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Global nonexistence of solutions for a system of viscoelastic wave equations with weak damping terms

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Abstract

This paper deals with the initial boundary value problem for the viscoelastic wave equations

$$\begin{cases} u_{tt}-\triangle u+\int_0^t g_1(t-\tau)\triangle u(\tau)\,d\tau+u_t=f_1(u,v),\\ v_{tt}-\triangle v+\int_0^t g_2(t-\tau)\triangle v(\tau)\,d\tau+v_t=f_2(u,v)\end{cases}$$

in a bounded domain. We obtain the global nonexistence of solutions by applying a lemma due to Y. Zhou [Global existence and nonexistence for a nonliear wave equation with damping and source terms, Math. Nacht, 278 (2005) 1341–1358].

Keywords: Global nonexistence, viscoelastic wave equation.

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1 Introduction

In this paper we consider the following initial boundary value problem

$$\begin{cases} u_{tt}-\triangle u+\int_0^t g_1(t-\tau)\triangle u(\tau)\,d\tau+u_t=f_1(u,v), & (x,t)\in\Omega\times(0,T),\\ v_{tt}-\triangle v+\int_0^t g_2(t-\tau)\triangle v(\tau)\,d\tau+v_t=f_2(u,v), & (x,t)\in\Omega\times(0,T),\\ u(x,t)=v(x,t)=0, & (x,t)\in\partial\Omega\times(0,T),\\ u(x,0)=u_0(x),\ u_t(x,0)=u_1(x), & x\in\Omega,\\ v(x,0)=v_0(x),\ v_t(x,0)=v_1(x), & x\in\Omega,\end{cases} \tag{1.1}$$

where Ω is a bounded domain with smooth boundary $\partial\Omega$ in R^n, $n=1,2,3$; and $g_i(.) : R^+ \longrightarrow R^+$, $f_i(.,.) : R^2 \longrightarrow R$ are given functions to be specified later.

The single viscoelastic wave equation of the form

$$u_{tt}-\triangle u+\int_0^t g(t-\tau)\triangle u(\tau)\,d\tau+h(u_t)=f(u),\ x\in\Omega,\ t>0, \tag{1.2}$$

has been extensively studied and many results concerning nonexistence have been proved. See in this regard [5, 8, 9, 17].

The equation (1.2) without the viscoelastic term (i.e., $g=0$) can be written in the following form

$$u_{tt}-\triangle u+h(u_t)=f(u),\ x\in\Omega,\ t>0. \tag{1.3}$$

The local existence, global existence, and blow up in finite time of solution for (1.3) were established (see [3, 6, 7, 10, 11] and references therein).

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Agre and Rammaha [2] studied the global existence and blow up of the solution of the problem (1.1) with $g_i = 0$ $(i = 1, 2)$ using the same techniques as in [3]. After that, the blow up result was improved by Said-Houari [15]. Also, he showed that the local solution obtained in [2] is global and decay of solutions [14].

Recently, Han and Wang [4] obtained the local existence, global existence and blow up of the solution of the problem (1.1) under some suitable conditions. Messaoudi and Houari [12] considered problem (1.1) and improved the blow up result in [4], for positive initial energy, using the some techniques as in [15]. Also, Houari et. al. [16] studied the general decay of the solution of the problem (1.1) by using the Lyapunov functional method.

In this paper, we consider the problem (1.1) and prove a global nonexistence result of solutions.

This paper is organized as follows. In section 2, we present some lemmas. In section 3, we state the local existence result. In section 4, we show the global nonexistence of solutions.

2 Preliminaries

In this section, we shall give some assumptions and lemmas which will be used throughout this work. Let $\|.\|$ and $\|.\|_p$ denote the usual $L^2(\Omega)$ norm and $L^p(\Omega)$ norm, respectively. Firstly, we make the following assumptions:

(A1) $g_i : R^+ \longrightarrow R^+$ $(i = 1, 2)$ nonincreasing differentiable function satisfying

$$1 - \int_0^\infty g_i(s)\, ds = l_i > 0.$$

(A2) $g_i(t) \geq 0,\ \forall t \geq 0.$

Concerning the functions $f_1(u, v)$ and $f_2(u, v)$, we take

$$f_1(u, v) = a|u+v|^{2(p+1)}(u+v) + b|u|^p |v|^{p+2} u,$$

$$f_2(u, v) = a|u+v|^{2(p+1)}(u+v) + b|u|^{p+2} |v|^p v,$$

where $a, b > 0$ are constants and p satisfies

$$\begin{cases} -1 < p & \text{if } n = 1, 2, \\ -1 < p \leq 1 & \text{if } n = 3. \end{cases} \tag{2.1}$$

According to the above equalities we can easily verify that

$$u f_1(u, v) + v f_2(u, v) = 2(p+2) F(u, v),\ \forall (u, v) \in R^2, \tag{2.2}$$

where

$$F(u, v) = \frac{1}{2(p+2)}\left[a|u+v|^{2(p+2)} + 2b|uv|^{p+2}\right]. \tag{2.3}$$

We have the following result.

Lemma 2.1 [12]. There exist two positive constants c_0 and c_1 such that

$$\frac{c_0}{2(p+2)}\left(|u|^{2(p+2)} + |v|^{2(p+2)}\right) \leq F(u, v) \leq \frac{c_1}{2(p+2)}\left(|u|^{2(p+2)} + |v|^{2(p+2)}\right) \tag{2.4}$$

is satisfied.

Lemma 2.2 [13]. For any $\phi \in C^1(R)$ we have

$$\begin{aligned} \int_\Omega \int_0^t g(t-\tau) \triangle \phi(\tau) \phi'(t)\, d\tau dx &= -\frac{1}{2}\left(g' \circ \nabla\phi\right)(t) + \frac{1}{2} g(t) \|\nabla\phi\|^2 \\ &+ \frac{1}{2}\frac{d}{dt}\left[\left(g \circ \nabla\phi\right)(t) - \int_0^t g(\tau) \|\nabla\phi\|^2 d\tau\right]. \end{aligned}$$

Lemma 2.3 (Sobolev-Poincare inequality) [1]. Let p be a number with $2 \leq p < \infty$ $(n = 1, 2)$ or $2 \leq p \leq 2n/(n-2)$ $(n \geq 3)$, then there is a constant $C_* = C_*(\Omega,\ p)$ such that

$$\|u\|_p \leq C_* \|\nabla u\| \text{ for } u \in H_0^1(\Omega).$$

Lemma 2.4 [18]. Suppose that $\psi(t)$ is a twice continuously differentiable function satisfying

$$\begin{cases} \psi''(t) + \psi'(t) \geq C_0 \psi^{1+\alpha}(t), \quad t > 0, \\ \psi(0) > 0, \quad \psi'(0) \geq 0, \end{cases}$$

where $C_0 > 0$, $\alpha > 0$ are constants. Then, $\psi(t)$ blows up in finite time.

3 Local existence

In this section we state local existence and the uniqueness of the solution of the problem (1.1).

Definition 3.1. A pair of functions (u, v) is said to be a weak solution of (1.1) on $[0, T]$ if

$$\begin{aligned} u, v &\in C\left([0,T); H_0^1(\Omega)\right) \cap C^1\left([0,T); L^2(\Omega)\right), \\ u_t &\in L^2(\Omega \times (0,T)), v_t \in L^2(\Omega \times (0,T)), \\ u'' &\in L^2\left(0,T; H^{-1}(\Omega) + L^2(\Omega)\right), \\ v'' &\in L^2\left(0,T; H^{-1}(\Omega) + L^2(\Omega)\right), \end{aligned}$$

where $H^{-1}(\Omega) + L^2(\Omega)$ is the dual space of $H_0^1(\Omega) \cap L^2(\Omega)$. In additon, (u, v) satisfies

$$\begin{aligned} &\int_\Omega u'(t)\phi dx - \int_\Omega u_1(t)\phi dx + \int_\Omega \nabla u \nabla \phi dx \\ &- \int_0^t \int_\Omega (g_1 * \nabla u)\nabla \phi dx d\tau + \int_0^t \int_\Omega u' \phi dx d\tau \\ = &\int_0^t \int_\Omega f_1(u(\tau), v(\tau))\phi dx d\tau, \end{aligned}$$

$$\begin{aligned} &\int_\Omega v'(t)\varphi dx - \int_\Omega v_1(t)\varphi dx + \int_\Omega \nabla v \nabla \varphi dx \\ &- \int_0^t \int_\Omega (g_2 * \nabla v)\nabla \varphi dx d\tau + \int_0^t \int_\Omega v' \varphi dx d\tau \\ = &\int_0^t \int_\Omega f_2(u(\tau), v(\tau))\varphi dx d\tau, \end{aligned}$$

for all test functions $\phi \in H_0^1(\Omega) \cap L^2(\Omega)$, $\varphi \in H_0^1(\Omega) \cap L^2(\Omega)$ and for almost all $t \in [0, T]$.

Now, we state the local existence theorem that is proved in [4].

Theorem 3.1 (Local existence)**.** Assume that (2.1), (A1) and (A2) hold. Then for any initial data $(u_0, u_1) \in H_0^1(\Omega) \times L^2(\Omega)$, $(v_0, v_1) \in H_0^1(\Omega) \times L^2(\Omega)$, there exists a unique local weak solution (u, v) of problem (1.1) (in the sense of Definition 3.1) defined on $[0, T]$ for some $T > 0$, and satisfies the energy identity

$$\begin{aligned} &E(t) + \int_0^t \left(\|u_\tau(\tau)\|^2 + \|v_\tau(\tau)\|^2\right) d\tau - \frac{1}{2}\int_0^t \left(\left(g_1' \circ \nabla u\right)(\tau) + \left(g_2' \circ \nabla v\right)(\tau)\right) d\tau \\ &\frac{1}{2}\int_0^t \left(g_1(\tau)\|\nabla u(\tau)\|^2 + g_2(\tau)\|\nabla v(\tau)\|^2\right) d\tau \\ = \; &E(0) \end{aligned}$$

where $E(t)$ is defined in (4.3).

4 Global nonexistence result

In this section, we prove the global nonexistence of the solution of the problem (1.1). In order to do so, let us first introduce the following functionals,

$$\begin{aligned} J(t) = \; &\frac{1}{2}\left(1 - \int_0^t g_1(\tau) d\tau\right)\|\nabla u\|^2 + \frac{1}{2}\left(1 - \int_0^t g_2(\tau) d\tau\right)\|\nabla v\|^2 \\ &+ \frac{1}{2}\left[(g_1 \circ \nabla u)(t) + (g_2 \circ \nabla v)(t)\right] - \int_\Omega F(u, v) dx, \end{aligned} \tag{4.1}$$

and

$$\begin{aligned} I(t) &= \left(1-\int_0^t g_1(\tau)\,d\tau\right)\|\nabla u\|^2 + \left(1-\int_0^t g_2(\tau)\,d\tau\right)\|\nabla v\|^2 \\ &\quad + (g_1\circ\nabla u)(t) + (g_2\circ\nabla v)(t) - (p+1)\int_\Omega F(u,v)\,dx. \end{aligned} \tag{4.2}$$

We also define the energy function as follows

$$\begin{aligned} E(t) &= \frac{1}{2}\left(\|u_t\|^2+\|v_t\|^2\right) + \frac{1}{2}\left(1-\int_0^t g_1(\tau)\,d\tau\right)\|\nabla u\|^2 + \frac{1}{2}\left(1-\int_0^t g_2(\tau)\,d\tau\right)\|\nabla v\|^2 \\ &\quad + \frac{1}{2}\left[(g_1\circ\nabla u)(t) + (g_2\circ\nabla v)(t)\right] - \int_\Omega F(u,v)\,dx, \end{aligned} \tag{4.3}$$

where

$$(\Phi\circ\Psi)(t) = \int_0^t \Phi(t-\tau)\int_\Omega |\Psi(t)-\Psi(\tau)|\,dxd\tau.$$

Finally, we define

$$W = \left\{(u,v) : (u,v)\in H_0^1(\Omega)\times H_0^1(\Omega),\ I(u,v)>0\right\}\cup\{(0,0)\}. \tag{4.4}$$

The next lemma shows that our energy functional (4.3) is a nonincreasing function along the solution of the problem (1.1).

Lemma 4.1. $E(t)$ is a decreasing function for $t\geq 0$ and

$$\begin{aligned} E'(t) &\leq -\left(\|u_t\|^2+\|v_t\|^2\right) + \frac{1}{2}\left[\left(g_1'\circ\nabla u\right)(t) + \left(g_2'\circ\nabla v\right)(t)\right] \\ &\leq 0,\ \forall t\geq 0. \end{aligned} \tag{4.5}$$

Proof. Multiplying the first equation of (1.1) by u_t and the second equation by v_t, integrating over Ω, and using (2.5) and the assumption (A1)-(A2), we obtain (4.5).

Theorem 4.1. Under the conditions of Theorem 3.1, assume that initial conditions satisfies

$$E(0)\leq 0,\quad \int_\Omega (u_0u_1+v_0v_1)\,dx \geq 0,$$

and

$$\max\left\{\int_0^t g_1(s)\,ds,\ \int_0^t g_2(s)\,ds\right\} \leq \frac{p+1}{p+3-\frac{1}{4(p+2)}}$$

then the corresponding solution blows up in finite time. In other words, there exists a positive constant T^* such that $\lim\limits_{t\longrightarrow T^*}\left(\|u\|^2+\|v\|^2\right)=\infty$.

Proof. To apply Lemma 2.4, we define

$$\psi(t) = \frac{1}{2}\int_\Omega \left(|u|^2+|v|^2\right)dx. \tag{4.6}$$

Therefore,

$$\psi'(t) = \int_\Omega (uu_t+vv_t)\,dx, \tag{4.7}$$

and

$$\psi''(t) = \int_\Omega \left(u_t^2+v_t^2\right)dx + \int_\Omega (uu_{tt}+vv_{tt})\,dx. \tag{4.8}$$

Then, eq (1.1) is used to estimate (4.8) as follows

$$\begin{aligned} \psi''(t) &= \int_\Omega \left(u_t^2+v_t^2\right)dx - \left(\|\nabla u\|^2+\|\nabla v\|^2\right) \\ &\quad + \int_\Omega\int_0^t g_1(t-s)\nabla u(t)\nabla u(s)\,dsdx \\ &\quad + \int_\Omega\int_0^t g_2(t-s)\nabla v(t)\nabla v(s)\,dsdx \\ &\quad - \int_\Omega (uu_t+vv_t)\,dx + 2(p+2)\int_\Omega F(u,v)\,dx. \end{aligned} \tag{4.9}$$

We then use Young's inequality to estimates third and fiveth terms in (4.9);

$$\begin{aligned}
&\int_{\Omega}\int_{0}^{t} g_1(t-s)\nabla u(t)\nabla u(s)\,dsdx \\
=\;& \int_{\Omega}\int_{0}^{t} g_1(t-s)\nabla u(t)\left[\nabla u(s)-\nabla u(t)\right]dsdx+\left(\int_{0}^{t} g_1(s)\,ds\right)\|\nabla u\|^2 \\
\leq\;& \delta\|\nabla u\|^2+\frac{1}{4\delta}\left(\int_{0}^{t} g_1(s)\,ds\right)(g_1\circ\nabla u)(t)+\left(\int_{0}^{t} g_1(s)\,ds\right)\|\nabla u\|^2 \qquad (4.10)
\end{aligned}$$

and

$$\begin{aligned}
&\int_{\Omega}\int_{0}^{t} g_2(t-s)\nabla v(t)\left[\nabla v(s)-\nabla v(t)\right]dsdx \\
\leq\;& \delta\|\nabla v\|^2+\frac{1}{4\delta}\left(\int_{0}^{t} g_2(s)\,ds\right)(g_2\circ\nabla v)(t)+\left(\int_{0}^{t} g_2(s)\,ds\right)\|\nabla v\|^2. \qquad (4.11)
\end{aligned}$$

Inserting (4.10), (4.11) into (4.9) to get

$$\begin{aligned}
\psi''(t)+\psi'(t) \;\geq\;& \left(\|u_t\|^2+\|v_t\|^2\right)-\left(1+\int_{0}^{t} g_1(s)\,ds+\delta\right)\|\nabla u\|^2 \\
&-\left(1+\int_{0}^{t} g_2(s)\,ds+\delta\right)\|\nabla v\|^2+2(p+2)\int_{\Omega}F(u,v)\,dx \\
&-\frac{1}{4\delta}\left(\int_{0}^{t} g_1(s)\,ds\right)(g_1\circ\nabla u)(t)-\frac{1}{4\delta}\left(\int_{0}^{t} g_2(s)\,ds\right)(g_2\circ\nabla v)(t). \qquad (4.12)
\end{aligned}$$

From the definition of $E(t)$, it follows that

$$\begin{aligned}
\|\nabla u\|^2+\|\nabla v\|^2 \;\leq\;& \frac{2}{1-l}E(t)+\frac{2}{1-l}\int_{\Omega}F(u,v)\,dx-\frac{1}{1-l}\left(\|u_t\|^2+\|v_t\|^2\right) \\
&-\frac{1}{1-l}\left[(g_1\circ\nabla u)(t)+(g_2\circ\nabla v)(t)\right], \qquad (4.13)
\end{aligned}$$

where $l=\max\left\{\int_0^t g_1(s)\,ds,\ \int_0^t g_2(s)\,ds\right\}$. Substituting (4.13) into (4.12), we have

$$\begin{aligned}
\psi''(t)+\psi'(t) \;\geq\;& \left(\frac{2+\delta}{1-l}\right)\left(\|u_t\|^2+\|v_t\|^2\right)-2\left(\frac{1+l+\delta}{1-l}\right)E(t) \\
&+\left[2(p+2)-2\left(\frac{1+l+\delta}{1-l}\right)\right]\int_{\Omega}F(u,v)\,dx \\
&+\left[\frac{1+l+\delta}{1-l}-\frac{1}{4\delta}\left(\int_{0}^{t} g_1(s)\,ds\right)\right](g_1\circ\nabla u)(t) \\
&+\left[\frac{1+l+\delta}{1-l}-\frac{1}{4\delta}\left(\int_{0}^{t} g_2(s)\,ds\right)\right](g_2\circ\nabla v)(t). \qquad (4.14)
\end{aligned}$$

At this point we choose $\delta>0$, so that

$$\frac{1+l+\delta}{1-l}-\frac{1}{4\delta}\left(\int_{0}^{t} g_1(s)\,ds\right)\geq 0,\quad \frac{1+l+\delta}{1-l}-\frac{1}{4\delta}\left(\int_{0}^{t} g_2(s)\,ds\right)\geq 0.$$

Therefore, (4.14) becomes

$$\begin{aligned}
\psi''(t)+\psi'(t) \;\geq\;& \left[2(p+2)-2\left(\frac{1+l+\delta}{1-l}\right)\right]\int_{\Omega}F(u,v)\,dx \\
\geq\;& \frac{c_0}{2(p+2)}\left[2(p+2)-2\left(\frac{1+l+\delta}{1-l}\right)\right]\left(\|u\|_{2(p+2)}^{2(p+2)}+\|v\|_{2(p+2)}^{2(p+2)}\right) \\
\geq\;& \gamma\left(\|u\|_{2(p+2)}^{2(p+2)}+\|v\|_{2(p+2)}^{2(p+2)}\right) \qquad (4.15)
\end{aligned}$$

where $\gamma=\frac{c_0}{2(p+2)}\left[2(p+2)-2\left(\frac{1+l+\delta}{1-l}\right)\right]$. Also, from assumption of the theorem $\gamma\geq 0$.

Now, Hölder inequality are used to estimates $\|u\|_{2(p+2)}^{2(p+2)}$ and $\|v\|_{2(p+2)}^{2(p+2)}$ as follows

$$\int_{\Omega} |u|^2 \, dx \leq \left(\int_{\Omega} |u|^{2(p+2)} \, dx \right)^{\frac{1}{p+2}} \left(\int_{\Omega} 1 dx \right)^{\frac{p+1}{p+2}}.$$

W_n is called the volume of the domain Ω, then

$$\|u\|_{2(p+2)}^{2(p+2)} \geq \left(\int_{\Omega} |u|^2 \, dx \right)^{p+2} (W_n)^{-(p+1)},$$

and similarly, we have

$$\|v\|_{2(p+2)}^{2(p+2)} \geq \left(\int_{\Omega} |v|^2 \, dx \right)^{p+2} (W_n)^{-(p+1)}.$$

Consequently, we have

$$\psi''(t) + \psi'(t) \geq \gamma (W_n)^{-(p+1)} \left[\left(\int_{\Omega} |u|^2 \, dx \right)^{p+2} + \left(\int_{\Omega} |v|^2 \, dx \right)^{p+2} \right]. \tag{4.16}$$

In order to estimate the right-hand side in (4.16), we make use of the following inequality

$$(X+Y)^{\rho} \leq 2^{\rho-1} (X^{\rho} + Y^{\rho}),$$

$X, Y \geq 0$, $1 \leq \rho < \infty$, applying the above inequality we have

$$2^{-(p+1)} \left(\int_{\Omega} |u|^2 \, dx + \int_{\Omega} |v|^2 \, dx \right)^{p+2} \leq \left(\int_{\Omega} |u|^2 \, dx \right)^{p+2} + \left(\int_{\Omega} |v|^2 \, dx \right)^{p+2}.$$

Consequently, (4.16) becomes

$$\begin{aligned} \psi''(t) + \psi'(t) &\geq 2^{-(p+1)} \gamma (W_n)^{-(p+1)} \left(\int_{\Omega} |u|^2 \, dx + \int_{\Omega} |v|^2 \, dx \right)^{p+2} \\ &= 2\gamma (W_n)^{-(p+1)} \psi^{p+2}(t). \end{aligned}$$

It is easy to verify that the requirements of Lemma 2.4 are satisfied by

$$C_0 = 2\gamma (W_n)^{-(p+1)} > 0, \quad \alpha = p+1 > 0.$$

Therefore $\psi(t)$ blows up in finite. The proof of Theorem 4.1 is completed.

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