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A stochastic model for a single grade system with backup resource of manpower

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Abstract

In this paper, an organization with single grade subjected to exodus of personnel due to policy decisions taken by it, is considered. In order to avoid the crisis of the organization reaching a breakdown point, a suitable univariate policy recruitment based on shock model approach and cumulative damage process is suggested. A mathematical model is constructed and a performance measure namely the mean time to recruitment is obtained. The analytical results are numerically illustrated and the influences of nodal parameters on the performance measures are studied and relevant conclusions are presented.

Keywords: Single grade system, Univariate policy of recruitment, shock model.

2010 MSC: 90B70, 91B40, 91D35.

1 Introduction

Frequent wastage or exit of personnel is common in many administrative and production oriented organization. Whenever the organization announces revised policies regarding sales target, revision of wages, incentives and perquisites, the exodus is possible. Reduction in the total strength of marketing personnel adversely affects the sales turnover in the organization. As the recruitment involves several costs, it is usual that the organization has the natural reluctance to go in for frequent recruitments. Once the total amount of wastage crosses a certain threshold level, the organization reaches an uneconomic status which otherwise be called the breakdown point and recruitment is done at this point of time. The time to attain the breakdown point is an important characteristic for the management of the organization. Many models could be seen in, Barthlomew [1] and Barthlomew and Forbes [2].Many researchers [3] [4] and[6] have considered the problem of time to recruitment in a marketing organization under different conditions. Srinivasan and Saavithri [5] have considered a single grade system under univariate policy of recruitment with the assumption that survival times follow geometric process and the threshold level as a non-negative constant. They have obtained mean time to recruitment and the long run average cost. Uma.et.al [7] have studied the work of Srinivasan and Saavithri [5] by considering the threshold level of the organization as continuous random variable following exponential distribution and having SCBZ property. Recently, Vijaysankar et.al [8] have constructed a stochastic model by assuming the threshold with two components namely the level of wastage which can be allowed and the manpower which is available from what is known as backup resource. The threshold can be treated now as the total of the maximum allowable attrition and the maximum available backup resource. The backup resource is similar to the manpower inventory on hand which can be utilized whenever it becomes necessary. The present paper studies the problem of time to recruitment for a single grade system with survival times follow geometric process and the threshold level has two components.

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2 Model Description

Consider a single grade organization with univariate policy of recruitment which takes decisions at random epoch. At every decision making epoch a random number of persons quit the organization. There is an associated loss of manhours to the organization if a person quits. The loss of manhours at any decision forms a sequence of independent and identically distributed random variables. The survival time process is a geometric process and it is independent of process of loss of manhour. There is a threshold level for the level of wastage and also a resource backup available. If the total loss of manhours crosses the sum of the threshold and the resource backup available the break down occurs. The process that generates the loss of manhours and the threshold put together with the backup are linearly independent. Recruitment takes place only at decision points and time of recruitment is negligible. . The recruitment is made whenever the cumulative loss of manhours exceeds its threshold.

3 Notations

S_n : survival time after $(n-1)^{th}$ decision.

X_n : the loss of manhours at then^{th} decision.

T_n : the cumulative loss of manhours in the first $n$decisions.

$K(\cdot)$: distribution function of S_n with mean $\frac{\lambda}{a^{n-1}}, \quad a > 1$.

$G(\cdot)$: distribution function of $X_n,\ n = 1, 2, 3, \ldots.$

$G_n(\cdot)$: distribution function ofT_n.

T : The threshold of manpower depletion and$T = Y_1 + Y_2$.

$(i) Y_1$ = the maximum allowable attrition.

$(ii) Y_2$ = the maximum available backup resource.

$F(\cdot)$: distribution function of$T$.

W : time to recruitment under the given recruitment policy.

4 Results

In this section the expected time to recruitment is derived.

By assumption the recruitment is made whenever the cumulative loss of manhours exceeds the threshold T. Accordingly the time to recruitment $W = S_1$, if $T_1 > T$. If $T_1 \leq T$ then no recruitment is made till the next decision. If T_2 exceeds T then recruitment is made and $W = S_1 + S_2$, otherwise no recruitment. In general, if $T_k > T$ then recruitment is made and $W = S_1 + S_2 + ... + S_k$ and if $T_k \leq T$ no recruitment is made till the next decision.

Consequently,

$$W = \sum_{i=0}^{\infty} \sum_{j=0}^{i} S_{j+1} \chi(T_i \leq T \leq T_{i+1}), \tag{1.1}$$

where

$$\chi(e) = \begin{cases} 1, & \text{if the event } e \text{ happens} \\ 0, & \text{if the event } e \text{ does not happen} \end{cases} \tag{1.2}$$

The expected time to recruitment $E(W)$ is given by

$$E(W) = \sum_{i=0}^{\infty}\sum_{j=0}^{i} E(S_{j+1})P(T_i \leq T \leq T_{i+1}) \tag{1.3}$$

$$= \sum_{i=0}^{\infty}\sum_{j=0}^{i} \left(\frac{\lambda}{a^j}\right) P(0 \leq T - T_i < T_{i+1} - T_i)$$

$$= \sum_{i=0}^{\infty}\sum_{j=0}^{i} \left(\frac{\lambda}{a^j}\right) \int_0^{\infty}\int_0^{\infty}\int_t^{t+s} dF(u)dG(s)dG_i(t). \tag{1.4}$$

Assume that the loss of manhours at the i^{th} decision X_i, follows exponential distribution with parameter θ_1. Then the cumulative loss of manhours T_i follows gamma distribution with parameter θ_1 and i. Hence

$$dG_i(t) = \theta_1^i t^{i-1}\frac{e^{-\theta_1 t}}{(i-1)!}dt, \quad i = 1, 2, 3, \ldots,$$

Let $Y_1 \sim exp(\theta_2)$ and $Y_2 \sim exp(\theta_3)$. Since the p.d.f of T is the convolution of $T_1 + T_2$ and it is given by

$$f(u) = \frac{\theta_2\theta_3}{\theta_2 - \theta_3}\left[e^{-\theta_3 u} - e^{-\theta_2 u}\right].$$

Now the time to recruitment in equation (1.4) becomes

$$E(W) = \sum_{i=0}^{\infty}\sum_{j=0}^{i} \left(\frac{\lambda}{a^j}\right) \int_0^{\infty}\int_0^{\infty}\int_t^{t+s} \frac{\theta_2\theta_3}{\theta_2 - \theta_3}\left[e^{-\theta_3 u} - e^{-\theta_2 u}\right] dudG(s)dG_i(t)$$

$$= \sum_{i=0}^{\infty}\left(\frac{a\lambda}{a-1}\right)\left(1 - \frac{1}{a^{i+1}}\right) \int_0^{\infty}\int_0^{\infty}\int_t^{t+s} \frac{\theta_2\theta_3}{\theta_2 - \theta_3}\left[e^{-\theta_3 u} - e^{-\theta_2 u}\right] dudG(s)dG_i(t)$$

$$= \sum_{i=0}^{\infty}\left(\frac{a\lambda}{a-1}\right)\left(1 - \frac{1}{a^{i+1}}\right)\left(\frac{\theta_2\theta_3}{\theta_2 - \theta_3}\right) \int_0^{\infty}\int_0^{\infty}\int_t^{t+s} \left[e^{-\theta_3 u} - e^{-\theta_2 u}\right] dudG(s)dG_i(t)$$

On simplification, the time to recruitment is

$$E(W) = \frac{a\lambda\theta_2\theta_3}{(a-1)(\theta_2 - \theta_3)}\left\{\frac{a(\theta_3 + \theta_1) - \theta_1 - \theta_3}{a\theta_3(\theta_1 + \theta_3) - \theta_1\theta_3} - \frac{a(\theta_2 + \theta_1) - \theta_2 - \theta_1}{a\theta_2(\theta_1 + \theta_2) - \theta_1\theta_2}\right\}.$$

5 Numerical Illustration

The value of $E(W)$ can be determined numerically using the above expression when the values of the various parameters are given. The changes in $E(W)$ consequent to the changes in each of these parameters when other parameters are kept fixed are also possible.

Effect of loss of manhours on performance measure:

θ_1	$E(W)$
0.1	14.7429
0.2	15.2800
0.3	15.6848
0.4	16.0000
0.5	16.2517
0.6	16.4571
0.7	16.6277
0.8	16.7714
0.9	16.8941
1.0	17.0000

Table 1.1
($\lambda = 2, a = 2, \theta_2 = 0.3, \theta_3 = 0.4$)

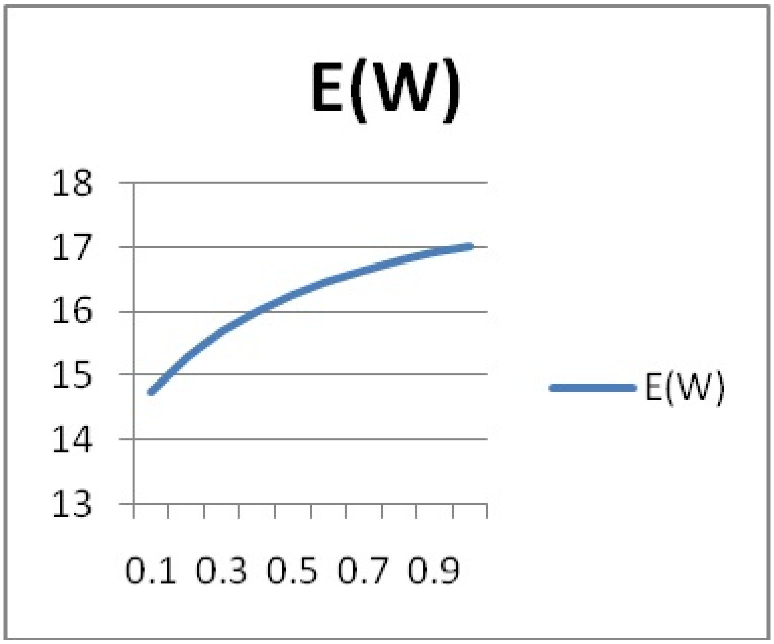

Figure 1: Figure 1.11

Effect of λ on performance measure

λ	$E(W)$
1.05	7.7400
1.10	8.1086
1.15	8.4771
1.20	8.8457
1.25	9.2143
1.30	9.5829
1.35	9.9514
1.40	10.3200
1.45	10.6886
1.50	11.0571

Table 1.2
$(\theta_1 = 0.1, a = 2, \theta_2 = 0.3, \theta_3 = 0.4)$

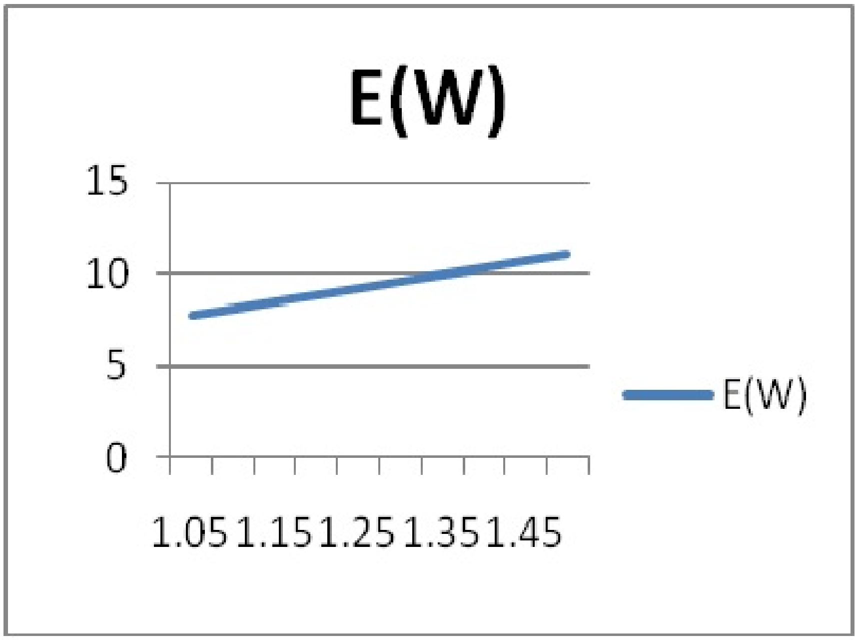

Figure 2: Figure 1.21

Effect of a on performance measure

a	$E(W)$
2	7.3714
3	5.5851
4	4.9825
5	4.6793
6	4.4967
7	4.3746
8	4.2873
9	4.2217
10	4.1706

Table 1.3
$(\theta_1 = 0.1, a = 2, \theta_2 = 0.3, \lambda = 2)$

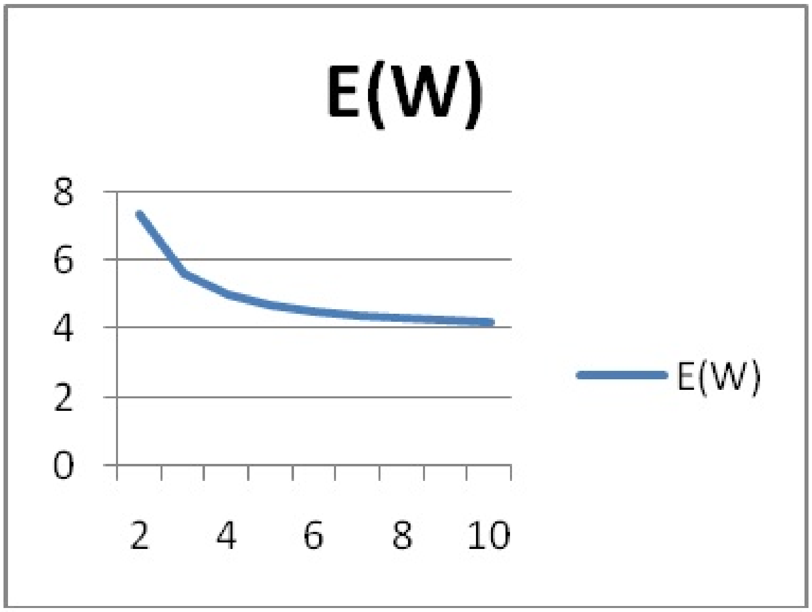

Figure 3: Figure 1.31

6 Conclusion

From the above tables we observe the following :

Case(i): If the value of the parameter θ_1 increases, the mean loss of manhours decreases and hence the expected time to recruitment increases as shown in Table 1.1 and Figure 1.11.

Case(ii): If the value of λ increases ,the expected time to recruitment $E(W)$ also increase as shown in Table 1.2 and Figure 1.21.

Case(iii): As a increases the mean survival time $\frac{\lambda}{a^{i-1}}$ decreases and hence the expected time to recruitment decreases as in Table 1.3 and Figure1.31.

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