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Milovanović bounds for minimum dominating energy of a graph

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Abstract

Recently Milovanović et.al gave a sharper lower bounds for energy of a graph. In this paper similar bounds for minimum dominating energy and Laplacian minimum dominating energy of a graph are established.

Keywords: Minimum dominating energy, Minimum covering energy, Laplacian minimum dominating energy, Laplacian Minimum covering energy

2010 MSC: 05C50, 05C69.

1 Introduction

The concept of energy of a graph was introduced by I. Gutman [3] in the year 1978. Let G be a graph with n vertices $\{v_1, v_2, ..., v_n\}$ and m edges. Let $|\lambda_1| \geq |\lambda_2| \geq ... \geq |\lambda_n|$ be the eigenvalues of adjacency matrix $A = (a_{ij})$ of the graph. Then the energy of a graph is defined by $E(G) = \sum_{i=1}^{n} |\lambda_i|$.

For details on the mathematical aspects of theory of graph energy see the papers [4, 5] and the references cited there in. The basic properties including various upper and lower bounds for energy of a graph have been established in [7, 8] and it has found remarkable chemical applications in the molecular orbital theory of conjugated molecules [2, 6].

Let $|\mu_1| \geq |\mu_2| \geq |\mu_3| \geq ... \geq |\mu_n|$ denotes eigenvalues of Laplacian matrix $L = (l_{ij})$ of a graph G. Then Laplacian energy is defined by $LE(G)=\sum_{i=1}^{n} \left|\mu_i - \frac{2m}{n}\right|$

Recently Milovanović [9] et.al gave a sharper lower bounds for energy of a graph. In this paper similar bounds for minimum dominating energy and Laplacian minimum dominating energy of a graph are established. Similar bounds for minimum covering energy and Laplacian minimum covering energy of a graph can also be derived.

2 Preliminaries

Definition 2.1. ***Minimum Dominating Energy of a Graph**: Let G be a simple graph of order n with vertex set $V = \{v_1, v_2, ..., v_n\}$ and edge set E. A subset D of V is called a dominating set of G if every vertex of $V - D$ is incident to some vertex of D. Any dominating set with minimum cardinality is called a minimum dominating set. For the graph G with minimum dominating set D, the minimum dominating matrix is defined by*

$$A_D(G) := (a_{ij}^D), \text{ where } a_{ij}^D = \begin{cases} 1 & if \;\; v_iv_j \in E \\ 1 & if \;\; i = j \;\; and \;\; v_i \in D \\ 0 & otherwise \end{cases}$$

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If $|\lambda_1| \geq |\lambda_2| \geq ... \geq |\lambda_n|$ are the eigenvalues of adjacency matrix $A_D(G)$ of the graph, then the minimum Dominating energy of the graph G is defined by $E_D(G) := \sum_{i=1}^{n} |\lambda_i|$.

Definition 2.2. ***Laplacian Minimum Dominating Energy of a Graph***: *If $D(G)$ denotes the diagonal matrix of vertex degree of the graph G, then $L_D(G)$=$D(G) - A_D(G)$ is called Laplacian dominating matrix of G. If $|\mu_1| \geq |\mu_2| \geq |\mu_3| \geq ... \geq |\mu_n|$ denotes eigenvalues of matrix $L_D(G)$, then Laplacian minimum dominating energy is defined by* $LE_D(G) :=\sum_{i=1}^{n} \left|\mu_i - \frac{2m}{n}\right|$.

For the basic properties on minimum covering energy, Laplacian minimum covering energy, minimum dominating energy, Laplacian minimum dominating energy, see the papers [1, 10, 11, 12] and the references cited there in.

3 Milovanović bounds for minimum dominating energy of a graph

Theorem 3.1. *Let G be a graph with n vertices and m edges. Let $|\lambda_1| \geq |\lambda_2| \geq \ldots \geq |\lambda_n|$ be a non-increasing order of eigenvalues of $A_D(G)$ and D is minimum dominating set then $E_D(G) \geq \sqrt{n(2m + |D|) - \alpha(n)(|\lambda_1| - |\lambda_n|)^2}$ where $\alpha(n)= n[\frac{n}{2}](1 - \frac{1}{n}[\frac{n}{2}])$ and $[x]$ denotes the integral part of a real number*

Proof. Let $a, a_1, a_2, \ldots a_n, A$ and $b, b_1, b_2, \ldots b_n, B$ be real numbers such that $a \leq a_i \leq A$ and $b \leq b_i \leq B$ $\forall\ i = 1, 2, \ldots n$ then the following inequality is valid. $\left|n\sum_{i=1}^{n} a_i b_i - \sum_{i=1}^{n} a_i \sum_{i=1}^{n} b_i\right| \leq \alpha(n)(A - a)(B - b)$ where α(n)= $n[\frac{n}{2}](1 - \frac{1}{n}[\frac{n}{2}])$ and equality holds if and only if $a_1 = a_2 = \ldots = a_n$ and $b_1 = b_2 = \ldots = b_n$.
If $a_i = |\lambda_i|$, $b_i = |\lambda_i|$, $a = b = |\lambda_n|$ and $A = B = |\lambda_1|$, then

$$\left|n\sum_{i=1}^{n}|\lambda_i|^2 - \left(\sum_{i=1}^{n}|\lambda_i|\right)^2\right| \leq \alpha(n)(|\lambda_1| - |\lambda_n|)^2$$

But $\sum_{i=1}^{n}|\lambda_i|^2$ =$2m + |D|$ and $E_D(G) \leq \sqrt{n(2m + |D|)}$ [10] then the above inequality becomes

$$n(2m + |D|) - (E_D(G))^2 \leq \alpha(n)(|\lambda_1| - |\lambda_n|)^2$$

$$\text{i,e., } E_D(G) \geq \sqrt{n(2m + |D|) - \alpha(n)(|\lambda_1| - |\lambda_n|)^2}$$

□

The above theorem is also true for the minimum covering energy of a graph. Hence we have the following result.
Let G be a graph with n vertices and m edges. Let$|\lambda_1| \geq |\lambda_2| \geq \ldots \geq |\lambda_n|$ be a non-increasing order of eigenvalues of $A_C(G)$ and C is minimum covering set, then $E_C(G) \geq \sqrt{n(2m + |C|) - \alpha(n)(|\lambda_1| - |\lambda_n|)^2}$ where α(n)= $n[\frac{n}{2}](1 - \frac{1}{n}[\frac{n}{2}])$ and $[x]$ denotes integral part of a real number

Theorem 3.2. *Let G be a graph with n vertices and m edges. Let $|\lambda_1| \geq |\lambda_2| \geq \ldots \geq |\lambda_n| > 0$ be a non-increasing order of eigenvalues of $A_D(G)$ then $E_D(G) \geq \frac{2m + |D| + n|\lambda_1||\lambda_n|}{(|\lambda_1| + |\lambda_n|)}$*

Proof. Let $a_i \neq 0$, b_i, r and R be real numbers satisfying $ra_i \leq b_i \leq Ra_i$, then the following inequality holds.[Theorem 2, [9]]

$$\sum_{i=1}^{n} b_i^2 + rR\sum_{i=1}^{n} a_i \leq (r + R)\sum_{i=1}^{n} a_i b_i$$

Put $b_i = |\lambda_i|$, a_i =1, $r = |\lambda_n|$ and $R = |\lambda_1|$ then

$$\sum_{i=1}^{n}|\lambda_i|^2 + |\lambda_1||\lambda_n|\sum_{i=1}^{n}1 \leq (|\lambda_1| + |\lambda_n|)\sum_{i=1}^{n}||\lambda_i|$$
$$i.e., 2m + |D| + |\lambda_1||\lambda_n|n \leq (|\lambda_1| + |\lambda_n|)E_D(G)$$
$$E_D(G) \geq \frac{2m + |D| + n|\lambda_1||\lambda_n|}{(|\lambda_1| + |\lambda_n|)}$$

□

This bound is similar for minimum covering energy of a graph.

4 Milovanović bounds for laplacian minimum dominating energy

Theorem 4.3. *Let G be a graph with n vertices and m edges. Let* $|\mu_1| \geq |\mu_2| \geq \ldots \geq |\mu_n|$ *be a non-increasing order of eigenvalues of* $L_D(G)$*. If D is minimum dominating set then* $LE_D(G) \geq \sqrt{2nM - \alpha(n)(|\mu_1| - |\mu_n|)^2} - 2m$*, where* $\alpha(n) = n[\frac{n}{2}](1 - \frac{1}{n}[\frac{n}{2}])$*, [x] denotes greatest integer part of real number and* $M = m + \frac{1}{2}\sum_{i=1}^{n}(d_i - c_i)^2$*.*

Here $c_i = \begin{cases} 1 & \text{if } v_i \in D \\ 0 & \text{if } v_i \notin D \end{cases}$

Proof. Let $a, a_1, a_2, \ldots a_n, A$ and $b, b_1, b_2, \ldots b_n, B$ be real numbers such that $a \leq a_i \leq A$ and $b \leq b_i \leq B$ $\forall\ i = 1, 2, \ldots n$ then the following inequality is valid.

$$\Big|n\sum_{i=1}^{n}a_ib_i - \sum_{i=1}^{n}a_i\sum_{i=1}^{n}b_i\Big| \leq \alpha(n)(A - a)(B - b)$$

If $a_i = |\mu_i|$, $b_i = |\mu_i|$, $a = b = |\mu_n|$ and $A = b = |\mu_1|$

$$\Big|n\sum_{i=1}^{n}|\mu_i|^2 - \Big(\sum_{i=1}^{n}|\mu_i|\Big)^2\Big| \leq \alpha(n)(|\mu_1| - |\mu_n|)^2$$

But $(\sum_{i=1}^{n}|\mu_i|)^2 \leq 2nM \quad \Rightarrow n2M - \Big(\sum_{i=1}^{n}|\mu_i|\Big)^2 \leq \alpha(n)(|\mu_1| - |\mu_n|)^2$

$$\Big(\sum_{i=1}^{n}|\mu_i|\Big) \geq \sqrt{2Mn - \alpha(n)(|\mu_1| - |\mu_n|)^2}$$

Since $LE_D(G) = \sum_{i=1}^{n}\Big|\mu_i - \frac{2m}{n}\Big| \geq \sum_{i=1}^{n}|\mu_i| - \Big|\frac{2m}{n}\Big|$

Hence $LE_D(G) \geq \sqrt{2nM - \alpha(n)(|\mu_1| - |\mu_n|)^2}$ -2m □

Theorem 4.4. *Let G be a graph with n vertices and m edges. Let* $|\mu_1| \geq |\mu_2| \geq \ldots \geq |\mu_n| > 0$ *be a non-increasing order of eigenvalues of* $LE_D(G)$ *and D is minimum dominating set then* $LE_D(G) \geq \frac{2M + n|\mu_1||\mu_n|}{(|\mu_1| + |\mu_n|)} - 2m$ *where* $M = m + \frac{1}{2}\sum_{i=1}^{n}(d_i - c_i)^2$*. Here* $c_i = \begin{cases} 1 & \text{if } v_i \in D \\ 0 & \text{if } v_i \notin D \end{cases}$

Proof. Let $a_i \neq 0$, b_i, r and R be real numbers satisfying $ra_i \leq b_i \leq Ra_i$, then we have the following inequality

$$\sum_{i=1}^{n}b_i^2 + rR\sum_{i=1}^{n}a_i \leq (r + R)\sum_{i=1}^{n}a_ib_i$$

Put $b_i = |\mu_i|$, a_i =1, $r = |\mu_n|$ and $R = |\mu_1|$

$$\sum_{i=1}^{n}|\mu_i|^2 + |\mu_1||\mu_n|\sum_{i=1}^{n}1 \leq (|\mu_1| + |\mu_n|)\sum_{i=1}^{n}|\mu_i|$$

$$i.e., 2M + |\mu_1||\mu_n|n \leq (|\mu_1| + |\mu_n|)\sum_{i=1}^{n}|\mu_i|$$

$$\Rightarrow \sum_{i=1}^{n}|\mu_i| \geq \frac{2M + n|\mu_1||\mu_n|}{(|\mu_1| + |\mu_n|)}$$

We know that $LE_D(G) = \sum_{i=1}^{n}\left|\mu_i - \frac{2m}{n}\right| \quad LE_D(G) \geq \sum_{i=1}^{n}\left|\mu_i\right| - \left|\frac{2m}{n}\right|$

$$\Rightarrow LE_D(G) \geq \frac{2M + n|\mu_1||\mu_n|}{(|\mu_1| + |\mu_n|)} - 2m$$

□

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