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Some new general integral inequalities for P-functions

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Abstract

In this paper, we derive new estimates for the remainder term of the midpoint, trapezoid, and Simpson formulae for functions whose derivatives in absolute value at certain power are P-functions. Some applications to special means of real numbers are also given.

Keywords: Convex function, P-function, Simpson's inequality, Hermite-Hadamard's inequality.

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1 Introduction

Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a convex function defined on the interval I of real numbers and $a, b \in I$ with $a < b$. The following inequality holds:

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}. \tag{1.1}$$

This double inequality is well known as Hermite-Hadamard integral inequality for convex functions in the literature .

In [2] Dragomir et al. defined the concept of P-function as the following:

Definition 1.1. *We say that $f : I \to \mathbb{R}$ is a P-function, or that f belongs to the class $P(I)$, if f is a non-negative function and for all $x, y \in I$, $\alpha \in [0,1]$, we have*

$$f(\alpha x + (1-\alpha)y) \leq f(x) + f(y).$$

$P(I)$ contain all nonnegative monotone convex and quasi convex functions.

In [2], Dragomir et al., proved following inequalities of Hadamard's type for P-function

Theorem 1.1. *Let $f \in P(I), a, b \in I$ with $a < b$ and $f \in L[a,b]$. Then the following inequality holds*

$$f\left(\frac{a+b}{2}\right) \leq \frac{2}{b-a}\int_a^b f(x)dx \leq 2\left[f(a)+f(b)\right]. \tag{1.2}$$

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The following inequality is well known in the literature as Simpson's inequality .

Let $f : [a, b] \to \mathbb{R}$ be a four times continuously differentiable mapping on (a, b) and $\left\| f^{(4)} \right\|_{\infty} = \sup_{x\in(a,b)} \left| f^{(4)}(x) \right| < \infty$. Then the following inequality holds:

$$\left| \frac{1}{3} \left[\frac{f(a) + f(b)}{2} + 2f\left(\frac{a+b}{2}\right) \right] - \frac{1}{b-a} \int_a^b f(x)dx \right| \le \frac{1}{2880} \left\| f^{(4)} \right\|_{\infty} (b-a)^4 .$$

In recent years many authors have studied error estimations for Simpson's inequality and Hermite-Hadamard inequalitiy; for refinements, counterparts, generalizations, see ([1]-[10]).

In [3], Iscan obtained a new generalization of some integral inequalities for differentiable convex mapping which are connected Simpson and Hadamard type inequalities by using the following lemma.

Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I° such that $f' \in L[a, b]$, where $a, b \in I$ with $a < b$ and $\alpha, \lambda \in [0, 1]$. Then the following equality holds:

$$\begin{aligned} & \lambda \left(\alpha f(a) + (1-\alpha) f(b)\right) + (1-\lambda) f(\alpha a + (1-\alpha) b) - \frac{1}{b-a} \int_a^b f(x)dx \\ = & (b-a) \left[\int_0^{1-\alpha} (t - \alpha\lambda) f'(tb + (1-t)a)\, dt \right. \\ & \left. + \int_{1-\alpha}^{1} (t - 1 + \lambda(1-\alpha)) f'(tb + (1-t)a)\, dt \right]. \end{aligned}$$

The aim of this paper is to establish some new general integral inequalities for functions whose derivatives in absolute value at certain power are P-functions. Some applications of these results to special means is to give as well.

Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a differentiable function on I°, the interior of I. Throughout this section we will take

$$\begin{aligned} & I_f(\lambda, \alpha, a, b) \\ = & \lambda \left(\alpha f(a) + (1-\alpha) f(b)\right) + (1-\lambda) f(\alpha a + (1-\alpha) b) - \frac{1}{b-a} \int_a^b f(x)dx \end{aligned}$$

where $a, b \in I^{\circ}$ with $a < b$ and $\alpha, \lambda \in [0, 1]$.

Theorem 1.2. *Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I° such that $f' \in L[a, b]$, where $a, b \in I^{\circ}$ with $a < b$ and $\alpha, \lambda \in [0, 1]$. If $|f'|^q$ is P-function on $[a, b]$, $q \ge 1$, then the following inequality holds:*

$$\begin{aligned} \left| I_f(\lambda, \alpha, a, b) \right| & \le (b-a) \left(\left|f'(b)\right|^q + \left|f'(a)\right|^q \right)^{\frac{1}{q}} \\ & \times \begin{cases} \gamma_2(\alpha,\lambda) + \gamma_2(1-\alpha,\lambda) & \alpha\lambda \le 1-\alpha \le 1-\lambda(1-\alpha) \\ \gamma_2(\alpha,\lambda) + \gamma_1(1-\alpha,\lambda) & \alpha\lambda \le 1-\lambda(1-\alpha) \le 1-\alpha \\ \gamma_1(\alpha,\lambda) + \gamma_2(1-\alpha,\lambda) & 1-\alpha \le \alpha\lambda \le 1-\lambda(1-\alpha) \end{cases}, \end{aligned} \tag{1.3}$$

where

$$\begin{aligned} \gamma_1(\alpha,\lambda) &= (1-\alpha) \left[\alpha\lambda - \frac{(1-\alpha)}{2} \right], \\ \gamma_2(\alpha,\lambda) &= (\alpha\lambda)^2 - \gamma_1(\alpha,\lambda) . \end{aligned} \tag{1.4}$$

Proof. Suppose that $q \ge 1$.Since $|f'|^q$ is P-function on $[a, b]$, from Lemma 1 and using the well known power mean inequality, we have

$$\begin{aligned} & \left| I_f(\lambda, \alpha, a, b) \right| \\ \le & (b-a) \left[\int_0^{1-\alpha} |t - \alpha\lambda| \left| f'(tb + (1-t)a) \right| dt + \int_{1-\alpha}^{1} |t - 1 + \lambda(1-\alpha)| \left| f'(tb + (1-t)a) \right| dt \right] \end{aligned}$$

$$\leq \quad (b-a)\left\{\left(\int_0^{1-\alpha}|t-\alpha\lambda|\,dt\right)^{1-\frac{1}{q}}\left(\int_0^{1-\alpha}|t-\alpha\lambda|\left|f'\left(tb+(1-t)a\right)\right|^q dt\right)^{\frac{1}{q}}\right.$$

$$\left.+\left(\int_{1-\alpha}^{1}|t-1+\lambda\,(1-\alpha)|\,dt\right)^{1-\frac{1}{q}}\left(\int_{1-\alpha}^{1}|t-1+\lambda\,(1-\alpha)|\left|f'\left(tb+(1-t)a\right)\right|^q dt\right)^{\frac{1}{q}}\right\}$$

$$\leq (b-a)\left(\left|f'(b)\right|^q+\left|f'(a)\right|^q\right)^{\frac{1}{q}}\left\{\int_0^{1-\alpha}|t-\alpha\lambda|\,dt+\int_{1-\alpha}^{1}|t-1+\lambda\,(1-\alpha)|\,dt\right\} \tag{1.5}$$

Additionally, by simple computation

$$\int_0^{1-\alpha}|t-\alpha\lambda|\,dt=\begin{cases}\gamma_2(\alpha,\lambda), & \alpha\lambda\leq 1-\alpha\\ \gamma_1(\alpha,\lambda), & \alpha\lambda\geq 1-\alpha\end{cases}, \tag{1.6}$$

$$\gamma_1(\alpha,\lambda)=(1-\alpha)\left[\alpha\lambda-\frac{(1-\alpha)}{2}\right],\ \gamma_2(\alpha,\lambda)=(\alpha\lambda)^2-\gamma_1(\alpha,\lambda)\,,$$

$$\int_{1-\alpha}^{1}|t-1+\lambda\,(1-\alpha)|\,dt=\int_0^{\alpha}|t-(1-\alpha)\,\lambda|\,dt \tag{1.7}$$

$$=\begin{cases}\gamma_1(1-\alpha,\lambda), & 1-\lambda\,(1-\alpha)\leq 1-\alpha\\ \gamma_2(1-\alpha,\lambda), & 1-\lambda\,(1-\alpha)\geq 1-\alpha\end{cases},$$

Thus, using (1.6) and (1.7) in (1.5), we obtain the inequality (1.3). This completes the proof. □

Corollary 1.1. *Under the assumptions of Theorem 1.2 with $q=1$, we have*

$$\left|I_f\left(\lambda,\alpha,a,b\right)\right|\leq (b-a)\left(\left|f'(b)\right|+\left|f'(a)\right|\right)$$
$$\times\begin{cases}\gamma_2(\alpha,\lambda)+\gamma_2(1-\alpha,\lambda) & \alpha\lambda\leq 1-\alpha\leq 1-\lambda\,(1-\alpha)\\ \gamma_2(\alpha,\lambda)+\gamma_1(1-\alpha,\lambda) & \alpha\lambda\leq 1-\lambda\,(1-\alpha)\leq 1-\alpha\\ \gamma_1(\alpha,\lambda)+\gamma_2(1-\alpha,\lambda) & 1-\alpha\leq\alpha\lambda\leq 1-\lambda\,(1-\alpha)\end{cases},$$

Corollary 1.2. *In Theorem 1.2 , if we take $\alpha=\frac{1}{2}$ and $\lambda=\frac{1}{3}$, then we have the following Simpson type inequality*

$$\left|\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]-\frac{1}{b-a}\int_a^b f(x)dx\right|\leq\frac{5\,(b-a)}{36}\left(\left|f'(b)\right|^q+\left|f'(a)\right|^q\right)^{\frac{1}{q}}$$

Corollary 1.3. *In Theorem 1.2 , if we take $\alpha=\frac{1}{2}$ and $\lambda=0$, then we have following midpoint inequality*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)dx\right|\leq\frac{b-a}{4}\left(\left|f'(b)\right|^q+\left|f'(a)\right|^q\right)^{\frac{1}{q}}$$

Corollary 1.4. *In Theorem 1.2 , if we take $\alpha=\frac{1}{2}$, and $\lambda=1$, then we get the following trapezoid inequality which is identical to the inequality in [1, Theorem 2.3].*

$$\left|\frac{f\,(a)+f\,(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right|\leq\frac{b-a}{4}\left(\left|f'(b)\right|^q+\left|f'(a)\right|^q\right)^{\frac{1}{q}}$$

Using Lemma 1 we shall give another result for convex functions as follows.

Theorem 1.3. *Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I° such that $f' \in L[a,b]$, where $a, b \in I^{\circ}$ with $a < b$ and $\alpha, \lambda \in [0,1]$. If $|f'|^q$ is P-function on $[a,b]$, $q > 1$, then the following inequality holds:*

$$\left| I_f(\lambda,\alpha,a,b)\right| \leq (b-a)\left(\frac{1}{p+1}\right)^{\frac{1}{p}} \tag{1.8}$$

$$\times\begin{cases} \left[\varepsilon_1^{1/p}(\alpha,\lambda,p)c_f^{1/q}(\alpha,q)+\varepsilon_1^{1/p}(1-\alpha,\lambda,p)k_f^{1/q}(\alpha,q)\right], & \alpha\lambda \leq 1-\alpha \leq 1-\lambda(1-\alpha) \\ \left[\varepsilon_1^{1/p}(\alpha,\lambda,p)c_f^{1/q}(\alpha,q)+\varepsilon_2^{1/p}(1-\alpha,\lambda,p)k_f^{1/q}(\alpha,q)\right], & \alpha\lambda \leq 1-\lambda(1-\alpha) \leq 1-\alpha \\ \left[\varepsilon_2^{1/p}(\alpha,\lambda,p)c_f^{1/q}(\alpha,q)+\varepsilon_1^{1/p}(1-\alpha,\lambda,p)k_f^{1/q}(\alpha,q)\right], & 1-\alpha \leq \alpha\lambda \leq 1-\lambda(1-\alpha) \end{cases},$$

where

$$\begin{aligned} c_f(\alpha,q) &= (1-\alpha)\left[\left|f'((1-\alpha)b+\alpha a)\right|^q+\left|f'(a)\right|^q\right], \\ k_f(\alpha,q) &= \alpha\left[\left|f'((1-\alpha)b+\alpha a)\right|^q+\left|f'(b)\right|^q\right], \end{aligned} \tag{1.9}$$

$$\varepsilon_1(\alpha,\lambda,p)=(\alpha\lambda)^{p+1}+(1-\alpha-\alpha\lambda)^{p+1},\ \varepsilon_2(\alpha,\lambda,p)=(\alpha\lambda)^{p+1}-(\alpha\lambda-1+\alpha)^{p+1},$$

and $\frac{1}{p}+\frac{1}{q}=1$.

Proof. Since $|f'|^q$ is P-function on $[a,b]$, from Lemma 1 and by Hölder's integral inequality, we have

$$\begin{aligned} &\left|I_f(\lambda,\alpha,a,b)\right| \\ \leq\ & (b-a)\left[\int_0^{1-\alpha}|t-\alpha\lambda|\left|f'(tb+(1-t)a)\right|dt+\int_{1-\alpha}^1|t-1+\lambda(1-\alpha)|\left|f'(tb+(1-t)a)\right|dt\right] \\ \leq\ & (b-a)\left\{\left(\int_0^{1-\alpha}|t-\alpha\lambda|^p dt\right)^{\frac{1}{p}}\left(\int_0^{1-\alpha}\left|f'(tb+(1-t)a)\right|^q dt\right)^{\frac{1}{q}}\right. \\ &\left.+\left(\int_{1-\alpha}^1|t-1+\lambda(1-\alpha)|^p dt\right)^{\frac{1}{p}}\left(\int_{1-\alpha}^1\left|f'(tb+(1-t)a)\right|^q dt\right)^{\frac{1}{q}}\right\}. \end{aligned} \tag{1.10}$$

By the inequality (1.2), we get

$$\begin{aligned} \int_0^{1-\alpha}\left|f'(tb+(1-t)a)\right|^q dt &= (1-\alpha)\left[\frac{1}{(1-\alpha)(b-a)}\int_a^{(1-\alpha)b+\alpha a}\left|f'(x)\right|^q dx\right] \\ &\leq (1-\alpha)\left[\left|f'((1-\alpha)b+\alpha a)\right|^q+\left|f'(a)\right|^q\right]. \end{aligned} \tag{1.11}$$

The inequality (1.11) also holds for $\alpha = 1$. Similarly, for $\alpha \in (0,1]$ by the inequality (1.2), we have

$$\begin{aligned} \int_{1-\alpha}^1\left|f'(tb+(1-t)a)\right|^q dt &= \alpha\left[\frac{1}{\alpha(b-a)}\int_{(1-\alpha)b+\alpha a}^b\left|f'(x)\right|^q dx\right] \\ &\leq \alpha\left[\left|f'((1-\alpha)b+\alpha a)\right|^q+\left|f'(b)\right|^q\right]. \end{aligned} \tag{1.12}$$

The inequality (1.12) also holds for $\alpha = 0$. By simple computation

$$\int_0^{1-\alpha}|t-\alpha\lambda|^p dt=\begin{cases}\frac{(\alpha\lambda)^{p+1}+(1-\alpha-\alpha\lambda)^{p+1}}{p+1}, & \alpha\lambda\leq 1-\alpha \\ \frac{(\alpha\lambda)^{p+1}-(\alpha\lambda-1+\alpha)^{p+1}}{p+1}, & \alpha\lambda\geq 1-\alpha\end{cases}, \tag{1.13}$$

and

$$\int_{1-\alpha}^{1}|t-1+\lambda(1-\alpha)|^{p}\,dt=\begin{cases}\frac{[\lambda(1-\alpha)]^{p+1}+[\alpha-\lambda(1-\alpha)]^{p+1}}{p+1}, & 1-\alpha\leq 1-\lambda(1-\alpha)\\ \frac{[\lambda(1-\alpha)]^{p+1}-[\lambda(1-\alpha)-\alpha]^{p+1}}{p+1}, & 1-\alpha\geq 1-\lambda(1-\alpha)\end{cases}, \tag{1.14}$$

thus, using (1.11)-(1.14) in (1.10), we obtain the inequality (1.8). This completes the proof. □

Corollary 1.5. *In Theorem 1.3, if we take $\alpha=\frac{1}{2}$ and $\lambda=\frac{1}{3}$, then we have the following Simpson type inequality*

$$\left|\frac{1}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]-\frac{1}{b-a}\int_{a}^{b}f(x)dx\right|\leq\frac{b-a}{12}\left(\frac{1+2^{p+1}}{3(p+1)}\right)^{\frac{1}{p}}$$
$$\times\left\{\left(\left|f'\left(\frac{a+b}{2}\right)\right|^{q}+\left|f'(a)\right|^{q}\right)^{\frac{1}{q}}+\left(\left|f'\left(\frac{a+b}{2}\right)\right|^{q}+\left|f'(b)\right|^{q}\right)^{\frac{1}{q}}\right\}.$$

Corollary 1.6. *In Theorem 1.3, if we take $\alpha=\frac{1}{2}$ and $\lambda=0$, then we have the following midpoint inequality*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_{a}^{b}f(x)dx\right|\leq\frac{b-a}{4}\left(\frac{1}{p+1}\right)^{\frac{1}{p}}$$
$$\times\left\{\left(\left|f'\left(\frac{a+b}{2}\right)\right|^{q}+\left|f'(a)\right|^{q}\right)^{\frac{1}{q}}+\left(\left|f'\left(\frac{a+b}{2}\right)\right|^{q}+\left|f'(b)\right|^{q}\right)^{\frac{1}{q}}\right\}.$$

We note that by inequality

$$\left|f'\left(\frac{a+b}{2}\right)\right|^{q}\leq\left|f'(a)\right|^{q}+\left|f'(b)\right|^{q}$$

we have

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_{a}^{b}f(x)dx\right|$$
$$\leq\frac{b-a}{4}\left(\frac{1}{p+1}\right)^{\frac{1}{p}}\left\{\left(\left|f'(b)\right|^{q}+2\left|f'(a)\right|^{q}\right)^{\frac{1}{q}}+\left(\left|f'(a)\right|^{q}+2\left|f'(b)\right|^{q}\right)^{\frac{1}{q}}\right\}.$$

Corollary 1.7. *In Theorem 1.3, if we take $\alpha=\frac{1}{2}$ and $\lambda=1$, then we have the following trapezoid inequality*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_{a}^{b}f(x)dx\right|\leq\frac{b-a}{4}\left(\frac{1}{p+1}\right)^{\frac{1}{p}}$$
$$\times\left\{\left(\left|f'\left(\frac{a+b}{2}\right)\right|^{q}+\left|f'(a)\right|^{q}\right)^{\frac{1}{q}}+\left(\left|f'\left(\frac{a+b}{2}\right)\right|^{q}+\left|f'(b)\right|^{q}\right)^{\frac{1}{q}}\right\}.$$

2 Some applications for special means

We now recall the following well-known concepts. For arbitrary real numbers a, b, $a\neq b$, we define

1. The unweighted arithmetic mean

$$A(a,b):=\frac{a+b}{2},\ a,b\in\mathbb{R}.$$

2. Then $n-$Logarithmic mean

$$L_{n}(a,b):=\left(\frac{b^{n+1}-a^{n+1}}{(n+1)(b-a)}\right)^{\frac{1}{n}},\ n\in\mathbb{N},\ n\geq 1,\ a,b\in\mathbb{R},\ a<b.$$

Now we give some applications of the new results derived in section 2 to special means of real numbers.

Proposition 2.1. *Let* $a, b \in \mathbb{R}$ *with* $a < b$ *and* $n \in \mathbb{N}, n \geq 2$. *Then*

$$\left|\frac{1}{3}A(a^n, b^n) + \frac{2}{3}A^n(a,b) - L_n^n(a,b)\right| \leq \frac{5n\,(b-a)}{36}\left(|b|^{(n-1)q} + |a|^{(n-1)q}\right)^{\frac{1}{q}}$$

Proof. The assertion follows from Corollary 1.2 applied to the function $f(x) = x^n$, $x \in \mathbb{R}$, because $|f'|^q$ is a P-function. □

Proposition 2.2. *Let* $a, b \in \mathbb{R}$ *with* $a < b$ *and* $n \in \mathbb{N}, n \geq 2$. *Then*

$$|A^n(a,b) - L_n^n(a,b)| \leq \frac{n\,(b-a)}{4}\left(|b|^{(n-1)q} + |a|^{(n-1)q}\right)^{\frac{1}{q}}$$

and

$$|A(a^n, b^n) - L_n^n(a,b)| \leq \frac{n\,(b-a)}{4}\left(|b|^{(n-1)q} + |a|^{(n-1)q}\right)^{\frac{1}{q}}$$

Proof. The assertion follows from Corollary 1.3 and Corollary 1.4 applied to the function $f(x) = x^n$, $x \in \mathbb{R}$, because $|f'|^q$ is a P-function. □

Proposition 2.3. *Let* $a, b \in \mathbb{R}$ *with* $a < b$ *and* $n \in \mathbb{N}, n \geq 2$. *Then*

$$\left|\frac{1}{3}A(a^n, b^n) + \frac{2}{3}A^n(a,b) - L_n^n(a,b)\right| \leq \frac{n\,(b-a)}{12}\left(\frac{1+2^{p+1}}{3\,(p+1)}\right)^{\frac{1}{p}}$$
$$\times\left\{\left(|A(a,b)|^{(n-1)q} + |a|^{(n-1)q}\right)^{\frac{1}{q}} + \left(|A(a,b)|^{(n-1)q} + |b|^{(n-1)q}\right)^{\frac{1}{q}}\right\}.$$

Proof. The assertion follows from Corollary 1.5 applied to the function $f(x) = x^n$, $x \in \mathbb{R}$, because $|f'|^q$ is a P-function. □

Proposition 2.4. *Let* $a, b \in \mathbb{R}$ *with* $a < b$ *and* $n \in \mathbb{N}, n \geq 2$. *Then*

$$|A^n(a,b) - L_n^n(a,b)| \leq \frac{n\,(b-a)}{4}\left(\frac{1}{p+1}\right)^{\frac{1}{p}}$$
$$\times\left\{\left(|A(a,b)|^{(n-1)q} + |a|^{(n-1)q}\right)^{\frac{1}{q}} + \left(|A(a,b)|^{(n-1)q} + |b|^{(n-1)q}\right)^{\frac{1}{q}}\right\}.$$

and

$$|A(a^n, b^n) - L_n^n(a,b)| \leq \frac{b-a}{4}\left(\frac{1}{p+1}\right)^{\frac{1}{p}}$$
$$\times\left\{\left(|A(a,b)|^{(n-1)q} + |a|^{(n-1)q}\right)^{\frac{1}{q}} + \left(|A(a,b)|^{(n-1)q} + |b|^{(n-1)q}\right)^{\frac{1}{q}}\right\}.$$

Proof. The assertion follows from Corollary 1.6 and Corollary 1.7 applied to the function $f(x) = x^n$, $x \in \mathbb{R}$, because $|f'|^q$ is a P-function. □

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