Malaya J. Mat. 2(3)(2014) 315–321

Malaya Journal of Matematik
MJM
an international journal of mathematical sciences with computer applications...
www.malayajournal.org
ISSN : 2319-3786

On pre-$\mathcal{I}_s$-open sets and pre-$\mathcal{I}_s$-continuous functions

R. Santhi[a,*] and M. Rameshkumar[b]

[a]Department of Mathematics, NGM College, Pollachi-642001, Tamil Nadu, India.

[b]Department of Mathematics, P. A College of Engineering and Technology, Pollachi-642002, Tamil Nadu, India.

Abstract

We study the notion of pre-$\mathcal{I}_s$-open and pre-$\mathcal{I}_s$-continuous and obtain some properties. Then, we introduce and investigate pre-$\mathcal{I}_s$-open functions and pre-$\mathcal{I}_s$-closed functions. Also we obtain a decomposition of continuity via idealization.

Keywords: pre-$\mathcal{I}_s$-open set, pre-$\mathcal{I}_s$-continuous function.

2010 MSC: 54C05, 54C10.

1 Introduction

Ideal in topological space have been considered since 1930 by Kuratowski[9] and Vaidyanathaswamy[15]. After several decades, in 1990, Jankovic and Hamlett[6] investigated the topological ideals which is the generalization of general topology. Where as in 2010, Khan and Noiri[7] introduced and studied the concept of semi-local functions. The notion of pre-open sets and pre-continuity was first introduced and investigated by Mashhour et. al. [11] in 1982. Finally in 1996, Dontchev [3] introduced the notion of pre-$\mathcal{I}$-open sets and pre-$\mathcal{I}$-continuity in ideal topological spaces. Recently we introduced pre-$\mathcal{I}_s$-open sets and pre-$\mathcal{I}_s$-continuity to obtain decomposition of continuity.

In this paper we study the notion of pre-$\mathcal{I}_s$-open and pre-$\mathcal{I}_s$-continuous and obtain some properties. We introduce and investigate pre-$\mathcal{I}_s$-open functions and pre-$\mathcal{I}_s$-closed functions. Also we obtain a decomposition of continuity via idealization.

2 Preliminaries

Let (X,τ) be a topological space with no separation properties assumed. For a subset A of a topological space (X,τ), $cl(A)$ and $int(A)$ denote the closure and interior of A in (X,τ) respectively.

An ideal $\mathcal{I}$ on a topological space (X,τ) is a nonempty collection of subsets of X which satisfies: (1) $A \in \mathcal{I}$ and $B \subseteq A$ implies $B \in \mathcal{I}$ (2)$A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$.

If $(X,\tau)$is a topological space and $\mathcal{I}$ is an ideal on X, then $(X,\tau,\mathcal{I})$ is called an ideal topological space or an ideal space.

Let $P(X)$ be the power set of X. Then the operator $()^* : P(X) \to P(X)$ called a local function [9] of A with respect to τ and $\mathcal{I}$, is defined as follows: for $A \subseteq X$, $A^*(\mathcal{I},\tau) = \{x \in X : U \cap A \notin \mathcal{I}$ for every open set U containing $x\}$. We simply write A^* instead of $A^*(\mathcal{I},\tau)$ in case there is no confusion. For every ideal topological space $(X,\tau,\mathcal{I})$ there exists topology τ^* finer than τ, generated by $\beta(\mathcal{I},\tau) = \{U \setminus J : U \in \tau$ and $J \in \mathcal{I}\}$ but in general $\beta(\mathcal{I},\tau)$ is not always a topology. Additionally $cl^*(A) = A \cup A^*$ defines Kuratowski closure operator for a topology τ^* finer than τ. Throughout this paper X denotes the ideal topological space $(X,\tau,\mathcal{I})$ and also $int^*(A)$ denotes the interior of A with respect to τ^*.

*Corresponding author.
E-mail address: santhifuzzy@yahoo.co.in (R. Santhi), rameshngm@gmail.com (M. Rameshkumar).

Definition 2.1. *Let (X, τ) be a topological space. A subset A of X is said to be semi-open [10] if there exists an open set U in X such that $U \subseteq A \subseteq cl(U)$. The complement of a semi-open set is said to be semi-closed. The collection of all semi-open (resp. semi-closed) sets in X is denoted by SO(X) (resp. SC(X)). The semi-closure of A in (X, τ) is denoted by the intersection of all semi-closed sets containing A and is denoted by $scl(A)$.*

Definition 2.2. *For $A \subseteq X$, $A_*(\mathcal{I}, \tau)$ = $\{ x \in X / U \cap A \notin \mathcal{I}$for every $U \in SO(X)\}$ is called the semi-local function [7] of A with respect to $\mathcal{I}$ and τ, where $SO(X, x) = \{U \in SO(X) : x \in U\}$. We simply write A_* instead of $A_*(\mathcal{I}, \tau)$ in this case there is no ambiguity.*

It is given in [2] that $\tau^{*s}(\mathcal{I})$ is a topology on X, generated by the sub basis $\{U - E : U \in SO(X)$ and $E \in I\}$ or equivalently $\tau^{*s}\mathcal{I} = \{U \subseteq X : cl^{*s}(X - U) = X - U\}$. The closure operator cl^{*s} for a topology $\tau^{*s}(\mathcal{I})$ is defined as follows: for A $\subseteq X, cl^{*s}(A) = A \cup A_*$ and int^{*s} denotes the interior of the set A in $(X, \tau^{*s}, \mathcal{I})$. It is known that $\tau \subseteq \tau^*(\mathcal{I}) \subseteq \tau^{*s}(\mathcal{I})$. A subset A of $(X, \tau, \mathcal{I})$ is called semi-$*$-perfect [8] if $A = A_*$. $A \subseteq (X, \tau, \mathcal{I})$ is called $*$-semi dense in-itself [8](resp. semi-$*$-closed [8]) if $A \subset A_*$ (resp. $A_* \subseteq A$).

Lemma 2.1. *[7] Let $(X, \tau, \mathcal{I})$ be an ideal topological space and A, B be subsets of X. Then for the semi-local function the following properties hold:*

1. *If $A \subseteq B$ then $A_* \subseteq B_*$.*
2. *If $U \in \tau$ then $U \cap A_* \subseteq (U \cap A)_*$*
3. *$A_* = scl(A_*) \subseteq scl(A)$ and A_* is semi-closed in X.*
4. *$(A_*)_* \subseteq A_*$.*
5. *$(A \cup B)_* = A_* \cup B_*$.*
6. *If $\mathcal{I} = \{\phi\}$, then $A_* = scl(A)$.*

Definition 2.3. *A subset A of a topological space X is said to be*

1. *α-open [12]if $A \subseteq int(cl(int(A)))$,*
2. *pre-open [11] if $A \subseteq int(cl(A))$,*
3. *β-open [1] if $A \subseteq cl(int(cl(A)))$.*

Definition 2.4. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be*

1. *α-$\mathcal{I}$-open [4]if $A \subseteq int(cl^*(int(A)))$,*
2. *pre-$\mathcal{I}$-open [3] if $A \subseteq int(cl^*(A))$,*
3. *semi-$\mathcal{I}$-open [4] if $A \subseteq cl^*(int(A))$.*

Definition 2.5. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is said to be*

1. *α-$\mathcal{I}_s$-open [13]if $A \subseteq int(cl^{*s}(int(A)))$,*
2. *pre-$\mathcal{I}_s$-open [13] if $A \subseteq int(cl^{*s}(A))$,*
3. *semi-$\mathcal{I}_s$-open [13] if $A \subseteq cl^{*s}(int(A))$.*

By $PISO(X, \tau)$, we denote the family of all pre-$\mathcal{I}_s$-open sets of a space $(X, \tau, \mathcal{I})$.

Remark 2.1. *In [13], the authors obtained the following diagram:*

$$\begin{array}{ccccccc}
open & \longrightarrow & \alpha\text{-}\mathcal{I}_s\text{-open} & \longrightarrow & \text{semi-}\mathcal{I}_s\text{-open} & \longrightarrow & \text{semi-open} \\
 & & \downarrow & & & & \downarrow \\
 & & \text{pre-}\mathcal{I}_s\text{-open} & \longrightarrow & \text{pre-open} & \longrightarrow & \beta\text{-open}
\end{array}$$

3 Pre-$\mathcal{I}_s$-open sets

Theorem 3.1. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space.*

1. *If $\{A_\alpha : \alpha \in \Delta\} \subseteq PISO(X)$, then $\cup\{A_\alpha : \alpha \in \Delta\} \in PISO(X)$*
2. *If $A \in PISO(X)$ and $U \in \tau$, then $A \cap U \in PISO(X)$.*
3. *If $A \in PISO(X)$ and $B \in \tau^\alpha$, then $A \cap B \in PO(X)$*

Proof. (1) Since $\{A_\alpha : \alpha \in \Delta\} \subseteq PISO(X)$, then $A_\alpha \subseteq int(cl^{*s}(A_\alpha))$ for every $\alpha \in \Delta$. Thus

$$\bigcup_{\alpha\in\Delta} A_\alpha \subseteq \bigcup_{\alpha\in\Delta} int(cl^{*s}(A_\alpha)) \subseteq int(\bigcup_{\alpha\in\Delta} cl^{*s}(A_\alpha)) = int(\bigcup_{\alpha\in\Delta}((A_\alpha)_* \cup A_\alpha))$$

$$= int(\bigcup_{\alpha\in\Delta}(A_\alpha)_* \cup \bigcup_{\alpha\in\Delta} A_\alpha) \subseteq int((\bigcup_{\alpha\in\Delta} A_\alpha)_* \cup \bigcup_{\alpha\in\Delta} A_\alpha) = int(cl^{*s}(\bigcup_{\alpha\in\Delta} A_\alpha)).$$

(2) By assumption $A \subseteq int(cl^{*s}(A))$ and $U \subseteq int(U)$. By Lemma 2.1, $A \cap U \subseteq int(cl^{*s}(A)) \cap int(U) \subseteq int(cl^{*s}(A) \cap U) = int((A_* \cup A) \cap U) = int((A_* \cap U) \cup (A \cap U)) \subseteq int((A \cap U)_* \cup (A \cap U)) = int(cl^{*s}(A \cap U))$.

(3) Every pre-$\mathcal{I}_s$-open set is pre-open and the intersection of pre-open set and α-set is always pre-open set. □

Remark 3.1. *Intersection of even two pre-$\mathcal{I}_s$-open sets need not be pre-$\mathcal{I}_s$-open set as shown in the following example.*

Example 3.1. *Let X = {a, b, c, d }, $\tau = \{\phi, X, \{b\}, \{c, d\}, \{b, c, d\}\}$ and $\mathcal{I} = \{\phi\}$. Then we put A = {a, b, c} and B = {a, b, d} are pre-$\mathcal{I}_s$-open but $A \cap B$ = {a, b} is not pre-$\mathcal{I}_s$-open.*

Definition 3.1. *A subset F of a space $(X, \tau, \mathcal{I})$ is said to be pre-$\mathcal{I}_s$-closed if its complement is pre-$\mathcal{I}_s$-open.*

Remark 3.2. *For a subset A of a space $(X, \tau, \mathcal{I})$, we have $X - cl^{*s}(int(A)) \neq int(cl^{*s}(X - A))$ as shown from the following example.*

Example 3.2. *Let X = {a, b, c, d }, $\tau = \{\phi, X, \{d\}, \{a, c\}, \{a, c, d\}\}$ and $\mathcal{I} = \{\phi, X, \{c\}, \{d\}, \{c, d\}\}$. Then we put A = {b, d}, we have $int(cl^{*s}(X - A)) = int(cl^{*s}(\{a, c\}) = int(\{a, c\}) = \{a, c\}$ but $X - cl^{*s}(int(A)) = X - cl^{*s}(\{d\}) = X - \{d\} = \{a, b, c\}$.*

Theorem 3.2. *If a subset A of a space $(X, \tau, \mathcal{I})$ is pre-$\mathcal{I}_s$-closed, then $cl^{*s}(int(A)) \subseteq A$.*

Proof. Since A is pre-$\mathcal{I}_s$-closed, $X - A \in PISO(X, \tau)$. Since $\tau^{*s}(\mathcal{I})$ is finer than τ, we have $X - A \subseteq int(cl^{*s}(X - A)) \subseteq int(cl(X - A)) = X - cl(int(A)) \subseteq X - cl^{*s}(int(A))$. Therefore we obtain $cl^{*s}(int(A)) \subseteq A$. □

Corollary 3.1. *Let A be a subset of a space $(X, \tau, \mathcal{I})$ such that $X - cl^{*s}(int(A)) = int(cl^{*s}(X - A))$. Then A is pre-$\mathcal{I}_s$-closed if and only if $cl^{*s}(int(A)) \subseteq A$*

Proof. This is an immediate consequence of Theorem 3.2. □

Theorem 3.3. *[8] Let $(X, \tau, \mathcal{I})$ be an ideal space and $A \subseteq Y \subseteq X$, where Y is α-open in X. Then $A_*(\mathcal{I}_Y, \tau|_Y) = A_*(\mathcal{I}, \tau) \cap Y$.*

Theorem 3.4. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. If $Y \in \tau$ and $W \in PISO(X)$, then $Y \cap W \in PISO(Y, \tau|_Y, \mathcal{I}_Y)$.*

Proof. Since Y is open, we have $int_Y(A) = int(A)$ for any subset A of Y. Now $Y \cap W \subseteq Y \cap int(cl^{*s}(W)) = Y \cap (int(W_* \cup W)) = Y \cap (int(W_*) \cup int(W)) = (Y \cap int(W_*)) \cup (Y \cap int(W)) = int_Y(Y \cap W_*) \cup int_Y(Y \cap W) = int_Y[(Y \cap W_*) \cup (Y \cap W)] = int_Y[((Y \cap W_*) \cup (Y \cap W)) \cap Y] = int_Y[Y \cap (Y \cap W_*) \cup Y \cap W] \subseteq int_Y[Y \cap (Y \cap W)_* \cup Y \cap W] = int_Y[(Y \cap W)_*(I_Y, \tau|_Y) \cup (Y \cap W)] = int_Y[cl^{*s}_Y(Y \cap W)]$. This shows that $Y \cap W \in PISO(Y, \tau|_Y, \mathcal{I}_Y)$. □

Theorem 3.5. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subseteq U \in \tau$. Then, A is pre-$\mathcal{I}_s$-open in $(X, \tau, \mathcal{I})$ if and only if A is pre-$\mathcal{I}_s$-open in $(U, \tau|_U, \mathcal{I}_U)$.*

Proof. Let A be pre-$\mathcal{I}_s$-open in $(X, \tau, \mathcal{I})$. Then we have $A = U \cap A \subseteq U \cap int(cl^{*s}(A)) \subseteq int_U(U \cap cl^{*s}(A)) \subseteq int_U(cl^{*s}_U(A))$. This shows that A is pre-$\mathcal{I}_s$-open in $(U, \tau|_U, \mathcal{I}_U)$.
Sufficiency. Let A be pre-$\mathcal{I}_s$-open in $(U, \tau|_U, \mathcal{I}_U)$. Then we have $A \subseteq int_U(cl^{*s}_U(A)) = int(cl^{*s}(A) \cap U) \subseteq int(cl^{*s}(A))$. This shows that A is pre-$\mathcal{I}_s$-open in $(X, \tau, \mathcal{I})$. □

4 Pre-$\mathcal{I}_s$-continuous functions

Definition 4.1. *A function $f : (X,\tau,\mathcal{I}) \to (Y,\sigma)$ is said to be pre-$\mathcal{I}_s$-continuous [13] (resp. pre-$\mathcal{I}$-continuous [3], pre-continuous [11]) if $f^{-1}(V)$ is pre-$\mathcal{I}_s$-open (resp. pre-$\mathcal{I}$-open, pre-open) in $(X,\tau,\mathcal{I})$ for each open set V of (Y,σ).*

Theorem 4.1. *Let $f : (X,\tau,\mathcal{I}) \to (Y,\sigma)$ be a function. Then the following holds:*

a) *Every continuous function is pre-$\mathcal{I}_s$-continuous.*

b) *Every pre-$\mathcal{I}_s$-continuous is pre-continuous.*

c) *Every pre-$\mathcal{I}_s$-continuous is pre-$\mathcal{I}$-continuous.*

Proof. The proof is obvious. □

Remark 4.1. *Converse of the Theorem 4.1 need not be true as seen from the following examples.*

Example 4.1. *Let $X = \{a, b, c, d\}$, $Y = \{a, b, c\}$ $\tau = \{\phi, X, \{c\}\{a,b\},\{a,b,c\}\}, \sigma = \{\phi, X, \{c\},\{d\},\{c,d\}\}$ and $\mathcal{I} = \{\phi,\{b\},\{c\},\{b,c\}\}$. Define a function $f : (X,\tau,\mathcal{I}) \to (Y,\sigma)$ as follows $f(a) = f(b) = c, f(c) = b, f(d) = a$. Then f is pre-$\mathcal{I}_s$-continuous but not continuous.*

Example 4.2. *Let $X = \{a,b,c,d\}, \tau = \{\phi, X, \{a\},\{b\}\{a,b\}\}, \sigma = \{\phi, X, \{a\},\{a,b\},\{a,b,c\}\}$ and $\mathcal{I} = \{\phi,\{a\},\{b\},\{a,b\}\}$. The identity function $f : (X,\tau,\mathcal{I}) \to (X,\sigma)$ is pre-continuous and pre-$\mathcal{I}$-continuous, but it is not pre-$\mathcal{I}_s$-continuous.*

Theorem 4.2. *For a function $f : (X,\tau,\mathcal{I}) \to (Y,\sigma)$, the following are equivalent:*

1. *f is pre-$\mathcal{I}_s$-continuous,*
2. *for each $x \in X$ and each $V \in \sigma$ containing $f(x)$, then there exists $W \in PISO(X,\tau)$ containing x such that $f(W) \subseteq V$,*
3. *for each $x \in X$ and each $V \in \sigma$ containing $f(x), cl^{*s}(f^{-1}(V))$ is neighborhood of X,*
4. *the inverse image of each closed set in (Y,σ) is pre-$\mathcal{I}_s$-closed.*

Proof. (1) ⇒ (2). Let $x \in X$ and V be any open set of Y containing $f(x)$. Set $W = f^{-1}(V)$, then by(1), W is pre-$\mathcal{I}_s$-open and clearly $x \in W$ and $f(W) \subseteq V$.

(2) ⇒ (3). Since $V \in \sigma$ and $f(x) \in V$. Then by(2) there exists $W \in PISO(X)$ containing x such that $f(W) \subseteq V$. Thus, $x \in W \subseteq int(cl^{*s}(W)) \subseteq int(cl^{*s}(f^{-1}(V))) \subseteq cl^{*s}(f^{-1}(V))$. Hence $cl^{*s}(f^{-1}(V))$ is a neighborhood of X.

(3) ⇒ (4) and (1) ⇔ (4) are obvious. □

Theorem 4.3. *Let $f : (X,\tau,\mathcal{I}) \to (Y,\sigma)$ be pre-$\mathcal{I}_s$-continuous and $U \in \tau$. Then the restriction $f|_U : (U,\tau|_U,\mathcal{I}_U) \to (Y,\sigma)$ is pre-$\mathcal{I}_s$-continuous.*

Proof. Let V be any open set of (Y, σ). Since f is pre-$\mathcal{I}_s$-continuous, $f^{-1}(V) \in PISO(X,\tau)$ and by Theorem 3.5, $(f|_U)^{-1}(V) = f^{-1}(V) \cap U \in PISO(U,\tau|_U)$. This shows that $f|_U : (U,\tau|_U,\mathcal{I}_U) \to (Y,\sigma)$ is pre-$\mathcal{I}_s$-continuous. □

Theorem 4.4. *Let $f : (X,\tau,\mathcal{I}) \to (Y,\sigma)$ be a function and $\{U_\alpha/\alpha \in \Delta\}$ be an open cover of X. Then f is pre-$\mathcal{I}_s$-continuous if and only if the restriction $f|_{U_\alpha} : (U_\alpha,\tau|_{U_\alpha},\mathcal{I}_{U_\alpha}) \to (Y,\sigma)$ is pre-$\mathcal{I}_s$-continuous for each $\alpha \in \Delta$.*

Proof. This follows from Theorem 4.3.
Sufficiency Let V be any open set in (Y,σ). Since $f|_{U_\alpha}$ is pre-$\mathcal{I}_s$-continuous for each $\alpha \in \Delta$. $(f|_{U_\alpha})^{-1}(V)$ is pre-$\mathcal{I}_s$-open set of $(U_\alpha,\tau|_{U_\alpha},\mathcal{I}_{U_\alpha})$ and hence by Theorem 3.5, $(f|_{U_\alpha})^{-1}(V)$ is pre-$\mathcal{I}_s$-open set in $(X,\tau,\mathcal{I})$ for each $\alpha \in \Delta$. Moreover, we have

$$f^{-1}(V) = \Big(\bigcup_{\alpha\in\Delta} U_\alpha\Big) \cap f^{-1}(V) = \bigcup_{\alpha\in\Delta} \Big(U_\alpha \cap f^{-1}(V)\Big) = \bigcup_{\alpha\in\Delta} \Big(f|_{U_\alpha}\Big)^{-1}(V).$$

□

Theorem 4.5. *Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ and $g : (Y, \sigma, \mathcal{J}) \to (Z, \eta)$, be two functions where $\mathcal{I}$ and $\mathcal{J}$ are ideals of X and Y respectively. Then:*

1. *$g \circ f$ is pre-$\mathcal{I}_s$-continuous if f is pre-$\mathcal{I}_s$-continuous and g is continuous.*

2. *$g \circ f$ is pre-continuous if f is pre-$\mathcal{I}_s$-continuous and g is continuous.*

Proof. It is obvious. □

Theorem 4.6. *A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is pre-$\mathcal{I}_s$-continuous if and only if the graph function $g : X \to X \times Y$, defined by g(x) = (x, f(X)) for each $x \in X$, is pre-$\mathcal{I}_s$-continuous.*

Proof. Suppose that f is pre-$\mathcal{I}_s$-continuous. Let $x \in X$ and W be any open set of $X \times Y$ containing $g(x)$. Then there exists a basic open set $U \times V$ such that $g(x) = (x, f(x)) \in U \times V \subseteq W$. Since f is pre-$\mathcal{I}_s$-continuous, then there exists a pre-$\mathcal{I}_s$-open set $U_\circ$ of X containing x such that $f(U_\circ) \subseteq V$. By Theorem 3.1 $U_\circ \cap U \in PISO(X, \tau)$ and $g(U_\circ \cap U) \subseteq U \times V \subseteq W$. This shows that g is pre-$\mathcal{I}_s$-continuous.
Sufficiency: Suppose that g is pre-$\mathcal{I}_s$-continuous. Let $x \in$ X and V be any open set of Y containing $f(x)$. Then $X \times V$ is open in $X \times Y$ and by pre-$\mathcal{I}_s$-continuity of g, there exists $U \in PISO(X, \tau)$ containing x such that $g(U) \subseteq X \times V$. Therefore we obtain $f(U) \subseteq V$. This shows that f is pre-$\mathcal{I}_s$-continuous. □

5 Pre-$\mathcal{I}_s$-open and pre-$\mathcal{I}_s$-closed functions

Definition 5.1. *[14] A function $f : (X, \tau) \to (Y, \sigma, \mathcal{J})$ is called pre-$\mathcal{I}_s$-open (resp. pre-$\mathcal{I}_s$-closed) if for each $U \in \tau$ (resp. U is closed)$f(U) \in PISO(Y, \sigma, \mathcal{J})$(resp. $f(U)$ is pre-$\mathcal{I}_s$-closed set).*

Definition 5.2. *[3] A function $f : (X, \tau) \to (Y, \sigma, \mathcal{J})$ is called pre-$\mathcal{I}$-open (resp. pre-$\mathcal{I}$-closed) if for each $U \in \tau$ (resp. U is closed) $f(U)$ is pre-$\mathcal{I}$-open (resp. $f(U)$ is pre-$\mathcal{I}$-closed) set in $(Y, \sigma, \mathcal{J})$.*

Remark 5.1.

1. *Every pre-$\mathcal{I}_s$-open (resp. pre-$\mathcal{I}_s$-closed) function is pre-open (resp. pre-closed) and the converses are false in general.*

2. *Every pre-$\mathcal{I}_s$-open (resp. pre-$\mathcal{I}_s$-closed) function is pre-$\mathcal{I}$-open (resp. pre-$\mathcal{I}$-closed) and the converses are false in general.*

3. *Every open function is pre-$\mathcal{I}_s$-open but the converse is not true in general.*

Example 5.1. *Let $X = \{a, b, c, d\}, \tau = \{\phi, X, \{a, b\}\}, \sigma = \{\phi, X, \{d\}, \{a, c\}, \{a, c, d\}\}$ and $\mathcal{J} = \{\phi, \{c\}, \{d\}, \{c, d\}\}$. Define a function $f : (X, \tau) \to (X, \sigma, \mathcal{J})$ as follows $f(a) = c, f(b) = d, f(c) = b, f(d) = a$. Then f is pre-open, but it is not pre-$\mathcal{I}_s$-open.*

Example 5.2. *Let $X = \{a, b, c, d\}, \tau = \{\phi, X, \{b\}, \{b, c, d\}\}, \sigma = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and $\mathcal{J} = \{\phi, \{b\}, \{c\}, \{b, c\}\}$. Define a function $f : (X, \tau) \to (X, \sigma, \mathcal{J})$ as follows $f(a) = f(d) = a, f(b) = b, f(c) = c$. Then f is pre-$\mathcal{I}$-open, but it is not pre-$\mathcal{I}_s$-open.*

Example 5.3. *Let $X = \{a, b, c, d\}, \tau = \{\phi, X, \{a\}, \{a, b, c\}\}, \sigma = \{\phi, X, \{d\}, \{a, c\}, \{a, c, d\}\}$ and $\mathcal{J} = \{\phi, \{c\}, \{d\}, \{c, d\}\}$. Define a function $f : (X, \tau) \to (X, \sigma, \mathcal{J})$ as follows $f(a) = a, f(b) = b, f(c) = d, f(d) = b$. Then f is pre-$\mathcal{I}_s$open, but it is not open.*

Theorem 5.1. *A function $f : (X, \tau) \to (Y, \sigma, \mathcal{J})$ is called pre-$\mathcal{I}_s$-open if and only if for each subset $W \subseteq Y$ and each closed set F of X containing $f^{-1}(W)$, there exists a pre-$\mathcal{I}_s$-closed set $H \subseteq Y$ containing W such that $f^{-1}(H) \subseteq F$.*

Proof. Let $H = Y - f(X - F)$. Since $f^{-1}(W) \subseteq F$, we have $f(X - F) \subseteq Y - W$. Since f is pre-$\mathcal{I}_s$-open, then H is pre-$\mathcal{I}_s$-closed and $f^{-1}(H) = X - f^{-1}(f(X - F)) \subseteq X - (X - F) = F$.
Sufficiency. Let U be any open set of X and $W = Y - f(U)$. Then $f^{-1}(W) = X - f^{-1}(f(U)) \subseteq X - U$ and $X - U$ is closed. By the hypothesis, there exists an pre-$\mathcal{I}_s$-closed set H of Y containing W such that $f^{-1}(H) \subseteq X - U$. Then we have $f^{-1}(H) \cap U = \phi$ and $H \cap f(U) = \phi$. Therefore we obtain $Y - f(U) \supseteq H \supseteq W = Y - f(U)$ and $f(U)$ is pre-$\mathcal{I}_s$-open in Y. This shows that f is pre-$\mathcal{I}_s$-open. □

Theorem 5.2. *For any bijective function $f : (X, \tau) \to (Y, \sigma, \mathcal{J})$, the following are equivalent:*

1. $f^{-1} : (X, \sigma, \mathcal{J}) \to (X, \tau)$ *is pre-*$\mathcal{I}_s$*-continuous,*
2. *f is pre-*$\mathcal{I}_s$*-open,*
3. *f is pre-*$\mathcal{I}_s$*-closed,*

Proof. Obvious. □

6 Decomposition of Continuity

Definition 6.1. *A subset A of an ideal topological space* $(x, \tau, \mathcal{I})$ *is called* $\mathcal{I}_s$*-locally closed if* $A = U \cap V$*, where* $U \in \tau$ *and V is semi-*$*$*-perfect.*

Proposition 6.1. *Let* $(x, \tau, \mathcal{I})$ *be an ideal topological space and A a subset of X. Then the following are equivalent:*

1. *A is open,*
2. *A is pre-*$\mathcal{I}_s$*-open and* $\mathcal{I}_s$*-locally closed.*

Proof. (1) ⇒ (2) Let A is open. Then A is pre-$\mathcal{I}_s$-open. On the other hand $A = A \cap X$, where $A \in \tau$ and X is semi-$*$-perfect.

(2) ⇒ (1) By assumption $A \subseteq int(cl^{*s}(A)) = int(cl^{*s}(U \cap V))$, where $U \in \tau$ and V is semi-$*$-perfect. Hence $A = U \cap A \subseteq U \cap int(cl^{*s}(U)) \cap int(cl^{*s}(V)) = U \cap int(V \cup V_*) = int(U) \cap int(V) = int(U \cap V) = int(A)$. Hence A is open. □

Definition 6.2. *A function* $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$*, is called* $\mathcal{I}_s$*-LC-continuous if for every* $V \in \sigma$*,* $f^{-1}(V)$ *is* $\mathcal{I}_s$*-locally closed.*

Proposition 6.2. *Let* $(x, \tau, \mathcal{I})$ *be an ideal topological spaces. Then, every continuous function* $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$*, is* $\mathcal{I}_s$*-LC-continuous.*

Proof. Obvious. □

Remark 6.1. *Converse of the Theorem 6.2 need not be true as seen from the following example.*

Example 6.1. *Let* $X = Y = \{a, b, c, d\}$*,* $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{a, b, d\}\}, \sigma = \{\phi, X, \{d\}\}$ *and* $\mathcal{J} = \{\phi, \{b\}, \{c\}, \{b, c\}\}$*. The identity function* $f : (X, \tau) \to (X, \sigma, \mathcal{J})$ *is* $\mathcal{I}_s$*-LC-continuous but it is not continuous.*

Theorem 6.1. *Let* $(x, \tau, \mathcal{I})$ *be an ideal topological spaces. For a function* $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$*, the following conditions are equivalent:*

1. *f is continuous,*
2. *f is pre-*$\mathcal{I}_s$*-continuous and* $\mathcal{I}_s$*-LC-continuous.*

Proof. This follows from Proposition 6.1. □

References

[1] M. E. Abd El-Monsef, S. N. El-Deep and R. A. Mahmoud, β-open sets and β-continuous mappings, *Bull. Fac. Sci. Assiut Univ.*, 12(1983), 77-90.

[2] M.E. Abd El-Monsef, E.F. Lashien and A.A. Nasef, Some topological operators via ideals, *Kyungpook Math. J.*, 32(2)(1992), 273-284.

[3] J. Dontchev, *Idealization of Ganster-Reilly decomposition theorems*, http://arxiv.org/abs/ Math. GN/9901017, 5 Jan. 1999(Internet).

[4] E. Hatir and T.Noiri, On decompositions of continuity via idealization, *Acta. Math. Hungar.*, 96(4)(2002), 341-349.

[5] E. Hatir and T. Noiri, On semi-I-open sets and semi-I-continuous functions, *Acta. Math. Hungar.*, 107(4)(2005), 345-353.

[6] D. Jankovic and T. R. Hamlett, New topologies from old via ideals, *Amer. Math. Monthly*, 97(4)(1990), 295-310.

[7] M. Khan and T. Noiri, Semi-local functions in ideal topological spaces, *J. Adv. Res. Pure Math.*, 2(1)(2010), 36-42.

[8] M. Khan and T. Noiri, On gI-closed sets in ideal topological spaces, *J. Adv. Stud. in Top.*, 1(2010), 29-33.

[9] K. Kuratowski, *Topology*, Vol. I, Academic press, New York, 1966.

[10] N. Levine, Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly*, 70(1963), 36-41.

[11] A. S. Mashhour, M. E. Abd. El-Monsef and S. N. El-deeb, On pre-continuous and weak pre-continuous mappings, *Proc. Math. Phys. Soc. Egypt*, 53(1982), 47-53.

[12] O. Njastad, On some classes of nearly open sets, *Pacific J. Math.*, 15(1965), 961-970.

[13] R. Santhi and M. Rameshkumar, *A decomposition of continuity in ideal by using semi-local functions*, (Submitted).

[14] R. Santhi and M. Rameshkumar, *On α-$\mathcal{I}_s$-open sets and α-$\mathcal{I}_s$-continuous functions*,(submitted).

[15] R. Vaidyanathaswamy, *Set Topology*, Chelsea Publishing Company, 1960.

Received: February 28, 2014; *Accepted*: May 2, 2014

UNIVERSITY PRESS

Website: http://www.malayajournal.org/