Malaya J. Mat. 3(2)(2015) 175–181

Malaya Journal of Matematik

MJM
an international journal of mathematical sciences with computer applications...

www.malayajournal.org

ISSN : 2319-3786

Further Results on Sum Cordial Graphs

M. I. Bosmia[a] V. R. Visavaliya[b] B. M. Patel[c,*]

[a]Department of Mathematics, Government Engineering College, Gandhinagar-382028, Gujarat, India.

[b]Department of Mathematics, Government Engineering College, Chandkheda-382424, Gujarat, India.

[c]Department of Mathematics, Government Science College, Gandhinagar-382016, Gujarat, India.

Abstract

In this paper, we prove that wheel, closed helm, quadrilateral snake, double quadrilateral snake and gear graphs are sum cordial graphs.

Keywords: Cordial labeling, Sum cordial labeling, Sum cordial graph.

2010 MSC: 54C05, 54C10.

1 Introduction

All graphs $G = (V(G), E(G))$ in this paper are finite, connected and undirected. For any undefined notations and terminology we follow [3]. If the vertices or edges or both of the graph are assigned valued subject to certain conditions it is known as graph labeling. A dynamic survey on graph labeling is regularly updated by Gallian [4]. Labeled graphs have variety of applications in graph theory, particularly for missile guidance code, design good radar type codes and convolution codes with optimal autocorrelation properties. Labeled graphs plays vital role in the study of X-ray crystallography, communication network and to determine optimal circuit layouts. A detailed study on variety of applications on graph labeling is carried out in Bloom and Golomb [1].

Definition 1.1. *A mapping $f : V(G) \longrightarrow \{0,1\}$ is called binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f.*

The induced edge labeling $f^* : E(G) \longrightarrow \{0,1\}$ is given by $f^*(e = uv) = |f(u) - f(v)|$. Let us denote $v_f(0)$, $v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f ad $e_f(0)$, $e_f(1)$ be the number of edges of G having labels 0 and 1 respectively under f^*.

Definition 1.2. *A binary vertex labeling of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called cordial if it admits labeling.*

The concept of cordial labeling was introduced by Cahit [2] in which he investigated several results on this newly defined concept. Also, some new graphs are investigated as product cordial graphs by Vaidya [6].

Definition 1.3. *A binary vertex labeling of a graph G with induce edge labeling $f^* : E(G) \longrightarrow \{0,1\}$ defined by $f^*(uv) = (f(u) + f(v))(mod 2)$ is called sum cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is sum cordial if it admits sum cordial labeling.*

Shiama [5] investigated the sum cordial labeling and proved that path P_n, cycle C_n, star $K_{1,n}$ etc are some cordial graphs.

*Corresponding author.
E-mail address: cosmicmohit@gmail.com(M. I. Bosmia), visavaliavijay@gmail.com(V. R. Visavaliya), bhavinramani@yahoo.com (B. M. Patel).

Definition 1.4. *The wheel graph W_n is defined as the join of $K_1 + C_n$. The vertex corresponding to K_1 is said to be apex vertex, the vertices corresponding to cycle are called rim vertices. The edges corresponding to cycle are called the rim edges and edges joining apex and vertices of the cycle are called spoke edges.*

Definition 1.5. *The helm H_n is the graph obtained from a wheel W_n by attaching a pendant edge to each rim vertex.*

Definition 1.6. *The closed helm CH_n is the graph obtained from a helm H_n by joining each pendant vertex to each rim vertex.*

Definition 1.7. *The quadrilateral snake Q_n is obtained from the path P_n by replacing every edge of a path by a cycle C_n.*

Definition 1.8. *The double quadrilateral snake DQ_n consists of two quadrilateral snakes that have a common path.*

Definition 1.9. *Let $e = uv$ be an edge of a graph G and w is not a vertex of G. The edge e is sub divided when it is replaced by the edges $e^{'} = uw$ and $e^{''} = wv$.*

Definition 1.10. *The gear graph G_n is obtained from the wheel W_n by sub dividing each of its rim edge.*

2 Main Results

Theorem 2.1. *The wheel W_n is a sum cordial graph except $n \equiv 3(mod4)$.*

Proof: Let v be an apex vertex and $v_1, v_2, \ldots, v_n$ are rim vertices for wheel W_n. Then $|V(W_n)| = n + 1$ and $|E(W_n)| = 2n$.
To define $f : V(W_n) \longrightarrow \{0, 1\}$, we consider the following cases,

For $n \equiv 0, 1, 2(mod4)$

$$f(v) = 0;$$
$$f(v_i) = \begin{cases} 1, & i \equiv 1 \text{ or } 2(mod4); \\ 0, & i \equiv 3 \text{ or } 4(mod4). \end{cases} \quad ; 1 \leq i \leq n$$

Therefore,

$$v_f(0) = \begin{cases} \left\lceil \frac{n+1}{2} \right\rceil, & n \equiv 0(mod4); \\ \frac{n+1}{2}, & n \equiv 1(mod4); \\ \left\lfloor \frac{n+1}{2} \right\rfloor, & n \equiv 2(mod4). \end{cases}$$
$$v_f(1) = \begin{cases} \left\lfloor \frac{n+1}{2} \right\rfloor, & n \equiv 0(mod4); \\ \frac{n+1}{2}, & n \equiv 1(mod4); \\ \left\lceil \frac{n+1}{2} \right\rceil, & n \equiv 2(mod4). \end{cases}$$
$$e_f(0) = e_f(1) = n$$

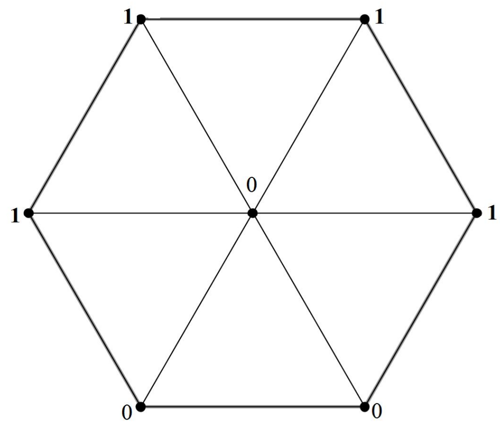
Therefore,

$$v_f(0) - v_f(1) = \begin{cases} 1, & n \equiv 0(mod4); \\ 0, & n \equiv 1(mod4); \\ -1, & n \equiv 2(mod4). \end{cases}$$

Hence, $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. So, wheel W_n is a sum cordial for $n \equiv 0, 1$ or $2(mod4)$.

For $n \equiv 3(mod4)$ In order to satisfy the vertex condition for the sum cordial graph it is necessary to assign 0 to $\frac{n+1}{2}$ vertices out of $n + 1$ vertices. The vertices having label 1 will give rise at least $\left\lceil \frac{2n+1}{2} \right\rceil$ edges with label 1 and at most $\left\lfloor \frac{2n-1}{2} \right\rfloor$ edges with label 0 out of $2n$ edges. Therefore, $|e_f(0) - e_f(1)| \geq 2$. Hence the edge condition for the sum cordial graph is not satisfied. So wheel W_n is not sum cordial for $n \equiv 3(mod4)$.

Example 2.1. *The wheel W_6 is a sum cordial graph.*

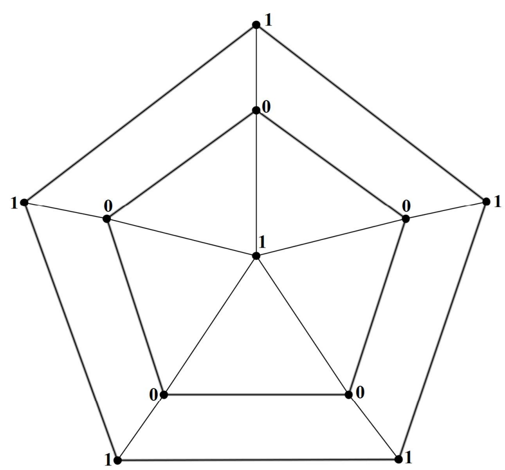


Sum cordial labeling of Wheel W_6

Theorem 2.2. *The closed Helm CH_n is a sum cordial graph.*

Proof: Let v be an apex vertex and $v_1, v_2, \ldots, v_n$ are rim vertices. We denote the pendant vertices by $v_1^{'}, v_2^{'}, \ldots, v_n^{'}$. Then $|V(CH_n)| = 2n + 1$ and $|E(CH_n)| = 4n$.
Define $f : V(CH_n) \to \{0, 1\}$ by $f(v) = 1,\ f(v_i) = 0,\ f(v_i^{'}) = 1$ for $1 \leq i \leq n$.
In view of the above labeling pattern, we have $v_f(0) = n,\ v_f(1) = n + 1,\ e_f(0) = 2n = e_f(1)$. Thus, we get $|v_f(0) - v_f(1)| \leq 1,\ |e_f(0) - e_f(1)| \leq 1$.
Hence, CH_n is a sum cordial graph.

Example 2.2. *The Closed helm CH_5 is a sum cordial graph.*



Sum cordial labeling of Closed helm CH_5

Theorem 2.3. *The quadrilateral snake Q_n is a sum cordial graph.*

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices and $e_1, e_2, \ldots, e_{n-1}$ be the edges of a path P_n. To construct a quadrilateral snake Q_n from the path P_n, we join v_i and v_{i+1} to new vertices w_i and $w_i^{'}$ by edges $e_{2i-1}^{'} = v_i w_i,\ e_{2i}^{'} = v_{i+1} w_i^{'}$ and $e_i^{''} = w_i w_i^{'}$ for $i = 1, 2, \ldots, n - 1$. Then $|V(Q_n)| = 3n - 2$ and $|E(Q_n)| = 4n - 4$.
To define $f : V(Q_n) \to \{0, 1\}$, we consider the following cases,

n is even

$$f(v_i) = 1: \ 1 \le i \le n$$
$$f(w_i) = \begin{cases} 0, & 1 \le i \le \frac{n}{2}; \\ 1, & \frac{n}{2} < i \le n-1. \end{cases}$$
$$f(w_i^{'}) = 0: \ 1 \le i \le n-1$$

Therefore, $v_f(0) = \dfrac{3n-2}{2} = v_f(1)$ and $e_f(0) = 2n-2 = e_f(1)$.
Therefore, $|v_f(0) - v_f(1)| = 0 = |e_f(0) - e_f(1)|$.

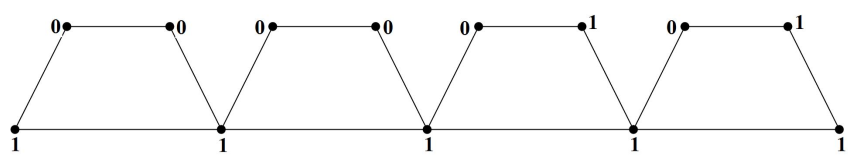
n is odd

$$f(v_i) = 1; \ 1 \le i \le n$$
$$f(w_i) = 0; \ 1 \le i \le n-1$$
$$f(w_i^{'}) = \begin{cases} 0, & 1 \le i \le \frac{n-1}{2}; \\ 1, & \frac{n-1}{2} < i \le n-1. \end{cases}$$

Therefore, $v_f(0) = \left\lfloor \dfrac{3n-2}{2} \right\rfloor$, $v_f(1) = \left\lceil \dfrac{3n-2}{2} \right\rceil$ and $e_f(0) = 2n-2 = e_f(1)$.
Therefore, $|v_f(0) - v_f(1)| = 1$ and $|e_f(0) - e_f(1)| = 0$.

Hence, Q_n is a sum cordial graph.

Example 2.3. *The quadrilateral snake Q_5 is a sum cordial graph.*



Sum cordial labeling of Quadrilateral snake Q_5

Theorem 2.4. *The double quadrilateral snake DQ_n is a sum cordial graph.*

Proof: Let $v_1, v_2, \dots, v_n$ be the vertices and $e_1, e_2, \dots, e_{n-1}$ be the edges of the path P_n. To construct a double quadrilateral snake DQ_n from the path P_n, we join v_i and v_{i+1} to new vertices u_i, $u_i^{'}$, w_i and $w_i^{'}$ by edges $e_{2i-1}^{u} = v_i u_i$, $e_{2i}^{u} = v_{i+1} u_i^{'}$, $e_i^{uu} = u_i u_i^{'}$, $e_{2i-1}^{w} = v_i w_i$, $e_{2i}^{w} = v_{i+1} w_i^{'}$ and $e_i^{ww} = w_i w_i^{'}$ for $i = 1, 2, \dots, n-1$. Then $|V(DQ_n)| = 5n-4$ and $|E(DQ_n)| = 7n-7$.
Define $f : V(DQ_n) \to \{0, 1\}$ such that

$$f(v_i) = \begin{cases} 1, & i \equiv 1 \text{ or } 2(mod 4); \\ 0, & i \equiv 0 \text{ or } 3(mod 4). \end{cases} \quad 1 \le i \le n$$
$$f(u_i) = f(u_i^{'}) = \begin{cases} 1, & i \equiv 3(mod 4); \\ 0, & \text{otherwise}. \end{cases} \quad 1 \le i \le n$$
$$f(w_i) = 1; \ 1 \le i \le n$$
$$f(w_i^{'}) = \begin{cases} 0, & i \equiv 1 \text{ or } 3(mod 4); \\ 1, & i \equiv 0 \text{ or } 2(mod 4). \end{cases} \quad 1 \le i \le n$$

Therefore,

For even n $v_f(0) = \dfrac{5n-4}{2} = v_f(1)$ and $e_f(0) = \left\lfloor \dfrac{7(n-1)}{2} \right\rfloor$, $e_f(1) = \left\lceil \dfrac{7(n-1)}{2} \right\rceil$.
Therefore, $|v_f(0) - v_f(1)| \le 1$ and $|e_f(0) - e_f(1)| \le 1$.

For odd n

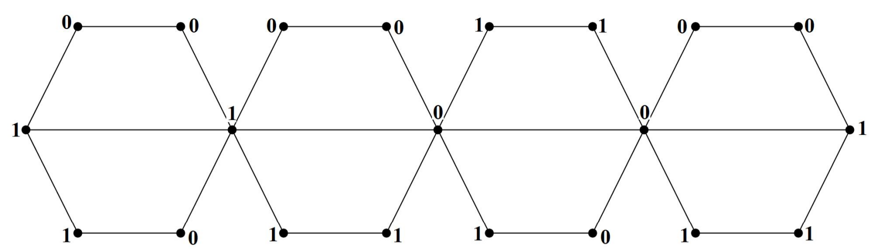
$$v_f(0) = \begin{cases} \left\lfloor \dfrac{5n-4}{2} \right\rfloor, & n \equiv 1(mod 4); \\ \left\lceil \dfrac{5n-4}{2} \right\rceil, & n \equiv 3(mod 4). \end{cases}$$

$$v_f(1) = \begin{cases} \left\lceil \dfrac{5n-4}{2} \right\rceil, & n \equiv 1(mod 4); \\ \left\lfloor \dfrac{5n-4}{2} \right\rfloor, & n \equiv 3(mod 4). \end{cases}$$

Also, $e_f(0) = \dfrac{7(n-1)}{2} = e_f(1)$.
Therefore, $|v_f(0) - v_f(1)| = 1$ and $|e_f(0) - e_f(1)| = 0$.

Hence, DQ_n is a sum cordial graph.

Example 2.4. *The double quadrilateral snake DQ_5 is a sum cordial graph.*



Sum cordial labeling of Double quadrilateral snake DQ_5

Theorem 2.5. *The gear graph G_n is a sum cordial graph.*

Proof: Let W_n be the wheel with an apex vertex v and rim vertices be $v_1, v_2, \dots, v_n$. To obtain the gear graph G_n, subdivide each rim edge of wheel by the vertices $u_1, u_2, \dots, u_n$, where each u_i sub divides the edge $v_i v_{i+1}$ for $i = 1, 2, \dots, n-1$ and u_n subdivides the edge $v_1 v_n$. Then $|V(G_n)| = 2n + 1$ and $|E(G_n)| = 3n$.
To define $f : V(G_n) \longrightarrow \{0, 1\}$, we consider the following two cases,

For even n Define

$$f(v) = 1$$

$$f(v_i) = \begin{cases} 1, & 1 \leq i \leq \dfrac{n}{2}; \\ 0, & \dfrac{n}{2} < i \leq n. \end{cases}$$

$$f(u_i) = \begin{cases} 1, & i \text{ is odd}; \\ 0, & i \text{ is even}. \end{cases}$$

Therefore, $v_f(0) = \left\lfloor \dfrac{2n+1}{2} \right\rfloor$, $v_f(1) = \left\lceil \dfrac{2n+1}{2} \right\rceil$, $e_f(0) = \dfrac{3n}{2} = e_f(1)$. Thus, we get $|v_f(0) - v_f(1)| \leq 1$, $|e_f(0) - e_f(1)| \leq 1$.

For odd n Define

$$f(v) = 1$$

$$f(v_1) = 1$$

$$f(v_i) = f(v_{n+2-i}) = \begin{cases} 1, & \text{if } i \text{ is odd}; \\ 0, & \text{if } i \text{ is even}. \end{cases} ;\ 2 \leq i \leq \frac{n+1}{2}$$

$$f(u_i) = \begin{cases} 1, & \text{if } i \text{ is odd except } i = \dfrac{n+1}{2}; \\ 0, & \text{otherwise}. \end{cases} ;\ 1 \leq i \leq n$$

Therefore, $v_f(0) = \left\lfloor \frac{2n+1}{2} \right\rfloor$, $v_f(1) = \left\lceil \frac{2n+1}{2} \right\rceil$ and

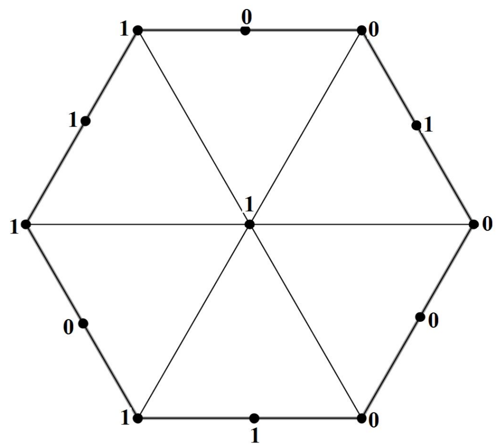
$$e_f(0) = \begin{cases} \left\lfloor \frac{n}{2} \right\rfloor, & \text{if } n \equiv 1(mod4); \\ \left\lceil \frac{n}{2} \right\rceil, & \text{if } n \equiv 3(mod4). \end{cases}$$

$$e_f(1) = \begin{cases} \left\lceil \frac{n}{2} \right\rceil, & \text{if } n \equiv 1(mod4); \\ \left\lfloor \frac{n}{2} \right\rfloor, & \text{if } n \equiv 3(mod4). \end{cases}$$

Therefore, $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence, the gear G_n is a sum cordial graph.

Example 2.5. *The Gear G_6 is a sum cordial graph.*



Sum cordial labeling of Gear G_6

3 Conclusion

We contribute some new results on sum cordial labeling. The labeling pattern is demonstrated by means of examples. To derive similar results for other graph families and in the context of different labeling problems is an open area of research.

References

[1] G S Bloom and S W Golomb, *Applications of Numbered Undirected Graph*, Proceedings of IEEE 65-4(1977), 562-570.

[2] I Cahit, *Cordial Graphs: A weaker version of graceful and harmonious graphs*, Ars Combinatoria 23(1987), 201-207.

[3] J Clark and D Holton, *A first look at Graph Theory*, Allied Publishers, New Delhi, 1995.

[4] J A Gallian, *A dynamic Survey of Graph labeling*, The Electronics Journal of Combinatorics, 17(2010), # D56.

[5] J Shiama, *Some Cordial labeling for Some Graphs*, International Journal of Mathematical Archive 3-9(2012), 3271-3276.

[6] Vaidya and N A Dani, *Some new Product Cordial Graphs*, Journal of Applied Computer Science Mathematics 8-4(2010), 62-65.

Received: March 3, 2014; *Accepted*: June 24, 2014

UNIVERSITY PRESS

Website: http://www.malayajournal.org/