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An adaptive integration scheme using a mixed quadrature of three different quadrature rules

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Abstract

In the present work,a mixed quadrature rule of precision seven is constructed blending Gauss-Legendre 2-point rule, Fejer's first and second 3-point rules each having precision three.The error analysis of the mixed rule is incorporated.An algorithm is designed for adaptive integration scheme using the mixed quadrature rule.Through some numerical examples,the effectiveness of adopting mixed quadrature rule in place of their constituent rules in the adaptive integration scheme is discussed.

Keywords: Gauss-Legendre quadrature, Fejer's quadrature, mixed quadrature and adaptive integration scheme

2010 MSC: 65D30, 65D32.

1 Introduction

In this article, we consider the following problem. Given a continuous function $f(x)$ over a bounded interval $[a, b]$ and a prescribed tolerance ϵ, we seek to find an approximation $Q(f)$ using a mixed quadrature rule to the integral

$$I(f) = \int_a^b f(x)dx \tag{1.1}$$

so that

$$|Q(f) - I(f)| \leq \epsilon \tag{1.2}$$

This can be done following adaptive integration scheme (AIS)[1] [2] [3].

Conte and Boor[3] evaluated real definite integral (1.1) in the adaptive integration scheme using Simpson's $\frac{1}{3}$ rule as a base rule.They fix a termination criterion for adaptive integration scheme using Simpon's $\frac{1}{3}$ two panel rule and Simpson's $\frac{1}{3}$ four panel rule (composite rule). Recently, R.B.Dash and D.Das[7] [8] [9] constructed some mixed quadrature rules and fix the termination criterion for adaptive integration using the mixed quadrature rule and evaluated successfully various real definite integrals. Mixed quadrature [5] [6] [7][8] [9] [10] [11] means a quadrature of higher precision which is formed by taking the linear/ convex combination of two or more quadrature rules of equal lower precision.

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The idea of mixed quadrature was first given by R.N. Das and G. Pradhan (1996) [5], who constructed a mixed quadrature rule of precision 5 blending Simpson's $\frac{1}{3}$ rule with Gauss- Legendre 2-point rule, each having precision 3. Evaluating some real definite integrals on the whole interval, they showed the superiority of the mixed quadrature rule over their constituent rules. N. Das and S.K. Pradhan(2004)[6] derived a mixed quadrature rule of precision 7 by taking a linear combination of Simpson's $\frac{1}{3}$ rule,Simpson's $\frac{3}{8}$ rule and Gauss-Legendre 2-point rule, each having precision 3. They also showed the superiority of the mixed quadrature rule over their constituent rules by evaluating some real definite integrals in the whole interval method.

In this paper, we have constructed a mixed quadrature rule of precision 7 by mixing Gauss-Legendre 2-point rule[4] with Fejer's first and second 3-point rules[2] [10] each having equal precision (i.e. precision 3) for approximating some real definite integrals in the adaptive integration scheme. The construction of mixed quadrature rule is outlined in the following section.

2 Construction of the mixed quadrature rule of precision seven

A mixed quadrature rule of precision seven is constructed by using the following three well-known quadrature rules.

(i) Gauss- Legendre 2-point rule

(ii) Fejer's first 3-point rule

(iii) Fejer's second 3- point rule

The Gauss-Legendre 2-point rule $(R_{GL_2}(f))$ is

$$I(f)=\int_a^b f(x)dx=\int_{-1}^1 f(x)dx\approx R_{GL_2}(f)=f(-\frac{1}{\sqrt{3}})+f(\frac{1}{\sqrt{3}}) \tag{2.3}$$

The Fejer's first 3-point rule $(R_{1_{F_3}}(f))$ is

$$I(f)=\int_a^b f(x)dx=\int_{-1}^1 f(x)dx\approx R_{1_{F_3}}(f)=\frac{1}{9}[4f(\frac{-\sqrt{3}}{2})+10f(0)+4f(\frac{\sqrt{3}}{2})] \tag{2.4}$$

The Fejer's second 3-point rule $(R_{2_{F_3}}(f))$ is

$$I(f)=\int_a^b f(x)dx=\int_{-1}^1 f(x)dx\approx R_{2_{F_3}}(f)=\frac{2}{3}[f(\frac{-1}{\sqrt{2}})+f(0)+f(\frac{1}{\sqrt{2}})] \tag{2.5}$$

Each of these rules (2.1), (2.2) and (2.3) is of precision 3. Let $E_{GL_2}(f), E_{1_{F_3}}(f), E_{2_{F_3}}(f)$ denote the errors in approximating the integral $I(f)$ by the rules (2.1), (2.2) and (2.3) respectively.

Then,

$$I(f)=R_{GL_2}(f)+E_{GL_2}(f) \tag{2.6}$$

$$I(f) = R_{1_{F_3}}(f) + E_{1_{F_3}}(f) \tag{2.7}$$

$$I(f) = R_{2_{F_3}}(f) + E_{2_{F_3}}(f) \tag{2.8}$$

Assuming $f(x)$ to be sufficiently differentiable in $-1 \leq x \leq 1$, and using Maclaurin's expansion of function $f(x)$, we can express the errors associated with the quadrature rules under reference as

$E_{GL_2}(f) = \frac{8}{5! \times 9} f^{(iv)}(0) + \frac{40}{7! \times 27} f^{(vi)}(0) + \frac{16}{9! \times 9} f^{(viii)}(0) + ...$

$E_{1_{F_3}}(f) = -\frac{1}{5! \times 2} f^{(iv)}(0) - \frac{5}{8!} f^{(vi)}(0) - \frac{17}{9! \times 32} f^{(viii)}(0) - ...$

$E_{2_{F_3}}(f) = \frac{1}{3 \times 5!} f^{(iv)}(0) + \frac{5}{6 \times 7!} f^{(vi)}(0) + \frac{5}{4 \times 9!} f^{(viii)}(0) + ...$

Now multiplying the Eqs (2.4), (2.5) and (2.6) by 27, 32 and -24 respectively, then adding the results we obtain,

$I(f) = \frac{1}{35}(27R_{GL_2}(f) + 32R_{1_{F_3}}(f) - 24R_{2_{F_3}}(f)) + \frac{1}{35}(27E_{GL_2}(f) + 32E_{1_{F_3}}(f) - 24E_{2_{F_3}}(f))$

$$I(f) = R_{GL_2 1_{F_3} 2_{F_3}}(f) + E_{GL_2 1_{F_3} 2_{F_3}}(f) \tag{2.9}$$

Where

$$R_{GL_2 1_{F_3} 2_{F_3}}(f) = \frac{1}{35}(27R_{GL_2}(f) + 32R_{1_{F_3}}(f) - 24R_{2_{F_3}}(f)) \tag{2.10}$$

And

$$E_{GL_2 1_{F_3} 2_{F_3}}(f) = \frac{1}{35}(27E_{GL_2}(f) + 32E_{1_{F_3}}(f) - 24E_{2_{F_3}}(f)) \tag{2.11}$$

Eq.(2.8) expresses the desired mixed quadrature rule for the approximate evaluation of $I(f)$ and Eq (2.9) expresses the error generated in this approximation.

Hence,

$$E_{GL_2 1_{F_3} 2_{F_3}}(f) = \frac{1}{9! \times 35} f^{(viii)}(0) + ... \tag{2.12}$$

As the first term of $E_{GL_2 1_{F_3} 2_{F_3}}(f)$ contains 8^{th} order derivative of the integrand, the degree of precision of the mixed quadrature rule is 7. It is called a mixed type rule as it is constructed from three different types of rules of equal precision.

3 Error analysis of the mixed quadrature rule

An asymptotic error estimate and an error bound of the rule (2.8) are given in theorems 3.1 and 3.2 respectively.

Theorem-3.1

Let $f(x)$ be a sufficiently differentiable function in the closed interval $[-1,1]$. Then the error $E_{GL_2 1_{F_3} 2_{F_3}}(f)$ associated with the mixed quadrature rule $R_{GL_2 1_{F_3} 2_{F_3}}(f)$ is given by

$|E_{GL_2 1_{F_3} 2_{F_3}}(f)| \approx \frac{1}{9! \times 35} |f^{(viii)}(0)|$

Proof The proof follows from the Eq (2.10).

Theorem 3.2

The bound for the truncation error $E_{GL_2 1_{F_3} 2_{F_3}}(f) = I(f) - R_{GL_2 1_{F_3} 2_{F_3}}(f)$ *is given by*

$E_{GL_2 1_{F_3} 2_{F_3}}(f) \leq \frac{2M}{175}$

$where M = max_{-1 \leq x \leq 1} |f^{(v)}(x)|$

Proof

$E_{GL_2}(f) = \frac{8}{5! \times 9} f^{(iv)}(\eta_1), \qquad \eta_1 \in [-1,1]$

$E_{1_{F_3}}(f) = -\frac{1}{5! \times 2} f^{(iv)}(\eta_2), \qquad \eta_2 \in [-1,1]$

$E_{2_{F_3}}(f) = \frac{1}{5! \times 3} f^{(iv)}(\eta_3), \qquad \eta_3 \in [-1,1]$

$E_{GL_2 1_{F_3} 2_{F_3}}(f) = \frac{1}{35}[27 E_{GL_2}(f) + 32 E_{1_{F_3}}(f) - 24 E_{2_{F_3}}(f)]$

$= \frac{24}{5! \times 35} f^{(iv)}(\eta_1) - \frac{16}{5! \times 35} f^{(iv)}(\eta_2) - \frac{8}{5! \times 35} f^{(iv)}(\eta_3)$

Let $K = max_{x \in [-1,1]} |f^{(iv)}(x)|$ *and* $k = min_{x \in [-1,1]} |f^{(iv)}(x)|$. As $f^{(iv)}(x)$ is continuous and $[-1,1]$ is compact, there exist points b and a in the interval $[-1,1]$ such that $K = f^{(iv)}(b)$ *and* $k = f^{(iv)}(a)$. Thus

$E_{GL_2 1_{F_3} 2_{F_3}}(f) \leq \frac{24}{5! \times 35} f^{(iv)}(b) - \frac{16}{5! \times 35} f^{(iv)}(a) - \frac{8}{5! \times 35} f^{(iv)}(a)$

$= \frac{24}{5! \times 35} [f^{(iv)}(b) - f^{(iv)}(a)]$

$= \frac{1}{175} \int_a^b f^{(v)}(x) dx$

$= \frac{1}{175}(b-a) f^{(v)}(\xi)$ *for some* $\xi \in [-1,1]$ by mean value theorem.

Hence by choosing $|(b-a)| \leq 2$

we have $E_{GL_2 1_{F_3} 2_{F_3}}(f) \leq \frac{1}{175} |(b-a)| |f^{(v)}(\xi)| \leq \frac{2M}{175}$

Where $M = max_{-1\leq x\leq 1}|f^{(v)}(x)|$

4 Algorithm for adaptive quadrature routine

Applying the constituent rules $(R_{GL_2}(f), R1_{F_3}(f), R2_{F_3}(f))$ and the mixed quadrature rule $(R_{GL_2 1_{F_3} 2_{F_3}}(f))$, one can evaluate real definite integrals of the type $I(f) = \int_a^b f(x)dx$ in adaptive integration scheme. In the adaptive integration scheme, the desired accuracy is sought by progressively subdividing the interval of integration according to the computed behavior of the integrand, and applying the same formula over each subinterval. A simple adaptive strategy is outlined using the mixed quadrature rule $(R_{GL_2 1_{F_3} 2_{F_3}}(f))$ in the following four step algorithm.

Input: Function $F : [a, b] \longrightarrow R$ and the prescribed tolerance ϵ .

Output: An approximation $Q(f)$ *to the integral* $I(f) = \int_a^b f(x)dx$ *such that* $|Q(f) - I(f)| \leq \epsilon$.

Step-1: The mixed quadrature rule $(R_{GL_2 1_{F_3} 2_{F_3}}(f))$ is applied to approximate the integral $I(f) = \int_a^b f(x)dx$.

The approximate value is denoted by $(R_{GL_2 1_{F_3} 2_{F_3}}[a, b])$.

Step-2 : The interval of integration $[a, b]$ is divided into two equal pieces, $[a, c]$ *and* $[c, b]$. The mixed

quadrature rule $(R_{GL_2 1_{F_3} 2_{F_3}}(f))$ is applied to approximate the integral $I_1(f) = \int_a^c f(x)dx$ and the approximate value is denoted by $(R_{GL_2 1_{F_3} 2_{F_3}}[a, c])$. Similarly, the mixed quadrature rule $(R_{GL_2 1_{F_3} 2_{F_3}}(f))$ is applied to approximate the integral $I_2(f) = \int_c^b f(x)dx$ and the approximate value is denoted by $(R_{GL_2 1_{F_3} 2_{F_3}}[c, b])$.

Step-3:$(R_{GL_2 1_{F_3} 2_{F_3}}[a, c] + (R_{GL_2 1_{F_3} 2_{F_3}}[c, b])$ is compared with $(R_{GL_2 1_{F_3} 2_{F_3}}[a, b])$ to estimate the error in

$(R_{GL_2 1_{F_3} 2_{F_3}}[a, c] + (R_{GL_2 1_{F_3} 2_{F_3}}[c, b])$.

Step-4: If $|estimated\ \ error| \leq \frac{\epsilon}{2}$ (termination criterion) then $(R_{GL_2 1_{F_3} 2_{F_3}}[a, c] + R_{GL_2 1_{F_3} 2_{F_3}}[c, b])$ is accepted as

an approximation to $I(f) = \int_a^b f(x)dx$. Otherwise the same procedure is applied to $[a, c]$ *and* $[c, b]$, allowing each pieces a tolerance of $\frac{\epsilon}{2}$. If the termination criterion is not satisfied on one or more of the sub intervals, then those sub-intervals must be further subdivided and the entire process repeated. When the process stops, the addition of all accepted values yields the desired approximate value $Q(f)$ of the integral $I(f)$ such that $|Q(f) - I(f)| \leq \epsilon$.

N:B: In this algorithm we can use any quadrature rule to evaluate real definite integrals in adaptive integration scheme.

5 Numerical verification

Table 5.1: Comparative study among the quadrature rule $R_{GL_2}(f), R_{1_{F_3}}(f)$ *and* $R_{2_{F_3}}(f)$ for approximation of some real definite integrals without using adaptive integration scheme

Integrals	Exact Value $I(f)$	**Approximate Value ($Q(f)$) by**		
		$R_{GL_2}(f)$	$R_{1_{F_3}}(f)$	$R_{2_{F_3}}(f)$
$I_1(f)=\int_0^1 \frac{4}{1+x^2}dx$	$\pi \approx 3.14159265358$	3.14754	3.1379	3.14336
$I_2(f)=\int_0^3 \frac{sin2x}{1+x^2}dx$	0.4761463020	0.7939	0.2752	0.5673
$I_3(f)=\int_0^3 (sin4x)e^{-2x}dx$	0.1997146621	0.2398	0.2955	0.3898
$I_4(f)=\int_{0.04}^1 \frac{1}{\sqrt{x}}dx$	1.6	1.5116	1.620	1.5419
$I_5(f)=\int_0^2 \frac{1}{x^2+\frac{1}{10}}dx$	4.4713993943	3.9753	4.9022	4.4155
$I_6(f)=\int_{\frac{1}{2\pi}}^2 sin(\frac{1}{x})dx$	1.1140744942	1.4263	0.8665	1.2698
$I_7(f)=\int_0^{\frac{\pi}{2}} (x^2+x+1)cosxdx$	2.038197427067	2.0366	2.0389	2.0375
$I_8(f)=\int_0^5 \frac{x^3}{e^x-1}dx$	4.8998922	4.6016	5.0588	4.7760
$I_9(f)=\int_0^1 e^{-x^2}dx$	0.7468241328	0.7465	0.7469	0.7467
$I_{10}(f)=\int_0^4 13(x-x^2)e^{-\frac{3x}{2}}dx$	-1.5487883725279	-0.5999	-1.7966	-0.8318
$I_{11}(f)=\int_0^2 \sqrt{4x-x^2}dx$	π	3.1844	3.1312	3.1683
$I_{12}(f)=\int_1^6 [2+sin(2\sqrt{x})]dx$	8.1834792077	8.2627	8.1420	8.2171
$I_{13}(f)=\int_0^1 \frac{1}{1+x^4}dx$	0.8669729870	0.8595	0.8715	0.8646
$I_{14}(f)=\int_0^1 sin(\sqrt{x})dx$	0.6023373578	0.6097	0.6005	0.6069

Table 5.2: Comparative study among the quadrature/mixed quadrature rules ($R_{GL_3}(f)$,$R_{2_{F_5}}(f)$ and $R_{GL_2 1_{F_3} 2_{F_3}}(f)$) for approximation of integrals (table 5.1) without using adaptive integration scheme

Integrals	Exact Value $I(f)$	**Approximate Value ($Q(f)$) by**		
		$R_{GL_3}(f)$	$R_{2_{F_5}}(f)$	$R_{GL_2 1_{F_3} 2_{F_3}}(f)$
$I_1(f)=\int_0^1 \frac{4}{1+x^2}dx$	$\pi \approx 3.14159265358$	3.14106	3.14147	3.1415979
$I_2(f)=\int_0^3 \frac{sin2x}{1+x^2}dx$	0.4761463020	0.4415	0.4659	0.4751
$I_3(f)=\int_0^3 (sin4x)e^{-2x}dx$	0.1997146621	0.3913	0.2326	0.1878
$I_4(f)=\int_{0.04}^1 \frac{1}{\sqrt{x}}dx$	1.6	1.5667	1.5844	1.5905
$I_5(f)=\int_0^2 \frac{1}{x^2+\frac{1}{10}}dx$	4.4713993943	4.6629	4.5628	4.5209
$I_6(f)=\int_{\frac{1}{2\pi}}^2 sin(\frac{1}{x})dx$	1.1140744942	1.1304	1.0498	1.0219
$I_7(f)=\int_0^{\frac{\pi}{2}} (x^2+x+1)cosxdx$	2.038197427067	2.03810	2.03817	2.03819762
$I_8(f)=\int_0^5 \frac{x^3}{e^x-1}dx$	4.8998922	4.8862	4.8968	4.90003
$I_9(f)=\int_0^1 e^{-x^2}dx$	0.7468241328	0.746814	0.746822	0.74682421
$I_{10}(f)=\int_0^4 13(x-x^2)e^{-\frac{3x}{2}}dx$	-1.5487883725279	-1.1196	-1.43307	-1.5350
$I_{11}(f)=\int_0^2 \sqrt{4x-x^2}dx$	π	3.1560	3.1492	3.1468
$I_{12}(f)=\int_1^6 [2+sin(2\sqrt{x})]dx$	8.1834792077	8.1882	8.1847	8.1836
$I_{13}(f)=\int_0^1 \frac{1}{1+x^4}dx$	0.8669729870	0.8675	0.8670	0.866965
$I_{14}(f)=\int_0^1 sin(\sqrt{x})dx$	0.6023373578	0.6048	0.6036	0.6032

$R_{GL_3}(f)$**:Gauss-Legendre 3-point rule**

$R_{2_{F_5}}(f)$**: Fejer's second 5-point rule**

Table 5.3: Comparison of the results following from the Gauss-Legendre 2-point rule, Fejer's first 3-point rule and Fejer's second 3-point rule for approximating integrals using the adaptive integration scheme

Integrals	**Approximate value ($Q(f)$) by**					
	$(R_{GL_2}(f))$	#steps	$(R_{1_{F_3}}(f))$	#steps	$(R_{2_{F_3}}(f))$	#steps
$I_1(f)=\int_0^1 \frac{4}{1+x^2}dx$	3.141592690	17	3.141592653573	15	3.14159265359	15
$I_2(f)=\int_0^3 \frac{sin2x}{1+x^2}dx$	0.47614627	41	0.476146256	35	0.476146332	35
$I_3(f)=\int_0^3 (sin4x)e^{-2x}dx$	0.199714693	51	0.199714686	43	0.19971459	39
$I_4(f)=\int_{0.04}^1 \frac{1}{\sqrt{x}}dx$	1.59999986	39	1.6000001	35	1.59999986	31
$I_5(f)=\int_0^2 \frac{1}{x^2+\frac{1}{10}}dx$	4.471399346	53	4.471399461	49	4.471399326	43
$I_6(f)=\int_{\frac{1}{2\pi}}^2 sin(\frac{1}{x})dx$	1.114074589	51	1.114074448	43	1.114074503	41
$I_7(f)=\int_0^{\frac{\pi}{2}} (x^2+x+1)cosxdx$	2.0381974132	23	2.0381974183	17	2.0381974106	15
$I_8(f)=\int_0^5 \frac{x^3}{e^x-1}dx$	4.899892102	43	4.899892237	39	4.899892026	29
$I_9(f)=\int_0^1 e^{-x^2}dx$	0.7468241276	15	0.746824114	13	0.746824120	11
$I_{10}(f)=\int_0^4 13(x-x^2)e^{-\frac{3x}{2}}dx$	-1.5487882018	57	-1.5487884508	51	-1.5487882663	47
$I_{11}(f)=\int_0^2 \sqrt{4x-x^2}dx$	3.1415929475	45	3.141592395	37	3.141592855	39
$I_{12}(f)=\int_1^6 [2+sin(2\sqrt{x})]dx$	8.1834793329	31	8.18347908	27	8.183479317	25
$I_{13}(f)=\int_0^1 \frac{1}{1+x^4}dx$	0.8669729661	15	0.86697299	15	0.866972942	13
$I_{14}(f)=\int_0^1 sin(\sqrt{x})dx$	0.602337696	29	0.602337112	25	0.602337592	25

N:B:The prescribed tolerance(ϵ)=0.000001

 # Steps: No. of Steps

Table 5.4: Comparison of the results following from the Gauss-Legendre 3-point rule, Fejer's second 5-point rule and mixed quadrature rule $R_{GL_2 1_{F_3} 2_{F_3}}(f)$ for approximating integrals (given in table 5.3) using the adaptive integration scheme

Integrals	Approximate Value ($Q(f)$) by					
	$(R_{GL_3}(f))$	#steps	$(R_{2_{F_5}}(f))$	# steps	$(R_{GL_2 1_{F_3} 2_{F_3}}(f))$	#steps
$I_1(f) = \int_0^1 \frac{4}{1+x^2} dx$	3.14159265347	7	3.141592651	3	3.141592653589621	3
$I_2(f) = \int_0^3 \frac{sin2x}{1+x^2} dx$	0.4761463032	15	0.4761463085	11	0.4761463008	5
$I_3(f) = \int_0^3 (sin4x)e^{-2x} dx$	0.1997146667	19	0.1997146587	13	0.1997146616	9
$I_4(f) = \int_{0.04}^1 \frac{1}{\sqrt{x}} dx$	1.599999987	17	1.599999985	13	1.599999998	9
$I_5(f) = \int_0^2 \frac{1}{x^2+\frac{1}{10}} dx$	4.4713993946	17	4.471399387	15	4.471399396	11
$I_6(f) = \int_{\frac{1}{2\pi}}^2 sin(\frac{1}{x}) dx$	1.114074506	21	1.114074477	19	1.114074495	11
$I_7(f) = \int_0^{\frac{\pi}{2}} (x^2+x+1) cosx dx$	2.0381974267	7	2.0381974227	3	2.03819742776	1
$I_8(f) = \int_0^5 \frac{x^3}{e^x-1} dx$	4.8998921534	13	4.8998921579	7	4.899892158	3
$I_9(f) = \int_0^1 e^{-x^2} dx$	0.7468241324	3	0.7468241327	3	0.7468241329	1
$I_{10}(f) = \int_0^4 13(x-x^2)e^{-\frac{3x}{2}} dx$	-1.5487883665	21	-1.548788353	13	-1.5487883721	9
$I_{11}(f) = \int_0^2 \sqrt{4x-x^2} dx$	3.1415928159	25	3.1415928990	19	3.141592813	19
$I_{12}(f) = \int_1^6 [2+sin(2\sqrt{x})] dx$	8.1834792212	9	8.1834792108	9	8.1834792081	5
$I_{13}(f) = \int_0^1 \frac{1}{1+x^4} dx$	0.8669729873	7	0.886972987	7	0.8669729873	3
$I_{14}(f) = \int_0^1 sin(\sqrt{x}) dx$	0.602337586	17	0.602337475	17	0.60233758	15

N:B:The prescribed tolerance(ϵ)=0.000001

All the computations are done using 'C' Program[8].

6 Conclusion

We observe from Tables-5.1 and 5.2, that the mixed quadrature rule gives more accurate result in comparison to their constituent rules. Gauss-Legendre 3-point rule and Fejer's second 5-point rule when integrals ($I_1 - I_{14}$) are evaluated without using adaptive integration scheme. Tables-5.3 and 5.4, reveal that when these integrals are evaluated using the adaptive integration scheme, the mixed qudrature rule reduces the number of steps to achieve the prescribed accuracy and gives more accurate result in comparison to the their constituent rules, Gauss-Legendre 3-point rule and Fejer's second 5-point rule.

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