y, A\right). \tag{4.3}$$

In view of equation (4.3), we conclude that all the properties of the 2VHMaP of the second form $\mathcal{H}_n(x,y;A)$ can be deduced from the corresponding ones for 2VHMaP $H_n(x,y;A)$. For example, replacing x by $\frac{x}{2}$ and y by $-y$ in the main results (2.1), (2.6) and (2.13), we get the following generating relations involving the 2VHMaP of the second form $\mathcal{H}_n(x,y;A)$:

$$\sum_{n=0}^{\infty} \mathcal{H}_{2n}(x,y,A)\frac{t^n}{n!} = \frac{1}{\sqrt{1-4yt}} \exp\left(\frac{Ax^2t}{2(1-4yt)}\right), \tag{4.4}$$

$$\sum_{n=0}^{\infty} \mathcal{H}_{n+k}(x,y,A)\frac{t^n}{n!} = \exp\left(xt\sqrt{\frac{A}{2}} + yt^2 I\right) \mathcal{H}_k\left(xI + 2yt\left(\sqrt{\frac{A}{2}}\right)^{-1}, y, A\right) \tag{4.5}$$

and

$$\sum_{n=0}^{\infty} \mathcal{H}_n(x,y,A)\, \mathcal{H}_n(z,w,A)\frac{t^n}{n!} = \frac{1}{\sqrt{1+4ywt^2}} \exp\left(\frac{A(4xzt - (x^2w - 4z^2y)t^2)}{1+4ywt^2}\right), \tag{4.6}$$

respectively. It is therefore clear that by making use of relation (4.3) in some other generating functions obtained in Section 2, we may get a number of interesting results for the 2VHMaP of the second form $\mathcal{H}_n(x,y;A)$.

In this article, generating relations involving the Hermite matrix polynomials are introduced by making use of operational identities for decoupling of exponential operators. The approach presented here can be explored further to derive the results for some other suitable families of special matrix functions.

5 Acknowledgment

The author is thankful to the referee for valuable comments for improving the presentation of the paper.

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Received: January 31, 2014; *Accepted*: March 23, 2014

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