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Remarks on rg-compact, gpr-compact and gpr-connected spaces

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Abstract

We give some characterizations of rg-compact, gpr-compact and gpr-connected spaces by utilizing rg-open, gpr-open and gpr-closed sets. The paper is closely related to [A.M.Al-Shibani, rg-compact spaces and rg-connected spaces, Mathematica Pannonica, 17/1 (2006), 61-68], [Y.Gnanambal and K.Balachandran, On gpr-continuous functions in topological spaces,Indian .J.Pure appl.Math., 30(6) (1999),581-593] and [P.Gnanachandra et. al., Ultra Scientist, 24(1) A (2012), 185-191]

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1 Introduction

In 1993, N.Palaniappan and K.Chandrasekhara Rao[8], introduced the concept of regular generalized closed(briefly, rg-closed) sets and regular generalized open (briefly, rg-open) sets in a topological space. They are also defined regular generalized continuous(briefly, rg-continuous) map and regular generalized irresolute(briefly, rg-irresolute) map between topological spaces and studied some of their properties. In 1999, Y.Gnanambal and K.Balachandran [5], introduced and investigated the concept of generalized pre-regular closed (briefly, gpr-closed)sets and generalized pre-regular open (briefly, gpr-open) sets in topological spaces.Further they introduced gpr-continuous functions, gpr-connected spaces and gpr-compact spaces[6]. A.M.Ai-Shibani[1] introduced and investigated rg-compact spaces and rg-connected spaces using rg-open sets.
The purpose of this paper is to characterize these spaces using the well known fact that " every singleton is rg-open and hence gpr-open"[3].

Throughout this paper, space X mean topological space (X, τ). For a subset A of X, the closure, rg-closure,gpr-closure, interior and the complement of A are denoted by cl(A), rg-cl(A),gpr-cl(A), int(A) and A^c respectively.

2 Definitions and Basic Properties

Definition 2.1. *(i) A subset A of a space X is said to be regular open if A= int(cl(A))and regular closed if A=cl(int(A))[9].*
(ii) A subset A of a space X is said to be pre-open if A⊆ int(cl(A)) and pre-closed if cl(int(A))⊆A[7].
The pre-closure of a subset A of X is the intersection of all pre-closed sets containing A and is denoted by pcl(A).

Definition 2.2. *A subset A of a space X is said to be regular generalized closed (briefly. rg-closed)[8] if cl(A)⊆U whenever A⊆U, where U is regular open.It is said to be be regular generalized open (briefly. rg-open) if A^c is rg-closed.(equivalently F⊆int(A) whenever F⊆A and F is regular closed.*

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Definition 2.3. *The intersection of all rg-closed sets containing a set A is called the regular generalized closure of A and is denoted by rg-cl(A).*

Definition 2.4. *A subset A of a space X is said to be generalized pre-regular closed (briefly. gpr-closed)[5] if pcl(A)⊆U whenever A⊆U, where U is regular open.It is said to be be generalized pre-regular open (briefly. gpr-open) if A^c is gpr-closed.*
The intersection of all gpr-closed sets containing a set A is called the generalized pre-regular closure of A and is denoted by gpr-cl(A).

Definition 2.5. *Let f : X → Y be a function. Then f is*
(i). rg-continuous[8] if f^{-1}(V) is rg-closed for every closed set V of Y.
(ii). rg-irresolute[8] if f^{-1}(G) is rg-closed in X for every rg-closed set G of Y.
(iii).gpr-continuous[6] if f^{-1}(V) is gpr-closed for every closed set V of Y.

Definition 2.6. *A collection $\{A_\alpha\colon \alpha \in \nabla\}$ of rg-open sets in a topological space X is called rg-open cover[1] of a subset B of X if B$\subseteq \cup\{A_\alpha\colon \alpha \in \nabla\}$ holds.*

Definition 2.7. *A topological space X is called regular generalized compact(briefly. rg-compact)[1] if every rg-open cover of X has a finite subcover.*

Definition 2.8. *A subset B of X is called rg-compact relative to X [1] if for every collection $\{A_\alpha\colon \alpha \in \nabla\}$ of rg-open subsets of X such that B$\subseteq \cup\{A_\alpha\colon \alpha \in \nabla\}$, there exist a finite subset$\bigtriangledown_\circ$ of $\bigtriangledown$ such that B$\subseteq \cup\{A_\alpha\colon \alpha \in \bigtriangledown_\circ\}$*

Definition 2.9. *A collection $\{A_\alpha\colon \alpha \in \nabla\}$ of gpr-open sets in a topological space X is called gpr-open cover[6] of a subset B of X if B$\subseteq \cup\{A_\alpha\colon \alpha \in \nabla\}$ holds.*

Definition 2.10. *A topological space X is called generalized pre-regular compact(briefly. gpr-compact)[6] if every gpr-open cover of X has a finite subcover.*

Definition 2.11. *A subset B of X is called gpr-compact relative to X[6] if for every collection $\{A_\alpha\colon \alpha \in \nabla\}$ of gpr-open subsets of X such that B$\subseteq \cup\{A_\alpha\colon \alpha \in \nabla\}$, there exist a finite subset$\bigtriangledown_\circ$ of $\bigtriangledown$ such that B$\subseteq \cup\{A_\alpha\colon \alpha \in \bigtriangledown_\circ\}$*

Lemma 2.13. *(i). If A⊆X, then A⊆rg-cl(A)⊆cl(A).*
(ii). If A⊆B, then rg-cl(A)⊆rg-cl(B).
(iii).If A is rg-closed an A⊆B⊆cl(A), then B is rg-closed.

Lemma 2.14. *In a topological space X, the following hold:[3]*
(i). $\{x\}$is rg-open for every x∈X.
(ii). rg-cl(A)=gpr-cl(A)=A, for every subset A of X.

Lemma 2.15. *For a topological space, the following are equivalent:[6]*
(i) X is gpr-connected.
(ii) The only subsets of X which are both gpr-open and gpr-closed are the empty set ϕ and X.
(iii) Each gpr-continuous mp of X into a discrete space Y with atleast two points is a constant map.

Lemma 2.16. *In a topological space X, $\{x\}$ is open or pre-closed for every x∈X.[4]*

3 rg-compact spaces

A.M.Al-Shibani[Theorem 3.4[1]] established the equivalence of the following statements in any topological space (X, τ).
(i). For each x∈X and each open set V in Y with f(x)∈V, there exists an rg-open set U in X such that x∈U, f(U)⊆V.
(ii).For every subset A of X, f(rg-cl(A))⊆ cl(f(A)).
(iii). For every subset B of Y, rg-cl(f^{-1}(B))⊆f^{-1}(cl(B)). However the above statements are always true in any topological space as shown in the next proposition.

Proposition 3.1. *If (X, τ) is a topological space, then the following hold: (1). For each x∈X and each open set V in Y with f(x)∈V, there exists an rg-open set U in X such that x∈U, f(U)⊆V.*
(2).For every subset A of X, f(rg-cl(A))⊆ cl(f(A)).
(3). For every subset B of Y, rg-cl(f^{-1}(B))⊆f^{-1}(cl(B)).

Proof. (1). Take U={x}, then by lemma 2.13, U is rg-open and f(U)=f({x}) ⊆V. (2) and (3) follows from the fact that rg-cl(A)=A, for any set A. □

Theorem 3.2. *A topological space X is rg-compact if and only if X is finite.*

Proof. Let X be a rg-compact space. Since {x} is rg-open for all x∈X, {{x}: x∈X} is an rg-open cover of X. Since X is rg-compact, there exists a finite subset $X_\circ$ of X such that X⊆∪{{x} : x∈$X_\circ$}=$X_\circ$⊆X. Hence X=$X_\circ$, which is finite. Converse is obivious. □

Remark 3.3. *A.M.Al-Shibani established that*
(1) If X is rg-compact and f:X→Y is rg-continuous and bijective, then Y is compact.
(2) If f:X →Y is rg-irresolute and B is rg-compact relative to X, then f(B) is rg-compact relative to Y.
But the conditions f:X→Y is rg-continuous, bijective in (1) and f:X→Y is rg-irresolute in (2) are not necessary as shown in the following theorem.

Theorem 3.4. *Let f:X→Y be a map.*
(1). If X is rg-compact and f is surjective, then Y is compact.
(2). If B is rg-compact relative to X, then f(B) is rg-compact relative to Y.

Proof. (1) Let f:X→Y be a surjective map. If X is rg-compact, then by theorem 3.2, X is finite. Since f is surjective, Y=f(X), which is also finite and hence Y is compact.
(2) If B is rg-compact relative to X, then B is a finite subset of X, by Theorem 3.2. Therefore f(B) is also a finite subset of Y and hence f(B) is rg-compact relative to Y. □

4 gpr-compact spaces

Theorem 4.1. *A topological space X is gpr-compact if and only if X is finite.*

Proof. Let X be a gpr-compact space. Since {x} is gpr-open for all x∈X, {{x}: x∈X} is an gpr-open cover of X. Since X is gpr-compact, there exists a finite subset $X_\circ$ of X such that X⊆∪{{x} : x∈$X_\circ$}=$X_\circ$⊆X. Hence X=$X_\circ$, which is finite. Converse is obivious. □

5 gpr-connected spaces

A topological space (X, τ) is said to be gpr-connected [2] if X cannot be written as the disjoint union of two non empty gpr-open sets.

Theorem 5.1. *No topological space is gpr-connected.*

Proof. Let (X, τ) be topological space.
Case(1): Suppose {x} is open for all x∈X. In this case,(X, τ) is a discrete space and hence every subset of X is both gpr-open and gpr-closed. Therefore by lemma 2.14, (X, τ) cannot be gpr-connected.
Case (2): Suppose {x} is not open for all x∈X. Then {y} is not open for some y∈X. By lemma 2.15,{y} is pre-closed and hence {y} is gpr-closed. Also by lemma 2.13, {y} is gpr-open. Hence {y}is both gpr-closed and gpr-open.Therefore by using lemma 2.14, (X, τ) is not gpr-connected. □

6 Conclusion

In this paper the following results are established:
1.A topological space X is rg-compact if and only if X is finite.
2.A topological space X is gpr-compact if and only if X is finite.
3.No topological space is gpr-connected.

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