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Some vector-valued statistical convergent sequence spaces

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Abstract

In the present paper we introduce some vector-valued statistical convergent sequence spaces defined by a sequence of modulus functions associated with multiplier sequences and we also make an effort to study some topological properties and inclusion relation between these spaces.

Keywords: Modulus function, paranorm space, difference sequence space, statistical convergence.

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1 Introduction and Preliminaries

The study on vector-valued sequence spaces was exploited by Kamthan [11], Ratha and Srivastava [18], Leonard [14], Gupta [9], Tripathy and Sen [26] and many others. The scope for the studies on sequence spaces was extended on introducing the notion of associated multiplier sequences. Goes and Goes [8] defined the differentiated sequence space dE and integrated sequence space $\int E$ for a given sequence space E, with the help of multiplier sequences (k^{-1}) and (k) respectively. Kamthan used the multiplier sequence $(k!)$ see [11]. The study on multiplier sequence spaces were carried out by Colak [2], Colak et al. [3], Srivastava and Srivastava [25], Tripathy and Mahanta [28] and many others. Let w be the set of all sequences of real or complex numbers and let l_∞, c and c_0 be the Banach spaces of bounded, convergent and null sequences $x = (x_k)$ respectively with the usual norm $||x|| = \sup |x_k|$, where $k \in \mathbb{N}$, is the set of positive integers.
Throughout the paper, for all $k \in \mathbb{N}$, E_k are seminormed spaces seminormed by q_k and X is a seminormed space seminormed by q. By $w(E_k)$, $c(E_k)$, $l_\infty(E_k)$ and $l_p(E_k)$ we denote the spaces of all, convergent, bounded and p-absoluetly summable E_k-valued sequences. In the case $E_k = \mathbb{C}$ (the field of complex numbers) for all $k \in \mathbb{N}$, one has the scalar valued sequence spaces respectively. The zero element of E_k is denoted by θ_k and the zero sequence is denoted by $\bar{\theta} = (\theta_k)$.
The notion of difference sequence spaces was introduced by Kizmaz [12], who studied the difference sequence spaces $l_\infty(\Delta)$, $c(\Delta)$ and $c_0(\Delta)$. The notion was further generalized by Et and Colak [4] by introducing the spaces $l_\infty(\Delta^n)$, $c(\Delta^n)$ and $c_0(\Delta^n)$. Let w be the space of all complex or real sequences $x = (x_k)$ and let m, s be non-negative integers, then for $Z = l_\infty,\ c,\ c_0$ we have sequence spaces

$$Z(\Delta_s^m) = \{x = (x_k) \in w : (\Delta_s^m x_k) \in Z\},$$

where $\Delta_s^m x = (\Delta_s^m x_k) = (\Delta_s^{m-1} x_k - \Delta_s^{m-1} x_{k+1})$ and $\Delta_s^0 x_k = x_k$ for all $k \in \mathbb{N}$, which is equivalent to the following binomial representation

$$\Delta_s^m x_k = \sum_{v=0}^{m} (-1)^v \binom{m}{v} x_{k+sv}.$$

Taking $s = 1$, we get the spaces which were studied by Et and Colak [4]. Taking $m = s = 1$, we get the spaces which were introduced and studied by Kizmaz [12].

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Definition 1.1. *A modulus function is a function* $f : [0,\infty) \to [0,\infty)$ *such that*

1. $f(x) = 0$ *if and only if* $x = 0$,
2. $f(x+y) \le f(x) + f(y)$ *for all* $x \ge 0$, $y \ge 0$,
3. f *is increasing,*
4. f *is continuous from right at* 0.

It follows that f must be continuous everywhere on $[0,\infty)$. The modulus function may be bounded or unbounded. For example, if we take $f(x) = \frac{x}{x+1}$, then $f(x)$ is bounded. If $f(x) = x^p$, $0 < p < 1$, then the modulus $f(x)$ is unbounded. Subsequently, modulus function has been discussed in ([1], [16], [19], [20], [23]) and many others.

Definition 1.2. *Let* X *be a linear metric space. A function* $p : X \to \mathbb{R}$ *is called paranorm, if*

1. $p(x) \ge 0$, *for all* $x \in X$,
2. $p(-x) = p(x)$, *for all* $x \in X$,
3. $p(x+y) \le p(x) + p(y)$, *for all* $x, y \in X$,
4. *if* (λ_n) *is a sequence of scalars with* $\lambda_n \to \lambda$ *as* $n \to \infty$ *and* (x_n) *is a sequence of vectors with* $p(x_n - x) \to 0$ *as* $n \to \infty$, *then* $p(\lambda_n x_n - \lambda x) \to 0$ *as* $n \to \infty$.

A paranorm p for which $p(x) = 0$ implies $x = 0$ is called total paranorm and the pair (X, p) is called a total paranormed space. It is well known that the metric of any linear metric space is given by some total paranorm (see [29], Theorem 10.4.2, P-183).

Let $p = (p_k)$ be a bounded sequence of positive real numbers, let $F = (f_k)$ be a sequence of modulus function. Also let $t = t_k = p_k^{-1}$ and suppose $u = (u_k)$ is a sequence of strictly positive real numbers. In this paper we define the following sequence spaces:

$$W_0(\Delta_s^m, F, Q, p, u, t) = \Big\{(x_k) : x_k \in E_k \text{ for all } k \in \mathbb{N} \text{ and there exists } r > 0 \text{ such that } \frac{1}{n}\sum_{k=1}^{n}\Big[f_k\big(q_k(p_k^{-t_k} u_k \Delta_s^m x_k r)\big)\Big]^{p_k} \to 0 \text{ as } n \to \infty\Big\},$$

$$W_1(\Delta_s^m, F, Q, p, u, t) = \Big\{(x_k) : x_k \in E_k \text{ for all } k \in \mathbb{N} \text{ and there exists } r > 0 \text{ such that } \frac{1}{n}\sum_{k=1}^{n}\Big[f_k\big(q_k(p_k^{-t_k} u_k \Delta_s^m x_k r - l)\big)\Big]^{p_k} \to 0 \text{ as } n \to \infty, l \in E_k\Big\}$$

and

$$W_\infty(\Delta_s^m, F, Q, p, u, t) = \Big\{(x_k) : x_k \in E_k \text{ for all } k \in \mathbb{N} \text{ and there exists } r > 0 \text{ such that } \sup_n \frac{1}{n}\sum_{k=1}^{n}\Big[f_k\big(q_k(p_k^{-t_k} u_k \Delta_s^m x_k r)\big)\Big]^{p_k} < \infty\Big\}.$$

In the case $f_k = f$ and $q_k = q$ for all $k \in \mathbb{N}$, we write $W_0(\Delta_s^m, f, q, p, u, t)$, $W_1(\Delta_s^m, f, q, p, u, t)$ and $W_\infty(\Delta_s^m, f, q, p, u, t)$ instead of $W_0(\Delta_s^m, F, Q, p, u, t)$, $W_1(\Delta_s^m, F, Q, p, u, t)$ and
$W_\infty(\Delta_s^m, F, Q, p, u, t)$ respectively.
Throughout the paper Z denotes any of the values $0, 1$ and ∞. If $x = (x_k) \in W_1(\Delta_s^m, f, q, p, u, t)$, we say that x is strongly $u_{q,t}$ Cesaro summable with respect to the modulus function f and write $x_k \to l \;\; W_1(\Delta_s^m, f, q, p, u, t)$; l is called the $u_{q,t}$ limit of x with respect to the modulus function f.
The main aim of this paper is to introduced the sequence spaces $W_Z(\Delta_s^m, F, Q, p, u, t)$, $\;Z = 0, 1$ and ∞. We also make an effort to study some topological properties and inclusion relations between these spaces.

2 Main Results

Theorem 2.1. *Let $F = (f_k)$ be a sequence of modulus functions and $p = (p_k)$ be a bounded sequence of positive real numbers. Then the spaces $W_Z(\Delta_s^m, F, Q, p, u, t)$, $Z = 0, 1, \infty$ are linear spaces over the complex field $\mathbb{C}$.*

Proof. We shall prove the result for $Z = 0$. Let $x = (x_k) \in W_0(\Delta_s^m, F, Q, p, u, t)$. Then there exists $r > 0$ such that $\frac{1}{n}\Big[f_k\big(q_k\big(p_k^{-t_k}u_k\Delta_s^m x_k r\big)\big)\Big]^{p_k} \to 0$ as $n \to \infty$. Let $\lambda \in \mathbb{C}$. Without loss of generality we can take $\lambda \neq 0$. Let $\rho = r(|\lambda|)^{-1} > 0$, then we have

$$\frac{1}{n}\Big[f_k\big(q_k\big(p_k^{-t_k}u_k\Delta_s^m \lambda x_k\big)\rho\big)\Big]^{p_k} = \frac{1}{n}\Big[f_k\big(q_k\big(p_k^{-t_k}u_k\Delta_s^m x_k r\big)\big)\Big]^{p_k} \to 0 \text{ as } n \to \infty.$$

Therefore $\lambda x \in W_0(\Delta_s^m, F, Q, p, u, t)$, for all $\lambda \in \mathbb{C}$ and for all $x = (x_k) \in W_0(\Delta_s^m, F, Q, p, u, t)$. Next, suppose that $x = (x_k),\ y = (y_k) \in W_0(\Delta_s^m, F, Q, p, u, t)$. Then there exists $r_1, r_2 > 0$ such that

$$\frac{1}{n}\sum_{k=1}^{n}\Big[f_k\big(q_k\big(p_k^{-t_k}u_k\Delta_s^m x_k r_1\big)\big)\Big]^{p_k} \to 0 \text{ as } n \to \infty$$

and

$$\frac{1}{n}\sum_{k=1}^{n}\Big[f_k\big(q_k\big(p_k^{-t_k}u_k\Delta_s^m y_k r_2\big)\big)\Big]^{p_k} \to 0 \text{ as } n \to \infty.$$

Thus given $\varepsilon > 0$, there exists $k_1, k_2 > 0$ such that

$$\frac{1}{n}\sum_{k=1}^{n}\Big[f_k\big(q_k\big(p_k^{-t_k}u_k\Delta_s^m x_k r_1\big)\big)\Big]^{p_k} < \varepsilon p_k, \text{ for all } k \geq k_1$$

and

$$\frac{1}{n}\sum_{k=1}^{n}\Big[f_k\big(q_k\big(p_k^{-t_k}u_k\Delta_s^m y_k r_2\big)\big)\Big]^{p_k} < \varepsilon p_k, \text{ for all } k \geq k_2.$$

Let $r = r_1 r_2 (r_1 + r_2)^{-1}$ and $k_0 = \max(k_1, k_2)$. Then we have for all $k \geq k_0$,

$$\frac{1}{n}\sum_{k=1}^{n}\Big[f_k\big(q_k\big(p_k^{-t_k}u_k\Delta_s^m (x_k + y_k) r\big)\big)\Big]^{p_k}$$

$$\leq \frac{1}{n}\sum_{k=1}^{n}\Big[f_k\big(q_k\big(p_k^{-t_k}u_k\Delta_s^m x_k r_1\big) r_2 (r_1 + r_2)^{-1} + f_k\big(q_k\big(p_k^{-t_k}u_k\Delta_s^m y_k r_2\big) r_1 (r_1 + r_2)^{-1}\Big]^{p_k} < \varepsilon p_k.$$

Hence $x + y \in W_0(\Delta_s^m, F, Q, p, u, t)$. Thus $W_0(\Delta_s^m, F, Q, p, u, t)$ is a linear space. Similarly we can prove that $W_1(\Delta_s^m, F, Q, p, u, t)$ and $W_\infty(\Delta_s^m, F, Q, p, u, t)$ are linear spaces. □

Theorem 2.2. *Let $F = (f_k)$ be a sequence of modulus functions and $p = (p_k)$ be a bounded sequence of positive real numbers. Then the space $W_0(\Delta_s^m, F, Q, p, u, t)$ is a complete paranormed space with paranorm defined by*

$$g(x) = \sup_n \Big(\frac{1}{n}\sum_{k=1}^{n}\Big[f_k\big(q_k\big(p^{-t_k}u_k\Delta_s^m x_k r\big)\big)\Big]^{p_k}\Big)^{\frac{1}{M}},$$

where $M = \max\{1, \sup p_k\}$.

Proof. Let $(x^{(i)})$ be a Cauchy sequence in $W_0(\Delta_s^m, F, Q, p, u, t)$. Then for a given $\varepsilon > 0$, there exists n_0 such that $g(x^i - x^j) < \varepsilon$, for all $i, j \geq n_0$. Thus, we have

$$\Big[\sum_{k=1}^{\infty}\big(f_k\big(q_k\big(p_k^{-t_k}u_k\Delta_s^m (x_k^i - x_k^j) r\big)\big)\big)^{p_k}\Big]^{\frac{1}{M}} < \varepsilon, \text{ for all } i, j \geq n_0. \quad (2.1)$$

$$\Longrightarrow \Big(f_k\big(q_k\big(p_k^{-t_k}u_k\Delta_s^m (x_k^i - x_k^j) r\big)\big)\Big) < \varepsilon, \text{ for all } i, j \geq n_0.$$
$$\Longrightarrow \Delta_s^m (x_k^i - x_k^j) < \varepsilon, \text{ for all } i, j \geq n_0, \text{ for all } k \in \mathbb{N}.$$

Hence $(x_k^i)_{i=1}^{\infty}$ is a Cauchy sequence in E_k, for each $k \in \mathbb{N}$. Since $E_k's$ are complete for each $k \in \mathbb{N}$, so $(x_k^i)_{i=1}^{\infty}$ converges in E_k, for each $k \in \mathbb{N}$. On taking limit as $j \to \infty$ in (2.1), we have

$$\Big[\sum_{k=1}^{\infty}\big(f_k\big(q_k\big(p_k^{-t_k}u_k\Delta_s^m(x_k^i - x_k)r\big)\big)\big)^{p_k}\Big]^{\frac{1}{M}} < \varepsilon, \text{ for all } i \geq n_0.$$

$$\Longrightarrow \Delta_s^m(x_k^i - x) \in W_0(\Delta_s^m, F, Q, p, u, t).$$

Since $W_0(\Delta_s^m, F, Q, p, u, t)$ is a linear space, so we have $x = x^{(i)} - (x^{(i)} - x) \in W_0(\Delta_s^m, F, Q, p, u, t)$. Thus $W_0(\Delta_s^m, F, Q, p, u, t)$ is a complete paranormed space. This completes the proof of the theorem. □

Theorem 2.3. *Let $F = (f_k)$ be a sequence of modulus functions and $p = (p_k)$ be a bounded sequence of positive real numbers. Then*

$$W_0(\Delta_s^m, F, Q, p, u, t) \subset W_1(\Delta_s^m, F, Q, p, u, t) \subset W_\infty(\Delta_s^m, F, Q, p, u, t).$$

Proof. It is easy to prove so we omit the details. □

Theorem 2.4. *Let $F = (f_k)$ and $G = (g_k)$ be any two sequences of modulus functions. For any bounded sequences $p = (p_k)$ and $t = (t_k)$ of strictly positive real numbers and any two sequences of seminorms $Q = (q_k)$, $V = (v_k)$, the following are true:*
(i) $W_Z(\Delta_s^m, f, Q, u, t) \subset W_Z(\Delta_s^m, f \circ g, Q, u, t)$,
(ii) $W_Z(\Delta_s^m, F, Q, p, u, t) \cap W_Z(\Delta_s^m, F, V, p, u, t) \subset W_Z(\Delta_s^m, F, Q + V, p, u, t)$,
(iii) $W_Z(\Delta_s^m, F, Q, p, u, t) \cap W_Z(\Delta_s^m, G, Q, p, u, t) \subset W_Z(\Delta_s^m, F + G, Q, p, u, t)$,
(iv) if q is stronger than v, then $W_Z(\Delta_s^m, F, Q, p, u, t) \subset W_Z(\Delta_s^m, F, V, p, u, t)$,
(v) if q is equivalent v, then $W_Z(\Delta_s^m, F, Q, p, u, t) = W_Z(\Delta_s^m, F, V, p, u, t)$,
(vi) $W_Z(\Delta_s^m, F, Q, p, u, t) \cap W_Z(\Delta_s^m, F, V, p, u, t) \neq \varphi$.

Proof. We shall prove (i) for the case $Z = 0$. Let $\varepsilon > 0$. We choose δ, $0 < \delta < 1$, such that $f(t) < \varepsilon$ for $0 \leq t \leq \delta$ and all $k \in \mathbb{N}$. We write $y_k = g\big(q_k\big(p_k^{-t_k}u_k\Delta_s^m x_k r\big)\big)$ and consider

$$\sum_{k=1}^{n}\big[f(y_k)\big] = \sum_1\big[f(y_k)\big] + \sum_2\big[f(y_k)\big],$$

where the first summation is over $y_k \leq \delta$ and the second summation is over $y_k > \delta$. Since f is continuous, we have

$$\sum_1\big[f(y_k)\big] < n\varepsilon. \tag{2.2}$$

By the definition of f, we have the following relation for $y_k > \delta$:

$$f(y_k) < 2f(1)\frac{y_k}{\delta}.$$

Hence

$$\frac{1}{n}\sum_2\big[f(y_k)\big] \leq 2\delta^{-1}f(1)\frac{1}{n}\sum_{k=1}^{n} y_k. \tag{2.3}$$

It follows from (2.2) and (2.3) that $W_Z(\Delta_s^m, f, Q, u, t) \subset W_Z(\Delta_s^m, f \circ g, Q, u, t)$. Similarly, we can prove the result for other cases. □

Theorem 2.5. *Let f be a modulus function. Then $W_Z(\Delta_s^m, Q, u, t) \subset W_Z(\Delta_s^m, f, Q, u, t)$.*

Proof. It is easy to prove in view of Theorem 2.4(i). □

Theorem 2.6. *Let $0 < p_k < r_k$ and $\left(\frac{r_k}{p_k}\right)$ be bounded. Then*

$$W_Z(\Delta_s^m, F, Q, r, u, t) \subset W_Z(\Delta_s^m, F, Q, p, u, t).$$

Proof. By taking $y_k = \Big[f_k\big(q_k\big(p_k^{-t_k}u_k\Delta_s^m x_k r\big)\big)\Big]^{r_k}$ for all k and using the same technique as in Theorem 5 of Maddox [15], one can easily prove the theorem. □

Theorem 2.7. *Let f be a modulus function. If $\lim\limits_{m\to\infty}\dfrac{f(m)}{m} = \beta > 0$, then $W_1(\Delta_s^m, Q, p, u, t) \subset W_1(\Delta_s^m, f, Q, p, u, t)$.*

Proof. It is easy to prove so we omit the details. □

3 $u_{q,t}$-Statistical Convergence

The notion of statistical convergence was introduced by Fast [6] and Schoenberg [24] independently. Over the years and under different names, statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory and number theory. Later on, it was further investigated from the sequence space point of view and linked with summability theory by Fridy [7], Connor [5], Salat [21], Murasaleen [17], Isik [10], Savas [22], Malkowsky and Savas [16], Kolk [13], Maddox [15], Tripathy and Sen [27] and many others. In recent years, generalizations of statistical convergence have appeared in the study of strong integral summability and the structure of ideals of bounded continuous functions on locally compact spaces. Statistical convergence and its generalizations are also connected with subsets of the Stone-Cech compactification of natural numbers. Moreover, statistical convergence is closely related to the concept of convergence in probability. The notion depends on the density of subsets of the set $\mathbb{N}$ of natural numbers.

Definition 3.3. *A subset E of $\mathbb{N}$ is said to have the natural density $\delta(E)$ if the following limit exists:*

$$\delta(E) = \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \chi_E(k),$$

where χ_E is the characteristic function of E. It is clear that any finite subset of $\mathbb{N}$ has zero natural density and $\delta(E^c) = 1 - \delta(E)$.

Definition 3.4. *A sequence $x = (x_k)$ is said to be $u_{q,t}$-statistical convergent to l if for every $\varepsilon > 0$,*

$$\delta\Big(\big\{k \in \mathbb{N} : q\big(p_k^{-t_k} u_k \Delta_s^m x_k r - l\big) \geq \varepsilon\big\}\Big) = 0.$$

In this case we write $x_k - l\big(S_{u,t}^q\big)$. The set of all $u_{q,t}$-statistical convergent sequences is denoted by $S_{u,t}^q$. By S, we denote the set of all statistically convergent sequences.

If $q(x) = |x|$, $u_k = p_k = t_k = 1$ for all $k \in \mathbb{N}$ and $r = 1$, then $S_{u,t}^q$ is same as S. In case $l = 0$ we write $S_{0u,t}^q$ instead of $S_{u,t}^q$.

Theorem 3.8. *Let $p = (p_k)$ be a bounded sequence and $0 < h = \inf p_k \leq p_k \leq \sup p_k = H < \infty$ and let f be a modulus function. Then*

$$W_1(\Delta_s^m, f, q, p, u, t) \subset S_{u,t}^q.$$

Proof. Let $x \in W_1(\Delta_s^m, f, q, p, u, t)$ and let $\varepsilon > 0$ be given. Let $\sum_1$ and $\sum_2$ denote the sums over $k \leq n$ with $q(p_k^{-t_k} u_k \Delta_s^m x_k r - l) \geq \varepsilon$ and $q(p_k^{-t_k} u_k \Delta_s^m x_k r - l) < \varepsilon$, respectively. Then

$$\begin{aligned}
&\frac{1}{n}\sum_{k=1}^{n} \Big[f\big(q\big(p_k^{-t_k} u_k \Delta_s^m x_k r - l\big)\big)\Big]^{p_k} \\
&\geq \frac{1}{n}\sum_{1} \Big[f\big(q\big(p_k^{-t_k} u_k \Delta_s^m x_k r - l\big)\big)\Big]^{p_k} \\
&\geq \frac{1}{n}\sum_{1} [f(\varepsilon)]^{p_k} \\
&\geq \frac{1}{n}\sum_{1} \min\Big(\big[f(\varepsilon)\big]^h, \big[f(\varepsilon)\big]^H\Big) \\
&\geq \frac{1}{n}\Big|\{k \leq n : q(p_k^{-t_k} u_k \Delta_s^m x_k - l) \geq \varepsilon\}\Big| \min\Big(\big[f(\varepsilon)\big]^h, \big[f(\varepsilon)\big]^H\Big).
\end{aligned}$$

Hence, $x \in S_{u,t}^q$. □

Theorem 3.9. *Let f be a bounded modulus function. Then $S_{u,t}^q \subset W_1(\Delta_s^m, f, q, p, u, t)$.*

Proof. Suppose that f is bounded. Let $\epsilon > 0$ and let $\sum_1$ and $\sum_2$ be the sums introduced in the Theorem 3.1. Since f is bounded, there exists an integer K such that $f(x) < K$ for all $x \geq 0$. Then

$$\frac{1}{n}\sum_{k=1}^{n}\Big[f\big(q\big(p_k^{-t_k}u_k\Delta_s^m x_k r-l\big)\big)\Big]^{p_k}$$
$$\leq \frac{1}{n}\Big(\sum_1\Big[f\big(q\big(p_k^{-t_k}u_k\Delta_s^m x_k r-l\big)\big)\Big]^{p_k}+\sum_2\Big[f\big(q\big(p_k^{-t_k}u_k\Delta_s^m x_k r-l\big)\big)\Big]^{p_k}\Big)$$
$$\leq \frac{1}{n}\sum_1\max(K^h,K^H)+\frac{1}{n}\sum_2\big[f(\varepsilon)\big]^{p_k}$$
$$\leq \max(K^h,K^H)\frac{1}{n}\Big|\{k\leq n: q(p_k^{-t_k}u_k\Delta_s^m x_k-l)\geq\varepsilon\}\Big|+\max(f(\varepsilon)^h,f(\varepsilon)^H).$$

Hence, $x\in W_1(\Delta_s^m,f,q,p,u,t)$. □

Theorem 3.10. *$S_{u,t}^q=W_1(\Delta_s^m,f,q,p,u,t)$ if and only if f is bounded.*

Proof. Let f be bounded. By Theorems 3.1 and 3.2, we have $S_{u,t}^q=W_1(\Delta_s^m,f,q,p,u,t)$.
Conversely, suppose that f is unbounded. Then there exists a sequence (t_k) of positive numbers with $f(t_k)=k^2$ for $k=1,2,\cdots$. If we choose

$$p_k^{-t_k}u_i\Delta_s^m x_i r=\begin{cases} t_k, & i=k^2,\ k=1,2,\cdots\\ 0, & \text{otherwise.}\end{cases}$$

Then we have

$$\frac{1}{n}|\{k\leq n:|p_k^{-t_k}u_k\Delta_s^m x_k r|\geq\epsilon\}|\leq\frac{\sqrt{n}}{n}$$

for all n, and so $x\in S_{u,t}^q$ but $x\notin W_1(\Delta_s^m,f,q,p,u,t)$ for $X=\mathbb{C}$, $q(x)=|x|$ and $p_k=1$ for all $k\in\mathbb{N}$. This contradicts the assumption that $S_{u,t}^q=W_1(\Delta_s^m,f,q,p,u,t)$. □

References

[1] Altinok H., Altin Y., Isik M. *The Sequence Space $Bv_\sigma(M,P,Q,S)$ On Seminormed Spaces*, Indian J. Pure Appl. Math., **39** (2008), 49-58.

[2] Colak R. *On invariant sequence spaces*, Erc. Univ. J. Sci., **5** (1989), 881-887.

[3] Colak R. *On certain sequence spaces and their Kothe-Toeplitz duals*, Rend. Mat. Appl., Ser, **13** (1993), 27-39.

[4] Et M., Colak R. *On some generalized difference sequence spaces* , Soochow J. Math., **21** (1995), 377-386.

[5] Connor J. S. *A topological and functional analytic approach to statistical convergence*, Appl. Numer. Harmonic Anal.,(1999), 403-413.

[6] Fast H. *Sur la convergence statistique*, Colloq. Math., **2** (1951), 241-244.

[7] Fridy J. A. *On the statistical convergence*, Analysis, **5** (1985), 301-303.

[8] Goes G., Goes S. *Sequence of bounded variation and sequences of Fourier coefficients*, Math. Zeit, **118** (1970), 93-102.

[9] Gupta M. *The generalized spaces $l^1(X)$ and $m_0(X)$*, J. Math. Anal. Appl., **76** (1980), 357-366.

[10] Isik M. *On statistical convergence of generalized difference sequence spaces*, Soochow J. Math., **30** (2004), 197-205.

[11] Kamthan P. K. *Bases in a certain class of Frechet spaces*, Tamkang J. Math., **7** (1976), 41-49.

[12] Kizmaz H. *On certain sequence spaces* , Cand. Math. Bull., **24** (1981), 169-176.

[13] Kolk E. *The statistical convergence in Banach spaces*, Acta. Comment. Univ. Tartu, **928** (1991), 41-52.

[14] Leonard I. E. *Banach sequence spaces*, J. Math. Anal. Appl., **54** (1976), 245-265.

[15] Maddox I. J. *Statistical convergence in a locally convex space*, Math. Proc. Cambridge Phil. Soc., **104** (1988), 141-145.

[16] Malkowsky E., Savas E. *Some λ-sequence spaces defined by a modulus*, Arch. Math., **36** (2000), 219-228.

[17] Mursaleen M. *λ-statistical convergence*, Math. Slovaca, **50** (2000), 111-115.

[18] Ratha A., Srivastava P. D. *On some vector valued sequence spaces $\mathcal{L}_{\infty}^{(p)}(E_k, \Lambda)$*, Ganita, **47** (1996), 1-12.

[19] Raj K., Sharma S. K. *Difference sequence spaces defined by sequence of modulus function*, Proyecciones, **30** (2011), 189-199.

[20] Raj K., Sharma S. K. *Some difference sequence spaces defined by sequence of modulus function*, Int. J. Math. Archive, **2** (2011), pp. 236-240.

[21] Salat T. *On statictical convergent sequences of real numbers*, Math. Slovaca, **30** (1980), 139-150.

[22] Savas E. *Strong almost convergence and almost λ-statistical convergence*, Hokkaido Math. J., **29** (2000), 531-566.

[23] Savas E. *On some generalized sequence spaces defined by a modulus* , Indian J. pure Appl. Math., **30** (1999), 459-464.

[24] Schoenberg I. J. *The integrability of certain functions and related summability methods*, Amer. Math. Monthly, **66** (1959), 361-375.

[25] Srivastava J. K., Srivastava B. K. *Generalized sequence space $c_0(X, \lambda, p)$*, Indian J. pure Appl. Math., **27** (1996), 73-84.

[26] Tripathy B. C., Sen M. *On generalized statistically convergent sequences*, Indian J. Pure Appl. Math., **32** (2001), 1689-1694.

[27] Tripathy B. C., Sen M. *Vector valued paranormed bounded and null sequence spaces associated with multiplier sequences*, Soochow J. Math., **29** (2003), 379-391.

[28] Tripathy B. C., Mahanta S. *On a class of vector valued sequences associated with multiplier sequences*, Acta Math. Appl. Sinica, **20** (2004), 487-494.

[29] Wilansky A. *Summability through Functional Analysis*, North- Holland Math. Stud. (1984).

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