Malaya J. Mat. 3(1)(2015) 119–130

Malaya Journal of Matematik

MJM an international journal of mathematical sciences with computer applications...

www.malayajournal.org

ISSN : 2319-3786

Generalized Mizoguchi-Takahashi contraction in consideration of common tripled fixed point theorem for hybrid pair of mappings

Bhavana Deshpande[a,*] and Amrish Handa[b]

[a,b]Department of Mathematics, Government P. G. Arts & Science College, Ratlam-457001, India.

Abstract

We establish a common tripled fixed point theorem for hybrid pair of mappings under generalized Mizoguchi-Takahashi contraction. It is to be noted that to find tripled coincidence point, we do not employ the condition of continuity of any mapping involved therein. An example is also given to validate our result. We improve, extend and generalize several known results.

Keywords: Mizoguchi-Takahashi contraction, fixed point theorem.

2010 MSC: 34G20.

1 Introduction

Let (X, d) be a metric space and $CB(X)$ be the set of all non empty closed bounded subsets of X. Let $D(x, A)$ denote the distance from x to $A \subset X$ and H denote the Hausdorff metric induced by d, that is,

$$\begin{aligned} D(x,\ A) &= \inf_{a \in A} d(x,\ a) \\ \text{and } H(A,\ B) &= \max\left\{\sup_{a \in A} D(a,\ B),\ \sup_{b \in B} D(b,\ A)\right\}, \text{ for all } A, B \in CB(X). \end{aligned}$$

Nadler [27] extended the famous Banach Contraction Principle [9] from single-valued mapping to multi-valued mapping. Then after several authors studied the existence of fixed points for various multi-valued contractive mappings under different conditions. For more details, see ([1],[2],[4],[15],[16],[19],[22],[23],[25],[26],[30]) and the reference therein. The theory of multi-valued mappings has application in control theory, convex optimization, differential inclusion and economics.

Bhaskar and Lakshmikantham [12] established some coupled fixed point theorems and applied these to study the existence and uniqueness of solution for periodic boundary value problems. Lakshmikantham and Ciric [24] proved coupled coincidence and common coupled fixed point theorems for nonlinear contractive mappings in partially ordered complete metric spaces and extended the results of Bhaskar and Lakshmikantham [12].

Berinde and Borcut [10] introduced the concept of tripled fixed point for single valued mappings in partially ordered metric spaces. In [10], Berinde and Borcut established the existence of tripled fixed point of single-valued mappings in partially ordered metric spaces. Samet and Vetro [28] introduced the notion of fixed point of N order in case of single-valued mappings. In particular for N=3 (tripled case), we have the following definition:

*Corresponding author.

E-mail address: bhavnadeshpande@yahoo.com (Bhavana Deshpande), amrishhanda83@gmail.com(Amrish Handa).

Definition 1.1. *Let X be a non-empty set and $F : X \times X \times X \to X$ be a given mapping. An element $(x, y, z) \in X \times X \times X$ is called a tripled fixed point of the mapping F if*

$$F(x,\ y,\ z) = x,\ F(y,\ z,\ x) = y \text{ and } F(z,\ x,\ y) = z.$$

For more details on coupled and tripled fixed point theory, see ([3],[5],[6],[7],[8],[11], [13],[14],[17],[18],[20]). Very recently Samet et al. [29] claimed that most of the coupled fixed point theorems in the setting of single-valued mappings on ordered metric spaces are consequences of well-known fixed point theorems.

Tripled fixed point theory to multi-valued mappings were extended by Deshpande et al. [19] and obtained tripled coincidence point and common tripled fixed point theorems involving hybrid pair of mappings under generalized nonlinear contraction. Very few authors established coupled and tripled fixed point theorems for hybrid pair of mappings including [1],[2],[19],[25].

In [19], Deshpande et al. introduced the following for multi-valued mappings:

Definition 1.2. *Let X be a non empty set, $F : X \times X \times X \to 2^X$ (a collection of all non empty subsets of X) and g be a self-mapping on X. An element $(x, y, z) \in X \times X \times X$ is called*

(1) *a tripled fixed point of F if $x \in F(x, y, z)$, $y \in F(y, z, x)$ and $z \in F(z, x, y)$.*

(2) *a tripled coincidence point of hybrid pair $\{F, g\}$ if $g(x) \in F(x, y, z)$, $g(y) \in F(y, z, x)$ and $g(z) \in F(z, x, y)$.*

(3) *a common tripled fixed point of hybrid pair $\{F, g\}$ if $x = g(x) \in F(x, y, z)$, $y = g(y) \in F(y, z, x)$ and $z = g(z) \in F(z, x, y)$.*

We denote the set of tripled coincidence points of mappings F and g by $C\{F, g\}$. Note that if $(x, y, z) \in C\{F, g\}$, then (y, z, x) and (z, x, y) are also in $C\{F, g\}$.

Definition 1.3. *Let $F : X \times X \times X \to 2^X$ be a multi-valued mapping and g be a self-mapping on X. The hybrid pair $\{F, g\}$ is called $w-$compatible if $g(F(x, y, z)) \subseteq F(gx, gy, gz)$ whenever $(x, y, z) \in C\{F, g\}$.*

Definition 1.4. *Let $F : X \times X \times X \to 2^X$ be a multi-valued mapping and g be a self-mapping on X. The mapping g is called $F-$weakly commuting at some point $(x, y, z) \in X \times X \times X$ if $g^2x \in F(gx, gy, gz)$, $g^2y \in F(gy, gz, gx)$ and $g^2z \in F(gz, gx, gy)$.*

Lemma 1.1. *Let (X, d) be a metric space. Then, for each $a \in X$ and $B \in CB(X)$, there is $b_0 \in B$ such that $D(a, B) = d(a, b_0)$, where $D(a, B) = \inf_{b \in B} d(a, b)$.*

Mizoguchi and Takahashi [26] proved the following generalization of Nadler's fixed point theorem for a weak contraction:

Theorem 1.1. *Let (X, d) be a complete metric space and $T : X \to CB(X)$ be a multi-valued mapping. Assume that*

$$H(Tx,\ Ty) \leq \psi(d(x,\ y))d(x,\ y),$$

for all $x, y \in X$, where ψ is a function from $[0, \infty)$ into $[0, 1)$ satisfying $\limsup_{s \to t+} \psi(s) < 1$ for all $t \geq 0$. Then T has a fixed point.

Amini-Harandi and O'Regan [4] obtained a generalization of Mizoguchi and Takahashi's fixed point theorem. Recently Ciric et al. [13] proved coupled fixed point theorems for mixed monotone mappings satisfying a generalized Mizoguchi-Takahashi's condition in the setting of ordered metric spaces. Main results of Ciric et al. [13] extended and generalized the results of Bhaskar and Lakshmikantham [12], Du [20] and Harjani et al. [21].

In this paper, we prove a common tripled fixed point theorem for hybrid pair of mappings under generalized Mizoguchi-Takahashi contraction. We improve, extend and generalize the results of Amini-Harandi and O'Regan [4], Bhaskar and Lakshmikantham [12], Ciric et al. [13], Du [20], Harjani et al. [21] and Mizoguchi and Takahashi [26]. It is to be noted that to find tripled coincidence point, we do not employ the condition of continuity of any mapping involved therein. An example validate to our result has also been given.

2 Main results

Let Φ denote the set of all functions $\varphi : [0, +\infty) \to [0, +\infty)$ satisfying
(i_φ) φ is non-decreasing,
(ii_φ) $\varphi(t) = 0 \Leftrightarrow t = 0$,
(iii_φ) $\limsup_{t\to 0+} \frac{t}{\varphi(t)} < \infty$.
Let Ψ denote the set of all functions $\psi : [0, +\infty) \to [0, 1)$ which satisfies $\lim_{r\to t+} \psi(r) < 1$ for all $t \geq 0$.

Theorem 2.1. *Let (X, d) be a metric space, $F : X \times X \times X \to CB(X)$ and $g : X \to X$ be two mappings. Suppose that there exist some $\varphi \in \Phi$ and some $\psi \in \Psi$ such that*

$$\begin{aligned} & \varphi\left(H(F(x,\, y,\, z),\, F(u,\, v,\, w))\right) \\ \leq\ & \psi\left(\varphi\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\}\right]\right) \\ & \times \varphi\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\}\right], \end{aligned} \tag{2.1}$$

for all $x, y, z, u, v, w \in X$. Furthermore, assume that $F(X \times X \times X) \subseteq g(X)$ and $g(X)$ is a complete subset of X. Then F and g have a tripled coincidence point. Moreover, F and g have a common tripled fixed point, if one of the following conditions holds:

(a) *F and g are $w-$compatible. $\lim_{n\to\infty} g^n x = u$, $\lim_{n\to\infty} g^n y = v$ and $\lim_{n\to\infty} g^n z = w$ for some $(x, y, z) \in C\{F, g\}$ and for some $u, v, w \in X$ and g is continuous at u, v and w.*

(b) *g is $F-$weakly commuting for some $(x, y, z) \in C\{F, g\}$, gx, gy and gz are fixed points of g, that is, $g^2 x = gx$, $g^2 y = gy$ and $g^2 z = gz$.*

(c) *g is continuous at x, y and z. $\lim_{n\to\infty} g^n u = x$, $\lim_{n\to\infty} g^n v = y$ and $\lim_{n\to\infty} g^n w = z$ for some $(x, y, z) \in C\{F, g\}$ and for some $u, v, w \in X$.*

(d) *$g(C\{F, g\})$ is a singleton subset of $C\{F, g\}$.*

Proof. Let $x_0, y_0, z_0 \in X$ be arbitrary. Then $F(x_0, y_0, z_0)$, $F(y_0, z_0, x_0)$ and $F(z_0, x_0, y_0)$ are well defined. Choose $gx_1 \in F(x_0, y_0, z_0)$, $gy_1 \in F(y_0, z_0, x_0)$ and $gz_1 \in F(z_0, x_0, y_0)$, because $F(X \times X \times X) \subseteq g(X)$. Since $F : X \times X \times X \to CB(X)$, therefore by Lemma 1.1, there exist $u_1 \in F(x_1, y_1, z_1)$, $u_2 \in F(y_1, z_1, x_1)$ and $u_3 \in F(z_1, x_1, y_1)$ such that

$$\begin{aligned} d(gx_1,\, u_1) &\leq H(F(x_0,\, y_0,\, z_0),\, F(x_1,\, y_1,\, z_1)), \\ d(gy_1, u_2) &\leq H(F(y_0,\, z_0,\, x_0),\, F(y_1,\, z_1,\, x_1)), \\ d(gz_1,\, u_3) &\leq H(F(z_0,\, x_0,\, y_0),\, F(z_1,\, x_1,\, y_1)). \end{aligned}$$

Since $F(X \times X \times X) \subseteq g(X)$, there exist $x_2, y_2, z_2 \in X$ such that $u_1 = gx_2$, $u_2 = gy_2$ and $u_3 = gz_2$. Thus

$$\begin{aligned} d(gx_1,\, gx_2) &\leq H(F(x_0,\, y_0,\, z_0),\, F(x_1,\, y_1,\, z_1)), \\ d(gy_1,\, gy_2) &\leq H(F(y_0,\, z_0,\, x_0),\, F(y_1,\, z_1,\, x_1)), \\ d(gz_1,\, gz_2) &\leq H(F(z_0,\, x_0,\, y_0),\, F(z_1,\, x_1,\, y_1)). \end{aligned}$$

Continuing this process, we obtain sequences $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ in X such that for all $n \in \mathbb{N}$, we have $gx_{n+1} \in F(x_n, y_n, z_n)$, $gy_{n+1} \in F(y_n, z_n, x_n)$ and $gz_{n+1} \in F(z_n, x_n, y_n)$ such that

$$\begin{aligned} d(gx_n,\, gx_{n+1}) &\leq H(F(x_{n-1},\, y_{n-1},\, z_{n-1}),\, F(x_n,\, y_n,\, z_n)), \\ d(gy_n,\, gy_{n+1}) &\leq H(F(y_{n-1},\, z_{n-1},\, x_{n-1}),\, F(y_n,\, z_n,\, x_n)), \\ d(gz_n,\, gz_{n+1}) &\leq H(F(z_{n-1},\, x_{n-1},\, y_{n-1}),\, F(z_n,\, x_n,\, y_n)), \end{aligned}$$

which implies, by (i_φ), we have

$$\begin{aligned} & \varphi\left(d(gx_n,\, gx_{n+1})\right) \\ \leq\ & \varphi\left(H(F(x_{n-1},\, y_{n-1},\, z_{n-1}),\, F(x_n,\, y_n,\, z_n))\right) \\ \leq\ & \psi\left(\varphi\left[\max\left\{d(gx_{n-1},\, gx_n),\, d(gy_{n-1},\, gy_n),\, d(gz_{n-1},\, gz_n)\right\}\right]\right) \\ & \times \varphi\left[\max\left\{d(gx_{n-1},\, gx_n),\, d(gy_{n-1},\, gy_n),\, d(gz_{n-1},\, gz_n)\right\}\right], \end{aligned}$$

which, by the fact that $\psi < 1$, implies

$$\begin{aligned} &\varphi\left(d(gx_n,\, gx_{n+1})\right) \\ \leq\; &\varphi\left[\max\left\{d(gx_{n-1},\, gx_n),\, d(gy_{n-1},\, gy_n),\, d(gz_{n-1},\, gz_n)\right\}\right]. \end{aligned} \tag{2.2}$$

Similarly

$$\begin{aligned} &\varphi\left(d(gy_n,\, gy_{n+1})\right) \\ \leq\; &\varphi\left[\max\left\{d(gx_{n-1},\, gx_n),\, d(gy_{n-1},\, gy_n),\, d(gz_{n-1},\, gz_n)\right\}\right], \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} &\varphi\left(d(gz_n,\, gz_{n+1})\right) \\ \leq\; &\varphi\left[\max\left\{d(gx_{n-1},\, gx_n),\, d(gy_{n-1},\, gy_n),\, d(gz_{n-1},\, gz_n)\right\}\right]. \end{aligned} \tag{2.4}$$

Combining (2.2), (2.3) and (2.4), we get

$$\begin{aligned} &\max\left\{\varphi\left(d(gx_n,\, gx_{n+1})\right),\, \varphi\left(d(gy_n,\, gy_{n+1})\right),\, \varphi\left(d(gz_n,\, gz_{n+1})\right)\right\} \\ \leq\; &\varphi\left[\max\left\{d(gx_{n-1},\, gx_n),\, d(gy_{n-1},\, gy_n),\, d(gz_{n-1},\, gz_n)\right\}\right]. \end{aligned}$$

Since φ is non-decreasing, it follows that

$$\begin{aligned} &\varphi\left[\max\left\{d(gx_n,\, gx_{n+1}),\, d(gy_n,\, gy_{n+1}),\, d(gz_n,\, gz_{n+1})\right\}\right] \\ \leq\; &\varphi\left[\max\left\{d(gx_{n-1},\, gx_n),\, d(gy_{n-1},\, gy_n),\, d(gz_{n-1},\, gz_n)\right\}\right], \end{aligned} \tag{2.5}$$

for all $n \geq 0$. Now (2.5) shows that $\{\varphi[\max\{d(gx_n, gx_{n+1}), d(gy_n, gy_{n+1}), d(gz_n, gz_{n+1})\}]\}$ is a non-increasing sequence. Thus there exists $\delta \geq 0$ such that

$$\lim_{n\to\infty} \varphi\left[\max\left\{d(gx_n,\, gx_{n+1}),\, d(gy_n,\, gy_{n+1}),\, d(gz_n,\, gz_{n+1})\right\}\right] = \delta. \tag{2.6}$$

Since $\psi \in \Psi$, we have $\lim_{r\to\delta+} \psi(r) < 1$ and $\psi(\delta) < 1$. Then there exist $\alpha \in [0, 1)$ and $\varepsilon > 0$ such that $\psi(r) \leq \alpha$ for all $r \in [\delta, \delta + \varepsilon)$. From (2.6), we can take $n_0 \geq 0$ such that $\delta \leq \varphi[\max\{d(gx_n, gx_{n+1}), d(gy_n, gy_{n+1}), d(gz_n, gz_{n+1})\}] \leq \delta + \varepsilon$ for all $n \geq n_0$. Then from (2.1) and (i_φ), for all $n \geq n_0$, we have

$$\begin{aligned} &\varphi\left(d(gx_n,\, gx_{n+1})\right) \\ \leq\; &\psi\left(\varphi\left[\max\left\{d(gx_{n-1},\, gx_n),\, d(gy_{n-1},\, gy_n),\, d(gz_{n-1},\, gz_n)\right\}\right]\right) \\ &\times\varphi\left[\max\left\{d(gx_{n-1},\, gx_n),\, d(gy_{n-1},\, gy_n),\, d(gz_{n-1},\, gz_n)\right\}\right] \\ \leq\; &\alpha\varphi\left[\max\left\{d(gx_{n-1},\, gx_n),\, d(gy_{n-1},\, gy_n),\, d(gz_{n-1},\, gz_n)\right\}\right]. \end{aligned}$$

Thus, for all $n \geq n_0$, we have

$$\begin{aligned} &\varphi\left(d(gx_n,\, gx_{n+1})\right) \\ \leq\; &\alpha\varphi\left[\max\left\{d(gx_{n-1},\, gx_n),\, d(gy_{n-1},\, gy_n),\, d(gz_{n-1},\, gz_n)\right\}\right]. \end{aligned} \tag{2.7}$$

Similarly, for all $n \geq n_0$, we have

$$\begin{aligned} &\varphi\left(d(gy_n,\, gy_{n+1})\right) \\ \leq\; &\alpha\varphi\left[\max\left\{d(gx_{n-1},\, gx_n),\, d(gy_{n-1},\, gy_n),\, d(gz_{n-1},\, gz_n)\right\}\right], \end{aligned} \tag{2.8}$$

and

$$\begin{aligned} &\varphi\left(d(gz_n,\, gz_{n+1})\right) \\ \leq\; &\alpha\varphi\left[\max\left\{d(gx_{n-1},\, gx_n),\, d(gy_{n-1},\, gy_n),\, d(gz_{n-1},\, gz_n)\right\}\right]. \end{aligned} \tag{2.9}$$

Combining (2.7), (2.8) and (2.9), for all $n \geq n_0$, we get

$$\begin{aligned} &\max\left\{\varphi\left(d(gx_n,\, gx_{n+1})\right),\, \varphi\left(d(gy_n,\, gy_{n+1})\right),\, \varphi\left(d(gz_n,\, gz_{n+1})\right)\right\} \\ \leq\; &\alpha\varphi\left[\max\left\{d(gx_{n-1},\, gx_n),\, d(gy_{n-1},\, gy_n),\, d(gz_{n-1},\, gz_n)\right\}\right]. \end{aligned}$$

Since φ is non-decreasing, for all $n \geq n_0$, it follows that

$$\begin{aligned} & \varphi\left[\max\left\{d(gx_n,\, gx_{n+1}),\, d(gy_n,\, gy_{n+1}),\, d(gz_n,\, gz_{n+1})\right\}\right] \\ \leq\ & \alpha\varphi\left[\max\left\{d(gx_{n-1},\, gx_n),\, d(gy_{n-1},\, gy_n),\, d(gz_{n-1},\, gz_n)\right\}\right], \end{aligned} \tag{2.10}$$

Letting $n \to \infty$ in (2.10) and using (2.6), we obtain that $\delta \leq \alpha\delta$. Since $\alpha \in [0, 1)$, therefore $\delta = 0$. Thus

$$\lim_{n\to\infty} \varphi\left[\max\left\{d(gx_n,\, gx_{n+1}),\, d(gy_n,\, gy_{n+1}),\, d(gz_n,\, gz_{n+1})\right\}\right] = 0. \tag{2.11}$$

Since $\{\varphi[\max\{d(gx_n,\, gx_{n+1}),\, d(gy_n,\, gy_{n+1}),\, d(gz_n,\, gz_{n+1})\}]\}$ is a non-increasing sequence and φ is non-decreasing, then $\{\max\{d(gx_n,\, gx_{n+1}),\, d(gy_n,\, gy_{n+1}),\, d(gz_n,\, gz_{n+1})\}\}$ is also a non-increasing sequence of positive numbers. This implies that there exists $\theta \geq 0$ such that

$$\lim_{n\to\infty} \max\left\{d(gx_n,\, gx_{n+1}),\, d(gy_n,\, gy_{n+1}),\, d(gz_n,\, gz_{n+1})\right\} = \theta.$$

Since φ is non-decreasing, we have

$$\varphi\left[\max\left\{d(gx_n,\, gx_{n+1}),\, d(gy_n,\, gy_{n+1}),\, d(gz_n,\, gz_{n+1})\right\}\right] \geq \varphi[\theta]. \tag{2.12}$$

Letting $n \to \infty$ in (2.12), by using (2.11), we get $0 \geq \varphi[\theta]$, which, by (ii_φ), implies $\theta = 0$. Thus

$$\lim_{n\to\infty} \max\left\{d(gx_n,\, gx_{n+1}),\, d(gy_n,\, gy_{n+1}),\, d(gz_n,\, gz_{n+1})\right\} = 0. \tag{2.13}$$

Suppose that $\max\{d(gx_n, gx_{n+1}), d(gy_n, gy_{n+1}), d(gz_n, gz_{n+1})\} = 0$ for some $n \geq 0$. Then, we have $d(gx_n, gx_{n+1}) = 0$, $d(gy_n, gy_{n+1}) = 0$ and $d(gz_n, gz_{n+1}) = 0$ which implies that $gx_n = gx_{n+1} \in F(x_n, y_n, z_n)$, $gy_n = gy_{n+1} \in F(y_n, z_n, x_n)$ and $gz_n = gz_{n+1} \in F(z_n, x_n, y_n)$, that is, (x_n, y_n, z_n) is a tripled coincidence point of F and g. Now, suppose that $\max\{d(gx_n, gx_{n+1}), d(gy_n, gy_{n+1}), d(gz_n, gz_{n+1})\} \neq 0$, for all $n \geq 0$. Denote

$$a_n = \varphi\left[\max\left\{d(gx_n,\, gx_{n+1}),\, d(gy_n,\, gy_{n+1}),\, d(gz_n,\, gz_{n+1})\right\}\right], \text{ for all } n \geq 0.$$

From (2.10), we have $a_n \leq \alpha a_{n-1}$, for all $n \geq n_0$. Then, we have

$$\sum_{n=0}^{\infty} a_n \leq \sum_{n=0}^{n_0} a_n + \sum_{n=n_0+1}^{\infty} \alpha^{n-n_0} a_{n_0} < \infty. \tag{2.14}$$

On the other hand, by (iii_φ), we have

$$\limsup_{n\to\infty} \frac{\max\left\{d(gx_n,\, gx_{n+1}),\, d(gy_n,\, gy_{n+1}),\, d(gz_n,\, gz_{n+1})\right\}}{\varphi\left[\max\left\{d(gx_n,\, gx_{n+1}),\, d(gy_n,\, gy_{n+1}),\, d(gz_n,\, gz_{n+1})\right\}\right]} < \infty. \tag{2.15}$$

Thus, by (2.14) and (2.15), we have $\sum \max\{d(gx_n, gx_{n+1}), d(gy_n, gy_{n+1}), d(gz_n, gz_{n+1})\} < \infty$. It means that $\{gx_n\}_{n=0}^{\infty}$, $\{gy_n\}_{n=0}^{\infty}$ and $\{gz_n\}_{n=0}^{\infty}$ are Cauchy sequences in $g(X)$. Since $g(X)$ is complete, so there exist x, y, $z \in X$ such that

$$\lim_{n\to\infty} gx_n = gx,\ \lim_{n\to\infty} gy_n = gy \text{ and } \lim_{n\to\infty} gz_n = gz. \tag{2.16}$$

Now, since $gx_{n+1} \in F(x_n, y_n, z_n)$, $gy_{n+1} \in F(y_n, z_n, x_n)$ and $gz_{n+1} \in F(z_n, x_n, y_n)$, therefore by using condition (2.1), (i_φ) and by the fact that $\psi < 1$, we get

$$\begin{aligned} & \varphi\left(D(gx_{n+1},\, F(x,\, y,\, z))\right) \\ \leq\ & \varphi\left(H(F(x_n,\, y_n,\, z_n),\, F(x,\, y,\, z))\right) \\ \leq\ & \psi\left(\varphi\left[\max\left\{d(gx_n,\, gx),\, d(gy_n,\, gy),\, d(gz_n,\, gz)\right\}\right]\right) \\ & \times\varphi\left[\max\left\{d(gx_n,\, gx),\, d(gy_n,\, gy),\, d(gz_n,\, gz)\right\}\right] \\ \leq\ & \varphi\left[\max\left\{d(gx_n,\, gx),\, d(gy_n,\, gy),\, d(gz_n,\, gz)\right\}\right]. \end{aligned}$$

Since φ is non-decreasing, we have

$$D(gx_{n+1},\, F(x,\, y,\, z)) \leq \max\left\{d(gx_n,\, gx),\, d(gy_n,\, gy),\, d(gz_n,\, gz)\right\}. \tag{2.17}$$

Letting $n \to \infty$ in (2.17), by using (2.16), we obtain

$$D(gx,\ F(x,\ y,\ z)) = 0.$$

Similarly

$$D(gy,\ F(y,\ z,\ x)) = 0 \text{ and } D(gz,\ F(z,\ x,\ y)) = 0,$$

which implies that

$$gx \in F(x,\ y,\ z),\ gy \in F(y,\ z,\ x) \text{ and } gz \in F(z,\ x,\ y),$$

that is, (x, y, z) is a tripled coincidence point of F and g.

Suppose now that (a) holds. Assume that for some $(x, y, z) \in C\{F, g\}$,

$$\lim_{n\to\infty} g^n x = u,\ \lim_{n\to\infty} g^n y = v \text{ and } \lim_{n\to\infty} g^n z = w, \tag{2.18}$$

where $u, v, w \in X$. Since g is continuous at u, v and w. We have, by (2.18), that u, v and w are fixed points of g, that is,

$$gu = u,\ gv = v \text{ and } gw = w. \tag{2.19}$$

As F and g are $w-$compatible, so, for all $n \geq 1$,

$$\begin{aligned} g^n x &\in F(g^{n-1}x,\ g^{n-1}y,\ g^{n-1}z), \\ g^n y &\in F(g^{n-1}y,\ g^{n-1}z,\ g^{n-1}x), \\ g^n z &\in F(g^{n-1}z,\ g^{n-1}x,\ g^{n-1}y). \end{aligned} \tag{2.20}$$

Now, by using (2.1), (2.20), (i_φ) and by the fact that $\psi < 1$, we obtain

$$\begin{aligned} & \varphi\left(D(g^n x,\ F(u,\ v,\ w))\right) \\ \leq\ & \varphi\left(H(F(g^{n-1}x,\ g^{n-1}y,\ g^{n-1}z),\ F(u,\ v,\ w))\right) \\ \leq\ & \psi\left(\varphi\left[\max\left\{d(g^n x,\ gu),\ d(g^n y,\ gv),\ d(g^n z,\ gw)\right\}\right]\right) \\ & \times\varphi\left[\max\left\{d(g^n x,\ gu),\ d(g^n y,\ gv),\ d(g^n z,\ gw)\right\}\right] \\ \leq\ & \varphi\left[\max\left\{d(g^n x,\ gu),\ d(g^n y,\ gv),\ d(g^n z,\ gw)\right\}\right]. \end{aligned}$$

Since φ is non-decreasing, we have

$$D(g^n x,\ F(u,\ v,\ w)) \leq \max\left\{d(g^n x,\ gu),\ d(g^n y,\ gv),\ d(g^n z,\ gw)\right\}. \tag{2.21}$$

On taking limit as $n \to \infty$ in (2.21), by using (2.18) and (2.19), we get

$$D(gu,\ F(u,\ v,\ w)) = 0.$$

Similarly

$$D(gv,\ F(v,\ w,\ u)) = 0 \text{ and } D(gw,\ F(w,\ u,\ v)) = 0,$$

which implies that

$$gu \in F(u,\ v,\ w),\ gv \in F(v,\ w,\ u) \text{ and } gw \in F(w,\ u,\ v). \tag{2.22}$$

Now, from (2.19) and (2.22), we have

$$u =\ gu \in F(u,\ v,\ w),\ v = gv \in F(v,\ w,\ u) \text{ and } w = gw \in F(w,\ u,\ v),$$

that is, (u, v, w) is a common tripled fixed point of F and g.

Suppose now that (b) holds. Assume that for some $(x, y, z) \in C\{F, g\}$, g is $F-$weakly commuting, that is, $g^2x \in F(gx, gy, gz)$, $g^2y \in F(gy, gz, gx)$, $g^2z \in F(gz, gx, gy)$ and $g^2x = gx$, $g^2y = gy$, $g^2z = gz$. Thus $gx = g^2x \in F(gx, gy, gz)$, $gy = g^2y \in F(gy, gz, gx)$ and $gz = g^2z \in F(gz, gx, gy)$, that is, (gx, gy, gz) is a common tripled fixed point of F and g.

Suppose now that (c) holds. Assume that for some $(x, y, z) \in C\{F, g\}$ and for some $u, v, w \in X$,

$$\lim_{n\to\infty} g^n u = x,\ \lim_{n\to\infty} g^n v = y \text{ and } \lim_{n\to\infty} g^n w = z.$$

Since g is continuous at x, y and z. We have that x, y and z are fixed point of g, that is,

$$gx = x,\ gy = y \text{ and } gz = z.$$

Since $(x, y, z) \in C\{F, g\}$, therefore, we obtain

$$x = gx \in F(x,\ y,\ z),\ y = gy \in F(y,\ z,\ x) \text{ and } z = gz \in F(z,\ x,\ y),$$

that is, (x, y, z) is a common tripled fixed point of F and g.

Finally, suppose that (d) holds. Let $g(C\{F, g\}) = \{(x, x, x)\}$. Then $\{x\} = \{gx\} = F(x, x, x)$. Hence (x, x, x) is a common tripled fixed point of F and g. □

Example 2.1. *Suppose that $X = [0, 1]$, equipped with the metric $d : X \times X \to [0, +\infty)$ defined as $d(x, y) = \max\{x, y\}$ and $d(x, x) = 0$ for all $x, y \in X$. Let $F : X \times X \times X \to CB(X)$ be defined as*

$$F(x,\ y,\ z) = \begin{cases} \{0\},\ \text{for } x,\ y,\ z = 1, \\ \left[0,\ \frac{x^4}{4}\right],\ \text{for } x,\ y,\ z \in [0,\ 1), \end{cases}$$

and $g : X \to X$ be defined as

$$g(x) = x^2, \text{ for all } x \in X.$$

Define $\varphi : [0, +\infty) \to [0, +\infty)$ by

$$\varphi(t) = \begin{cases} \ln(t+1),\ \text{for } t \neq 1 \\ \frac{3}{4},\ \text{for } t = 1, \end{cases}$$

and $\psi : [0, +\infty) \to [0, 1)$ defined by

$$\psi(t) = \frac{\varphi(t)}{t}, \text{ for all } t \geq 0.$$

Now, for all $x, y, z, u, v, w \in X$ with $x, y, z, u, v, w \in [0, 1)$, we have

Case (a). If $x = u$, then

$$\begin{aligned} & H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \\ = \ & \frac{u^4}{4} \\ \leq \ & \ln(u^2+1) \\ \leq \ & \ln(\max\{x^2,\ u^2\}+1) \\ \leq \ & \ln(d(gx,\ gu)\ +1) \\ \leq \ & \ln\left[\max\left\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\right\}+1\right], \end{aligned}$$

which implies that

$$\begin{aligned} & \varphi\left(H(F(x,\ y,\ z),\ F(u,\ v,\ w))\right) \\ = \ & \ln\left[H(F(x,\ y,\ z),\ F(u,\ v,\ w))+1\right] \\ \leq \ & \ln\left[\ln\left[\max\left\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\right\}+1\right]+1\right] \\ \leq \ & \frac{\ln\left[\ln\left[\max\left\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\right\}+1\right]+1\right]}{\ln\left[\max\left\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\right\}+1\right]} \\ & \times\ln\left[\max\left\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\right\}+1\right] \\ \leq \ & \psi\left(\varphi\left[\max\left\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\right\}\right]\right) \\ & \times\varphi\left[\max\left\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\right\}\right]. \end{aligned}$$

Case (b). If $x \neq u$ with $x < u$, then

$$\begin{aligned} & H(F(x,\ y,\ z),\ F(u,\ v,\ w)) \\ = \ & \frac{u^4}{4} \\ \leq \ & \ln(u^2+1) \\ \leq \ & \ln(\max\{x^2,\ u^2\}+1) \\ \leq \ & \ln(d(gx,\ gu)\ +1) \\ \leq \ & \ln\left[\max\left\{d(gx,\ gu),\ d(gy,\ gv),\ d(gz,\ gw)\right\}+1\right], \end{aligned}$$

which implies that

$$\begin{aligned}
& \varphi\left(H(F(x,\, y,\, z),\, F(u,\, v,\, w))\right) \\
= \;& \ln\left[H(F(x,\, y,\, z),\, F(u,\, v,\, w)) + 1\right] \\
\leq \;& \ln\left[\ln\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\} + 1\right] + 1\right] \\
\leq \;& \frac{\ln\left[\ln\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\} + 1\right] + 1\right]}{\ln\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\} + 1\right]} \\
& \times \ln\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\} + 1\right] \\
\leq \;& \psi\left(\varphi\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\}\right]\right) \\
& \times \varphi\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\}\right].
\end{aligned}$$

Similarly, we obtain the same result for $u < x$. Thus the contractive condition (2.1) is satisfied for all $x, y, z, u, v, w \in X$ with $x, y, z, u, v, w \in [0, 1)$. Again, for all $x, y, z, u, v, w \in X$ with $x, y, z \in [0, 1)$ and $u, v, w = 1$, we have

$$\begin{aligned}
& H(F(x,\, y,\, z),\, F(u,\, v,\, w)) \\
= \;& \frac{x^4}{4} \\
\leq \;& \ln(x^2 + 1) \\
\leq \;& \ln(\max\{x^2,\, u^2\} + 1) \\
\leq \;& \ln(d(gx,\, gu)\; + 1) \\
\leq \;& \ln\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\} + 1\right],
\end{aligned}$$

which implies that

$$\begin{aligned}
& \varphi\left(H(F(x,\, y,\, z),\, F(u,\, v,\, w))\right) \\
= \;& \ln\left[H(F(x,\, y,\, z),\, F(u,\, v,\, w)) + 1\right] \\
\leq \;& \ln\left[\ln\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\} + 1\right] + 1\right] \\
\leq \;& \frac{\ln\left[\ln\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\} + 1\right] + 1\right]}{\ln\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\} + 1\right]} \\
& \times \ln\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\} + 1\right] \\
\leq \;& \psi\left(\varphi\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\}\right]\right) \\
& \times \varphi\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\}\right].
\end{aligned}$$

Thus the contractive condition (2.1) is satisfied for all $x, y, z, u, v, w \in X$ with $x, y, z \in [0, 1)$ and $u, v, w = 1$. Similarly, we can see that the contractive condition (2.1) is satisfied for all $x, y, z, u, v, w \in X$ with $x, y, z, u, v, w = 1$. Hence, the hybrid pair $\{F, g\}$ satisfies the contractive condition (2.1), for all $x, y, z, u, v, w \in X$. In addition, all the other conditions of Theorem 2.1 are satisfied and $z = (0, 0, 0)$ is a common tripled fixed point of hybrid pair $\{F, g\}$. The function $F : X \times X \times X \to CB(X)$ involved in this example is not continuous at the point $(1, 1, 1) \in X \times X \times X$.

Remark 2.1. *We improve, extend and generalize the results of Ciric et al. [13] in the sense that*

(i) We prove our result for hybrid pair of mappings.

(ii) We prove our result in the framework of non complete metric space (X, d) and the product set $X \times X \times X$ is not empowered with any order.

(iii) We prove our result without the assumption of continuity and mixed g-monotone property for mapping $F : X \times X \times X \to CB(X)$.

(iv) The functions $\varphi : [0, +\infty) \to [0, +\infty)$ and $\psi : [0, +\infty) \to [0, 1)$ involved in our theorem and example are discontinuous.

If we put $g = I$ (the identity mapping) in the Theorem 2.1, we get the following result:

Corollary 2.1. *Let (X, d) be a complete metric space, $F : X \times X \times X \to CB(X)$ be a mapping. Suppose that there exist some $\varphi \in \Phi$ and some $\psi \in \Psi$ such that*

$$\begin{aligned}
& \varphi\left(H(F(x,\, y,\, z),\, F(u,\, v,\, w))\right) \\
\leq \;& \psi\left(\varphi\left[\max\left\{d(x,\, u),\, d(y,\, v),\, d(z,\, w)\right\}\right]\right) \\
& \times \varphi\left[\max\left\{d(x,\, u),\, d(y,\, v),\, d(z,\, w)\right\}\right],
\end{aligned}$$

for all $x, y, z, u, v, w \in X$. Then F has a tripled fixed point.

If we put $\psi(t) = 1 - \frac{\widetilde{\psi}(t)}{t}$ for all $t \geq 0$ in Theorem 2.1, then we get the following result:

Corollary 2.2. *Let (X, d) be a metric space, $F : X \times X \times X \to CB(X)$ and $g : X \to X$ be two mappings. Suppose that there exist some $\varphi \in \Phi$ and some $\widetilde{\psi} \in \Psi$ such that*

$$\begin{aligned} & \varphi\left(H(F(x,\, y,\, z),\, F(u,\, v,\, w))\right) \\ \leq\ & \varphi\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\}\right] \\ & -\widetilde{\psi}\left(\varphi\left[\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\}\right]\right), \end{aligned}$$

for all $x, y, z, u, v, w \in X$. Furthermore, assume that $F(X \times X \times X) \subseteq g(X)$ and $g(X)$ is a complete subset of X. Then F and g have a tripled coincidence point. Moreover, F and g have a common tripled fixed point, if one of the following conditions holds:

(a) *F and g are $w-$compatible.* $\lim_{n\to\infty} g^n x = u$, $\lim_{n\to\infty} g^n y = v$ *and* $\lim_{n\to\infty} g^n z = w$ *for some $(x, y, z) \in C\{F, g\}$ and for some $u, v, w \in X$ and g is continuous at u, v and w.*

(b) *g is $F-$weakly commuting for some $(x, y, z) \in C\{F, g\}$, gx, gy and gz are fixed points of g, that is, $g^2x = gx$, $g^2y = gy$ and $g^2z = gz$.*

(c) *g is continuous at x, y and z.* $\lim_{n\to\infty} g^n u = x$, $\lim_{n\to\infty} g^n v = y$ *and* $\lim_{n\to\infty} g^n w = z$ *for some $(x, y, z) \in C\{F, g\}$ and for some $u, v, w \in X$.*

(d) *$g(C\{F, g\})$ is a singleton subset of $C\{F, g\}$.*

If we put $g = I$ (the identity mapping) in the Corollary 2.2, we get the following result:

Corollary 2.3. *Let (X, d) be a complete metric space, $F : X \times X \times X \to CB(X)$ be a mapping. Suppose that there exist some $\varphi \in \Phi$ and some $\widetilde{\psi} \in \Psi$ such that*

$$\begin{aligned} & \varphi\left(H(F(x,\, y,\, z),\, F(u,\, v,\, w))\right) \\ \leq\ & \varphi\left[\max\left\{d(x,\, u),\, d(y,\, v),\, d(z,\, w)\right\}\right] \\ & -\widetilde{\psi}\left(\varphi\left[\max\left\{d(x,\, u),\, d(y,\, v),\, d(z,\, w)\right\}\right]\right), \end{aligned}$$

for all $x, y, z, u, v, w \in X$. Then F has a tripled fixed point.

If we put $\varphi(t) = 2t$ for all $t \geq 0$ in Theorem 2.1, then we get the following result:

Corollary 2.4. *Let (X, d) be a metric space, $F : X \times X \times X \to CB(X)$ and $g : X \to X$ be two mappings. Suppose that there exists some $\psi \in \Psi$ such that*

$$\begin{aligned} & H(F(x,\, y,\, z),\, F(u,\, v,\, w)) \\ \leq\ & \psi\left(2\max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\}\right) \\ & \times \max\left\{d(gx,\, gu),\, d(gy,\, gv),\, d(gz,\, gw)\right\}, \end{aligned}$$

for all $x, y, z, u, v, w \in X$. Furthermore, assume that $F(X \times X \times X) \subseteq g(X)$ and $g(X)$ is a complete subset of X. Then F and g have a tripled coincidence point. Moreover, F and g have a common tripled fixed point, if one of the following conditions holds:

(a) *F and g are $w-$compatible.* $\lim_{n\to\infty} g^n x = u$, $\lim_{n\to\infty} g^n y = v$ *and* $\lim_{n\to\infty} g^n z = w$ *for some $(x, y, z) \in C\{F, g\}$ and for some $u, v, w \in X$ and g is continuous at u, v and w.*

(b) *g is $F-$weakly commuting for some $(x, y, z) \in C\{F, g\}$, gx, gy and gz are fixed points of g, that is, $g^2x = gx$, $g^2y = gy$ and $g^2z = gz$.*

(c) *g is continuous at x, y and z.* $\lim_{n\to\infty} g^n u = x$, $\lim_{n\to\infty} g^n v = y$ *and* $\lim_{n\to\infty} g^n w = z$ *for some $(x, y, z) \in C\{F, g\}$ and for some $u, v, w \in X$.*

(d) *$g(C\{F, g\})$ is a singleton subset of $C\{F, g\}$.*

If we put $g = I$ (the identity mapping) in the Corollary 2.4, we get the following result:

Corollary 2.5. *Let (X, d) be a complete metric space, $F : X \times X \times X \to CB(X)$ be a mapping. Suppose that there exists some $\psi \in \Psi$ such that*

$$\begin{aligned} & H(F(x, y, z), F(u, v, w)) \\ \leq \ & \psi\left(2 \max\left\{d(x, u), d(y, v), d(z, w)\right\}\right) \\ & \times \max\left\{d(x, u), d(y, v), d(z, w)\right\}, \end{aligned}$$

for all $x, y, z, u, v, w \in X$. Then F has a tripled fixed point.

If we put $\psi(t) = k$, where $0 < k < 1$, for all $t \geq 0$ in Corollary 2.4, then we get the following result:

Corollary 2.6. *Let (X, d) be a metric space. Assume $F : X \times X \times X \to CB(X)$ and $g : X \to X$ be two mappings satisfying*

$$\begin{aligned} & H(F(x, y, z), F(u, v, w)) \\ \leq \ & k \max\left\{d(gx, gu), d(gy, gv), d(gz, gw)\right\}, \end{aligned}$$

for all $x, y, z, u, v, w \in X$, where $0 < k < 1$. Furthermore, assume that $F(X \times X \times X) \subseteq g(X)$ and $g(X)$ is a complete subset of X. Then F and g have a tripled coincidence point. Moreover, F and g have a common tripled fixed point, if one of the following conditions holds:

(a) F and g are $w-$compatible. $\lim_{n\to\infty} g^n x = u$, $\lim_{n\to\infty} g^n y = v$ and $\lim_{n\to\infty} g^n z = w$ for some $(x, y, z) \in C\{F, g\}$ and for some $u, v, w \in X$ and g is continuous at u, v and w.

(b) g is $F-$weakly commuting for some $(x, y, z) \in C\{F, g\}$, gx, gy and gz are fixed points of g, that is, $g^2x = gx$, $g^2y = gy$ and $g^2z = gz$.

(c) g is continuous at x, y and z. $\lim_{n\to\infty} g^n u = x$, $\lim_{n\to\infty} g^n v = y$ and $\lim_{n\to\infty} g^n w = z$ for some $(x, y, z) \in C\{F, g\}$ and for some $u, v, w \in X$.

(d) $g(C\{F, g\})$ is a singleton subset of $C\{F, g\}$.

If we put $g = I$ (the identity mapping) in the Corollary 2.6, we get the following result:

Corollary 2.7. *Let (X, d) be a complete metric space. Assume $F : X \times X \times X \to CB(X)$ be a mapping satisfying*

$$\begin{aligned} & H(F(x, y, z), F(u, v, w)) \\ \leq \ & k \max\left\{d(x, u), d(y, v), d(z, w)\right\}, \end{aligned}$$

for all $x, y, z, u, v, w \in X$, where $0 < k < 1$. Then F has a tripled fixed point.

3 Acknowledgment

This research work was supported by M. P. Council of Science and Technology, Bhopal (MP), India.

References

[1] M. Abbas, H. Aydi and E. Karapinar, Tripled fixed point theorems for multi-valued nonlinear contraction mappings in partially ordered metric spaces, *Abstr. Appl. Anal.*, Volume 2011 (2011), Article ID 812690, 12 pages.

[2] M. Abbas, L. Ciric, B. Damjanovic and M. A. Khan, Coupled coincidence point and common fixed point theorems for hybrid pair of mappings, *Fixed Point Theory Appl.*, doi:10.1186/1687-1812-2012-4 (2012).

[3] S. M. Alsulami and A. Alotaibi, Tripled coincidence points for monotone operators in partially ordered metric spaces, *International Mathematical Forum*, 7(37)(2012), 1811-1824.

[4] A. Amini-Harandi and D. O'Regan, Fixed point theorems for set-valued contraction type mappings in metric spaces, *Fixed Point Theory Appl.*, 7(2010), Article ID 390183.

[5] H. Aydi, E. Karapinar and M. Postolache, Tripled coincidence point theorems for weak $\varphi-$contractions in partially ordered metric spaces, *Fixed Point Theory Appl.*, 44(2012), 1-15.

[6] H. Aydi and E. Karapinar, Triple fixed points in ordered metric spaces, *Bull. Math. Anal. Appl.*, 4(1)(2012), 197-207.

[7] H. Aydi and E. Karapinar, New Meir-Keeler type tripled fixed point theorems on partially ordered metric spaces, *Mathematical Problems in Engineering*, Volume 2012, Article ID 409872, 17 pages.

[8] H. Aydi, E. Karapinar and C. Vetro, Meir-Keeler type contractions for tripled fixed points, *Acta Mathematica Scientia*, 2012, 32B (6): 2119-2130.

[9] S. Banach, Sur les Operations dans les Ensembles Abstraits et leur. Applications aux Equations Integrals, *Fund. Math.*, 3(1922), 133-181.

[10] V. Berinde and M. Borcut, Tripled fixed point theorems for contractive type mappings in partially ordered metric spaces, *Nonlinear Anal.*, 74(15)(2011), 4889-4897.

[11] V. Berinde and M. Borcut, Tripled coincidence theorems of contractive type mappings in partially ordered metric spaces, *Appl. Math. Comput.*, 218(10), 5929-5936.

[12] T. G. Bhaskar and V. Lakshmikantham, Fixed point theorems in partially ordered metric spaces and applications, *Nonlinear Anal.*, 65(7) (2006), 1379-1393.

[13] L. Ciric, B. Damjanovic, M. Jleli and B. Samet, Coupled fixed point theorems for generalized Mizoguchi-Takahashi contractions with applications, *Fixed Point Theory Appl.*, (2012), 2012:51.

[14] P. Charoensawan, Tripled fixed points theorems of $\varphi-$contractive mixed monotone operators on partially ordered metric spaces, *Appl. Math. Sci.*, 6(105)(2012), 5229-5239.

[15] B. Deshpande, Common fixed point for set and single valued functions without continuity and compatibility, *Mathematica Moravica*, 11(2007), 27-38.

[16] B. Deshpande and R. Pathak, Fixed point theorems for non compatible discontinuous hybrid pairs of mappings on 2- metric spaces, *Demonstratio Mathematica*, XLV (1) 2012, 143-154.

[17] B. Deshpande and A. Handa, Nonlinear mixed monotone-generalized contractions on partially ordered modified intuitionistic fuzzy metric spaces with application to integral equations, *Afr. Mat.*, DOI 10.1007/s13370-013-0204-0.

[18] B. Deshpande and A. Handa, Application of coupled fixed point technique in solving integral equations on modified intuitionistic fuzzy metric spaces, *Adv. Fuzzy Syst.*, Volume 2014, Article ID 348069, 11 pages.

[19] B. Deshpande, S. Sharma and A. Handa, Tripled fixed point theorem for hybrid pair of mappings under generalized nonlinear contraction, *J. Korean Soc. Math. Educ. Ser. B: Pure Appl. Math.*, 21 (1) (2014), 23-38.

[20] W. S. Du, Coupled fixed point theorems for nonlinear contractions satisfied Mizoguchi-Takahashi's condition in quasi ordered metric spaces, *Fixed Point Theory Appl.*, 2010, 9 (2010) Article ID 876372.

[21] J. Harjani, B. Lopez and K. Sadarangani, Fixed point theorems for mixed monotone operators and applications to integral equations, *Nonlinear Anal.*, 74 (2011), 1749–1760.

[22] I. Kubiaczyk and B. Deshpande, A common fixed point theorem for multi-valued mappings through T-weak commutativity, *Mathematica Moravica*, 10 (2006), 55-60.

[23] I. Kubiaczyk and B. Deshpande, Common fixed point of multi-valued mappings without continuity, *Fasciculi Mathematici*, 37(9)(2007), 19-26.

[24] V. Lakshmikantham and L. Ciric, Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces, *Nonlinear Anal.*, 70(12)(2009), 4341-4349.

[25] W. Long, S. Shukla and S. Radenovic, Some coupled coincidence and common fixed point results for hybrid pair of mappings in 0-complete partial metric spaces, *Fixed Point Theory Appl.,* (2013), 2013:145.

[26] N. Mizoguchi and W. Takahashi, Fixed point theorems for multi-valued mappings on complete metric spaces, *J. Math. Anal. Appl.,* 141 (1989), 177–188.

[27] S. B. Nadler, Multi-valued contraction mappings, *Pacific J. Math.,* 30(1969), 475-488.

[28] B. Samet and C. Vetro, Coupled fixed point, F-invariant set and fixed point of N-order, *Ann. Funct. Anal.,* 1(2010), 46-56.

[29] B. Samet, E. Karapinar, H. Aydi and V. C. Rajic, Discussion on some coupled fixed point theorems, *Fixed Point Theory Appl.,* (2013), 2013:50.

[30] S. Sharma and B. Deshpande, Fixed point theorems for set and single valued mappings without continuity and compatibility, *Demonstratio Mathematica*, XL (3) (2007), 649-658.

Received: October 10, 2013; *Accepted*: August 23, 2014

UNIVERSITY PRESS

Website: http://www.malayajournal.org/