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On Quasi-weak Commutative Boolean-like Near-Rings

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Abstract

In this paper we establish a result that every quasi-weak commutative Boolean-like near-ring can be imbedded into a quasi-weak commutative Boolean-like commutative semi-ring with identity. Key words: Quasi-weak commutative near-ring, Boolean-like near-ring.

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1 Introduction

The concept of Boolean-like ring was coined by A.L.Foster[1]. Foster proved that if R is a Boolean ring with identity then ab(1-a)(1-b) = 0 for all a,b ϵ R. He generalized the concept of Boolean ring as Boolean-like ring as a ring R with identity satisfying (i) ab(1-a)(1-b) = 0 and (ii) 2a = 0 for all a,bϵ R. He also observed that the equation ab(1-a)(1-b) = 0 can be re-written as $(ab)^2 - ab^2 \;\; a^2b + ab = 0$. He re-defined a Boolean-like ring as a commutative ring with identity satisfying (i) $(ab)^2 - ab^2 \;\; a^2b + ab = 0$ and (ii) 2a = 0 for all a,b ϵ R. In 1962 Adil Yaqub [8] proved that the condition 'commutativity 'is not necessary in the definition of Boolean-like rings. He proved that any ring R with the conditions (i) $(ab)^2 - ab^2 \;\; a^2b + ab = 0$ and (ii) 2a = 0 for all a,b ϵ R is necessarily commutative.

Ketsela Hailu and others [4] have constructed the Boolean-like semi-ring of fractions of a weak commutative Boolean-like semi-ring. We have coined and studied the concept of quasi-weak commutative near-ring in [2]. In this paper we define Boolean-like near ring (right) and prove that every quasi-weak commutative. Boolean-like near ring can be imbedded into a quasi weak commutative semi ring with identity.

2 Preliminaries

In this section we recal some definitions and results which we use in the sequal.

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2.1. Definition

A non empty set R together with two binary operations + and · satisfying the following axioms is called a right near-ring
(i) (R,+) is a group
(ii) · is associative
(iii) · is right distributive w.r.to +
(ie) (a+b) c = a · c + b · c ∀ a,b,c ϵ R

2.2. Note

In a right near-ring R, 0 a = 0 ∀ a ϵ R.
If (R,+) is an abelian group, then (R,+, ·) is called a semi-ring.

2.3. Definition

A right near-ring (R,+, ·) is called a Boolean-like near ring if
(i) 2a = 0 ∀ a ϵ R and
(ii) (a+b-ab)ab = ab ∀ a,b ϵ R

2.4.Remark

If (R,+, ·) is a Boolean-like near ring,then (R,+) is always an abelian group for 2x = 0 ∀ x ϵ R implies x = -x ∀ x ϵ R. We know, a group in which every element is its own inverse is always commutative.

2.5. Definition [5]

A right near ring R is said to be weak commutative if xyz = xzy ∀ x,y,z ϵ R

2.6. Definition [8]

A right near ring R is said to be pseudo commutative if xyz = zyx ∀ x,y,z ϵ R

2.7. Definition [2]

A right near ring R is said to be quasi-weak commutative if xyz = yxz ∀ x,y,z ϵ R

2.8. Definition

Let R be a right near ring. A subset B R is said to be multiplicatively closed if a,b ϵ B implies ab ϵ B.

3.Main results

3.1. Lemma

In a Boolean-like near ring (right) R a · 0 = 0 ∀ a ϵ R

Proof:

Since R is Boolean-like near ring, $(a+b-ab)ab = ab \ \forall\ a,b \in R$
Taking a=0, we get
$(0 + b - 0b)\,0b = 0b$
(ie) $b \cdot 0 = 0$
Thus $a \cdot 0 = 0 \ \forall\ a \in R$.

3.2. Lemma

Let R be a quasi-weak commutative right near ring R. Then $(ab)^n = a^n b^n \ \forall\ a,b \in R$ and for all $n \geq 1$.

Proof:

Let $a,b \in R$.
Then $(ab)^2 = (ab)(ab) = a(bab)$
$= a(abb)$ (quasi weak)
$(ab)^2 = a^2b^2$
Assume $(ab)^m = a^m b^m$
Now $(ab)^{(m+1)} = (ab)^m ab$
$= a^m b^m ab$
$= a^m (ab^m b)$
$= a^{m+1}b^{m+1}$
Thus $(ab)^m = a^m b^m \ \forall\ a,b \in R$ and for all integer $m \geq 1$.

3.3 lemma

Let R be a quasi-weak commutative Boolean like near-ring.Then
$a^2b + ab^2 = ab + (ab)^2 \ \forall\ a,b \in R$.

Proof:

$a^2b + ab^2 = aab + abb$
$= aab + bab$
$= (a + b)ab$
$= (a + b \ \ ab + ab)ab$
$= (a + b \ \ ab)ab + (ab)^2$
$a^2b + ab^2 = ab + (ab)^2$ (R is Boolean-like near-ring)

3.4 Lemma

In a quasi-weak commutative Boolean like near ring (R,+, .),
$(a + a^2)(b + b^2)c = 0 \ \forall\ a,b,c \in R$.

Proof:

$(a + a^2)(b + b^2)c = \{a(b + b^2) + a^2(b + b^2)\}\, c$
$= a(b + b^2)c + a^2(b + b^2)c$

= (b + b^2)ac + (b + b^2) a^2c (R is quasi-weak commutative)
= $\{(b+b^2)a+(b+b^2)a^2\}$ c
=$\{ba+b^2a+ba^2+b^2a^2\}$c
=$\{ba+ba+(ba)^2+b^2a^2\}$ (using Lemma 3.3)
= $\{ba+ba+b^2a^2+b^2a^2\}$ (using Lemma 3.2)
=$\{2ba+2b^2a^2\}$
= 0 (R is Boolean-like near-ring).

3.5 Lemma

In a quasi-weak commutative Boolean like near ring R, (a - a^2) (b - b^2)c = 0 $\forall$ a,b,c ϵ R.

Proof:

(a - a^2)(b - b^2)c =$\{a(b-b^2)-a^2(b-b^2)\}$ c
= a(b - b^2)c - a^2(b - b^2)c
= (b - b^2)ac - (b - b^2) a^2c (quasi-weak commutative)
= $\{(b-b^2)a-(b-b^2)a^2\}$ c
=$\{ba-b^2a-ba^2-b^2a^2\}$c
=$\{ba-ba-(ba)^2-b^2a^2\}$
= $\{ba-ba-b^2a^2-b^2a^2\}$ (using Lemma 3.3)
= 0

3.6 Lemma

Let R be a quasi commutative Boolean like near-ring.Let S be a commutative subset of R which is multiplicatively closed.Define a relation N on R$\times$ S by (r_1 ,s_1) $\sim$ (r_2 ,s_2)if and only if there exists an element s ϵ S such that (r_1s_2 - r_2s_1)s = 0.Then N is an equivalence relation.

Proof:

(i) Let (r,s) ϵ R$\times$ S. Since rs-rs = 0,
we get (rs-rs)t=0 for all t ϵ S.
Hence $\sim$ is reflexive.

(ii) Let (r_1,s_1)$\sim$(r_2,s_2).Then there exists an element sϵS such that
(r_1,s_1-r_2,s_1)s=0. So (r_2,s_1-r_1,s_2)s = 0.
This proves $\sim$ is symmetric.

(iii) Let (r_1,s_1)$\sim$(r_2,s_2) and (r_2,s_2)$\sim$(r_3,s_3).
Then there exists p,qϵ S such that
(r_1s_2-r_2s_1)p=0 and (r_2s_3-r_3s_2)q = 0.
So s_3(r_1s_2-r_2s_1)p=0=s_1(r_2s_3-r_3s_2)q (By Lemma 3.1)
$\Longrightarrow$ (r_1s_2-r_2s_1)s_3p=0=(r_2s_3-r_3s_2)s_1q(R is quasi-weak commutative)
$\Longrightarrow$ (r_1s_2- r_2s_1)$s_3$pq=0=p(r_2s_3-r_3s_2)s_1q
$\Longrightarrow$(r_1s_2-r_2s_1)$s_3$pq=0=(r_2s_3-r_3s_2)ps_1q(R is quasi-weak commutative)
$\Longrightarrow$(r_1s_2-r_2s_1)$s_3$pq=0=(r_2s_3-r_3s_2)$s_1$pq(R is quasi-weak commutative)
$\Longrightarrow$ ($r_1s_2s_3$-$r_2s_1s_3$)pq=0=($r_2s_3s_1$-$r_3s_2s_1$)pq
$\Longrightarrow$ ($r_1s_2s_3$- $r_2s_1s_3$ +$r_2s_3s_1$-$r_3s_2s_1$) pq = 0.

$\Longrightarrow$ (r_1s_3 s_2-$r_2s_1s_3$+$r_2s_1s_3$-$r_3s_1s_2$)pq=0.(S is commutative)
$\Longrightarrow$ (r_1s_3-r_3s_1)$s_2$pq=0
$\Longrightarrow$ (r_1s_3-r_3s_1)r=0 where r = s_2 pqϵ S.
This implies (r_1,s_1)$\sim$(r_3,s_3).
Hence $\sim$ is transitive.
Hence the Lemma.

3.6 Remark

We denote the equivalence class containing (r,s)ϵ R $\times$ S by $\frac{r}{s}$ and the set of all equivalence classes by $S^{-1}R$.

3.8 Lemma

Let R be a quasi weak commutative Boolean like near-ring. Let S be a commutative subset of R which is also multiplicatively closed. If 0$\notin$ S and R has no zero divisors,then
(r_1,s_1)$\sim$ (r_2,s_2) if and only if r_1s_2=r_2s_1.

Proof:

Assume (r_1,s_1)$\sim$ (r_2,s_2).Then there exists an element sϵS such that (r_1s_2-r_2s_1)s=0.
Since 0$\notin$S and R has zero divisors,we get(r_1s_2-r_2s_1)= 0.
(i.e) r_1s_2 = r_2s_1
Conversely assume r_1s_2 = r_2s_1.
Then r_1s_2 - r_2s_1 = 0 and so (r_1s_2-r_2s_1) s = 0 for all sϵS.
Hence (r_1s_1) $\notin$ (r_2s_2).

3.9 Lemma:

Let R be a quasi weak commutative Boolean like near-ring. Let S be a commutative subset of R,which is also multiplicatively closed.
Then (i) $\frac{r}{s}$ = $\frac{rt}{st}$ = $\frac{tr}{st}$= $\frac{tr}{ts}$ for all rϵR and for all s,tϵS.
(ii) $\frac{rs}{s}$ = $\frac{rs'}{s'}$ for all rϵR and for all s,s' ϵ S.
(iii) $\frac{s}{s}$ = $\frac{s'}{s'}$ for all s,s′ ϵ S.
(iv) If 0ϵS,then $S^{-1}R$ contains exactly one element.

Proof:

The proof of (i),(ii) and (iii) are routine.
(iv) Since 0ϵS , (r_1s_2 - r_2s_1)0 = 0 $\forall\frac{r_1}{s_1},\frac{r_2}{s_2}$ ϵ $S^{-1}R$.
and so $\frac{r_1}{s_1}$= $\frac{r_2}{s_2}$.
Then $S^{-1}R$ contains exactly one-element.

3.10 Theorem:

Let R be a quasi weak commutative Boolean like near ring.Let S be a commutative subset of R which is also multiplicatively closed. Define binary operation + and on $S^{-1}R$ as follows :
$\frac{r_1}{s_1}$ +$\frac{r_2}{s_2}$ =$\frac{r_1s_2+r_2s_1}{s_1s_2}$ and

$\frac{r_1}{s_1} \cdot \frac{r_2}{s_2} = \frac{r_1 r_2}{s_1 s_2}$

Then $S^{-1}R$ is a commutative Boolean like semi-ring with identity.

Proof:

Let us first prove that + and · are well defined. Let $\frac{r_1}{s_1} = \frac{r_1'}{r_1'}$ and $\frac{r_2}{s_2} = \frac{r_2'}{r_2'}$ Then there exists t_1, $t_2 \epsilon S$ such that

$(r_1 s_1' - r_1' s_1)t = 0 \ldots\ldots\ldots(1)$

and $(r_2 s_2' - r_2' s_2)t = 0 \ldots\ldots\ldots(2)$

Now$[(r_1 s_2 + r_2 s_1) s_1' s_2' - (r_1' s_2' + r_2' s_1') s_1 s_2] t_1 t_2$

$= [r_1 s_2 s_1' s_2' + r_2 s_1 s_1' s_2' - r_1' s_2' s_1 s_2 - r_2' s_1' s_1 s_2]\ t_1 t_2$

$= [r_1 s_1' s_2 s_2' - r_1' s_1 s_2 s_2' + r_2 s_2' s_1 s_1' - r_2' s_2 s_1 s_1'] t_1 t_2$

$= [(r_1 s_1' - r_1' s_1) s_2 s_2' + (r_2 s_2' - r_2' s_2) s_1 s_1'] t_1 t_2$

$= (r_1 s_1' - r_1' s_1) t_1 s_2 s_2' t_2 + (r_2 s_2' - r_2' s_2) t_2 s_1 s_1' t_1$

$= 0 \cdot s_2 s_2' t_2 + 0 \cdot s_1 s_1' t_1$

$= 0$

Hence $\frac{r_1 s_2 + r_2 s_1}{s_1 s_2} = \frac{r_1' s_2' + r_2' s_1'}{s_1' s_2'}$

(i.e) $\frac{r_1}{s_1} + \frac{r_2}{s_2} = \frac{r_1'}{s_1'} + \frac{r_2'}{s_2'}$

Hence + is well defined.

From (1) we get

$(r_1 s_1' - r_1' s_1) t_1 t_2 r_2 s_2' = 0$

$t_1 t_2 (r_1 s_1' - r_1' s_1) r_2 s_2' = 0$ (quasi weak commutative)

$t_1 t_2 (r_1 s_1' r_2 - r_1' s_1 r_2) s_2' = 0$

$(r_1 s_1' r_2 - r_1' s_1 r_2) s_2' t_1 t_2 = 0$ (S is commutative subset)

$(r_1 s_1' r_2 s_2' - r_1' s_1 r_2 s_2') t_1 t_2 = 0$

$(r_1 r_2 s_1' s_2' - r_1' r_2 s_1 s_2') t_1 t_2 = 0$

$r_1 r_2 s_1' s_2' t_1 t_2 - r_1' r_2 s_1 s_2' t_1 t_2 = 0 \ldots\ldots\ldots(3)$

From (2) we get

$(r_2 s_2' - r_2' s_2) t_2 t_1 r_1' s_1 = 0$

$(r_2 s_2' - r_2' s_2) t_1 t_2 r_1' s_1 = 0$ (S is commutative subset)

$t_1 t_2 (r_2 s_2' - r_2' s_2) r_1' s_1 = 0$ (quasi weak commutative)

$t_1 t_2 (r_2 s_2' r_1' - r_2' s_2 r_1') s_1 = 0$

$(r_2 s_2' r_1' - r_2' s_2 r_1') t_1 t_2 s_1 = 0$ (quasi weak commutative)

$(r_2 s_2' r_1' - r_2' s_2 r_1') s_1 t_1 t_2 = 0$ (S is commutative subset)

$(r_2 s_2' r_1' s_1 - r_2' s_2 r_1' s_1) t_1 t_2 = 0$

$(r_2 r_1' s_2' s_1 - r_2' r_1' s_2 s_1) t_1 t_2 = 0$ (quasi weak commutative)

$(r_1' r_2 s_2' s_1 - r_1' r_2' s_2 s_1) t_1 t_2 = 0$ (quasi weak commutative)

$r_1' r_2 s_1 s_2' t_1 t_2 - r_1' r_2' s_1 s_2 t_1 t_2 = 0$(S is commutative subset)$\ldots\ldots\ldots(4)$

(3) + (4) gives

$r_1 r_2 s_1' s_2' t_1 t_2 - r_1' r_2' s_1 s_2 t_1 t_2 = 0$

$(r_1 r_2 s_1' s_2' - r_1' r_2' s_1 s_2) t_1 t_2 = 0$

This means $\frac{r_1 r_2}{s_1 s_2} = \frac{r_1' r_2'}{s_1' s_2'}$

Hence is well-defined.

We note that $\frac{r_1}{s_1} + \frac{r_2}{s_2} = \frac{r_1 s_2 + r_2 s_1}{s_1 s_2} = \frac{(r_1 + r_3)s}{s^2}$

$= \frac{r_1 + r_2}{s}$ (by lemma 3.9)$\ldots\ldots\ldots(5)$

Claim:1$(S^{-1}R, +)$is an abelian group.

Let $\frac{r_1}{s_1}, \frac{r_2}{s_2}, \frac{r_3}{s_3} \epsilon S^{-1}R$.

Then

$\frac{r_1}{s_1}+(\frac{r_2}{s_2}+\frac{r_3}{s_3})=\frac{r_1}{s_1}+(\frac{r_2s_3+r_3s_2}{s_2s_3})$
$=\frac{r_1s_2s_3+(r_2s_3+r_3s_2)s_1}{s_1s_2s_3}$
$=\frac{r_1s_2s_3+r_2s_3s_1+r_3s_2s_1}{s_1s_2s_3}$

Also $(\frac{r_1}{s_1}+\frac{r_2}{s_2})+\frac{r_3}{s_3}=(\frac{r_1s_2+r_2s_1}{s_1s_2})+\frac{r_3}{s_3}$
$=\frac{(r_1s_2+r_2s_1)s_3+r_3s_1s_2}{s_1s_2s_3}$
$=\frac{r1s_2s_3+r_2s_3s_1+r_3s_1s_2}{s_1s_2s_3}$

$\frac{r_1}{s_1}+(\frac{r_2}{s_2}+\frac{r_3}{s_3})=(\frac{r_1}{s_1}+\frac{r_2}{s_2})+\frac{r_3}{s_3}$

So + is associative.

For any $\frac{r}{s}\epsilon$ R,we have

$\frac{r}{s}+\frac{0}{s}=\frac{r+0}{s}=\frac{r}{s}$

Also $\frac{0}{s}+\frac{r}{s}=\frac{0+r}{s}=\frac{r}{s}$

Hence $\frac{0}{s}$ is the additive identity of $\frac{r}{s}\ \epsilon S^{-1}R$ for all r ϵ R

Clearly + is commutative.

Thus (R,+) is an abelian group.

Claim:2 $\cdot$ is associative.

Now $\frac{r_1}{s_1}\cdot(\frac{r_2}{s_2}\cdot\frac{r_3}{s_3})=\frac{r_1}{s_1}\cdot(\frac{r_2r_3}{s_2s_3})=\frac{r_1(r_2r_3)}{s_1(s_2s_3)}$
$=\frac{(r_1r_2)r_3}{(s_1s_2)s_3}$
$=(\frac{r_1}{s_1}\cdot\frac{r_2}{s_2})\cdot\frac{r_3}{s_3}$

So $\cdot$ is associative.

Claim:3 $\cdot$ is right distributive with respect to +.

Let $\frac{r_1}{s_1},\frac{r_2}{s_2},\frac{r_3}{s_3}\ \epsilon\ S^{-1}R$.

Now $(\frac{r_1}{s_1}+\frac{r_2}{s_2})\cdot\frac{r_3}{s_3}=(\frac{r_1s_2+r_2s_1}{s_1s_2})\cdot\frac{r_3}{s_3}$
$=\frac{r_1s_2r_3+r_2s_1r_3}{s_1s_2s_3}$
$=\frac{s_2r_1r_3+s_1r_2r_3}{s_1s_2s_3}$ (quasi weak commutative)
$=\frac{s_2r_1r_3}{s_1s_2s_3}+\frac{s_1r_2r_3}{s_1s_2s_3}$ (using (5))
$=\frac{s_2r_1r_3}{s_2s_1s_3}+\frac{s_1r_2r_3}{s_1s_2s_3}$
$=\frac{r_1r_3}{s_1s_3}+\frac{r_2r_3}{s_2s_3}$
$=\frac{r_1}{s_1}\cdot\frac{r_3}{s_3}+\frac{r_2}{s_2}\cdot\frac{r_3}{s_3}$

This proves right - distributive law.

Claim:4 $S^{-1}R$ is a Boolean-like ring.

It is already proved in **claim 1** that

$2(\frac{r}{s})=0$ for all $\frac{r}{s}\ \epsilon S^{-1}R$

Let a = $\frac{r_1}{s_1}$ and b = $\frac{r_2}{s_2}$ be any two elements of $S^{-1}R$ Let t ϵ S be any element.

Now by Lemma 3.5

$(a - a^2)(b - b^2)$ t = 0

$\Rightarrow (\frac{r_1}{s_1}-\frac{r_1^2}{s_1^2})(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2})t = 0$

$[\frac{r_1}{s_1}(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2})-\frac{r_1^2}{s_1^2}(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2})]t = 0$

$\frac{r_1}{s_1}(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2})t-\frac{r_1^2}{s_1^2}(\frac{r_2}{s_2}\frac{r_2^2}{s_2^2})t = 0$

$(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2})\frac{r_1}{s_1}t-(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2})\frac{r_1^2}{s_1^2}t=0$(quasi weak commutative)

$[(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2})\frac{r_1}{s_1}-(\frac{r_2}{s_2}-\frac{r_2^2}{s_2^2})\frac{r_1^2}{s_1^2}]t=0$

$[(\frac{r_2s_2-r_2^2}{s_2^2})\frac{r_1}{s_1}-(\frac{r_2s_2-r_2^2}{s_2^2})\frac{r_1^2}{s_1^2}]t=0$

$[(\frac{r_2s_2-r_2^2}{s_2^2})\frac{r_1s_1}{s_1^2}-(\frac{r_2s_2-r_2^2}{s_2^2})\frac{r_1^2}{s_1^2}]t=0$ (using Lemma 3.9)

$[(\frac{r_2s_2r_1s_1-r_2^2r_1s_1}{s_2^2s_1^2})-\frac{r_2s_2r_1^2-r_2^2r_1^2}{s_2^2s_1^2}]t=0$

$[(\frac{r_2r_1s_2s_1-r_2^2r_1s_1}{s_2^2s_1^2})$-$\frac{s_2r_2r_1^2-r_2^2r_1^2}{s_2^2s_1^2}]$t=0(quasi weak commutative)

$[(\frac{r_2r_1s_2s_1}{s_2^2s_1^2}$-$\frac{r_2^2r_1s_1}{s_2^2s_1^2}$ -$\frac{s_2r_2r_1^2}{s_2^2s_1^2}+\frac{r_2^2r_1^2}{s_2^2s_1^2}]$t=0

$[\frac{r_2r_1}{s_2s_1}$-$\frac{r_2^2}{s_2^2}\frac{r_1}{s_1}$-$\frac{r_2}{s_2}\frac{r_1^2}{s_1^2}+\frac{r_2^2r_1^2}{s_2^2s_1^2}]$t=0

$[ba-b^2a-ba^2+b^2a^2]$t=0

⇒ba = b^2a - ba^2 + b^2a^2

= b^2a + ba^2 - $(ba)^2$ (using Lemma 3.2)

ba = ba(b+a-ba)

This proves $S^{-1}R$ is Boolean-like near ring.

Claim :5 multiplication in $S^{-1}R$ is commutative

Let $\frac{r_1}{s_1},\frac{r_2}{s_2}$ be any two elements of $S^{-1}R$.

Then $\frac{r_1}{s_1}\cdot\frac{r_2}{s_2}=\frac{r_1r_2}{s_1s_2}=\frac{r_1r_2s}{s_1s_2s}$ ∀ sϵS $\frac{r_2r_1s}{s_1s_2s}$(quasi weak commutative)

$=\frac{r_2r_1s}{s_2s_1s}$(S is commutative subset)

$=\frac{r_2}{s_2}\frac{r_1}{s_1}$(using Lemma 3.9)

Hence multiplication in $S^{-1}R$ is commutative.

Claim:6 Existence of multiplicative identity in $S^{-1}R$

Let $\frac{r}{s}S^{-1}R$ be any element.

Then $\frac{r}{s}\cdot\frac{s}{s}=\frac{rs}{ss}=\frac{r}{s}$

Also $\frac{s}{s}\cdot\frac{r}{s}=\frac{sr}{ss}=\frac{r}{s}$

Hence $\frac{s}{s}$ ϵ S^{-1} R is the multiplicative identity of $S^{-1}R$

Thus $S^{-1}R$ is a commutative Boolean-like near-ring with identity.

3.11 Theorem

$S^{-1}R$ is quasi-weak commutative near-ring.

Proof:

Let a = $\frac{r_1}{s_1}$, b = $\frac{r_2}{s_2}$, c = $\frac{r_3}{s_3}$be any three elements of $S^{-1}R$

Now abc = $\frac{r_1}{s_1}\cdot\frac{r_2}{s_2}\cdot\frac{r_3}{s_3}=\frac{r_1r_2r_3}{s_1s_2s_3}$

$=\frac{r_2r_1r_3}{s_1s_2s_3}$(R is quasi-weak commutative)

$=\frac{r_2r_1r_3}{s_2s_1s_3}$ (S is commutative)

$=\frac{r_2}{s_2}\frac{r_1}{s_1}\frac{r_3}{s_3}$

Then abc = bac ∀a,b,c ϵ $S^{-1}R$.

This proves $S^{-1}R$ is quasi-weak commutative near-ring.

3.12 Theorem

Let R be a quasi-weak commutative Boolean-like near ring.Let S be a commutative subset of R which is multiplicatively closed. Let 0≠ s ϵS. Define a map f_s: R→ S^{-1} R as $f_s(r)=\frac{rs}{s}$ ∀ rϵ R. Then f_s is a near-ring monomorphism.

Proof:

Let r_1,r_2 ϵ R.

Then f_s $(r_1+r_2)=\frac{(r_1+r_2)s}{s}=\frac{r_1s+r_2s}{s}$

$=\frac{r_1s}{s}+\frac{r_2s}{s}$(By (5) of Theorem 3.11)

= $f(r_1)$ + $f(r_2)$

Also f_s $(r_1\cdot r_2)$ = $\frac{(r_1r_2)s}{s}$

$=\frac{r_1r_2s^2}{s^2}$

$=\frac{r_1r_2ss}{s^2}$

$=\frac{r_1(sr_2s)}{s^2}$
$=\frac{r_1s}{s} \cdot \frac{r_2s}{s}$(quasi weak commutative)
$=f_s(r_1) \cdot f_s(r_2)$
Also $f_s(r_1) = f_s(r_2) \Rightarrow \frac{r_1s}{s} = \frac{r_2s}{s}$
$\Rightarrow \frac{r_1s}{s} - \frac{r_2s}{s} = 0$
$\Rightarrow \frac{(r_1s - r_2s)}{s} = 0$
$\Rightarrow \frac{(r_1 - r_2)s}{s} = 0$
$\Rightarrow (\frac{r_1}{s} - \frac{r_2}{s}) = 0$
$\Rightarrow \frac{r_1}{s} = \frac{r_2}{s}$
Hence f_s is a monomorphism

3.13 Theorem

Let R be a quasi-weak commutative Boolean-like near-ring. Then R be embedded into a quasi-weak commutative. Boolean like commutative semi ring with identity.
Proof:
Follows from Theorem 3.11 and 3.12.

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