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Effect of Magnetic field on Herschel-Bulkley fluid through multiple stenoses

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Abstract

A mathematical model for electrically conducting flow of Herschel-Bulkley fluid through a uniform tube of multiple stenoses has been studied. Analytical solutions of resistance to the flow and wall shear stress have been calculated. It is found that the resistance to the flow increases with the heights of the stenoses, power law index, volumetric flow rate, radius of the plug core-region and yield stress, but decreases with induced magnetic field and shear stress. It is also observed that the wall shear stress is increasing with the heights of the stenoses and radius of the plug core-region.

Keywords: Multiple stenoses, Herschel-Bulkley fluid, Magnetic field.

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1 Introduction

Diseases in the blood vessels and in the heart, such as heart attacks and strokes, are the major mortality worldwide. The underlying cause for these events is the formation of lesions, known as atherosclerosis. These lesions and plaques can grow and occlude the artery and hence prevent blood supply to the distal bed. Plaques with calcium in them can also rupture and initiate formation of blood clots (thrombus). The clots can form as emboli and occlude the smaller vessels that can also result in interruption of blood supply to the distal bed. Plaques formed in coronary arteries can lead to heart attacks and clots in the risk factors for the presence of atherosclerotic lesions.

Hence the formation of stenosis/ atherosclerosis is found to be largely responsible for the cause of several vascular diseases. Thus a proper knowledge of the flow characteristics of blood in such blood vessels may lead to better understanding of the development of these diseases. This in turn may help in proper diagnosis of such diseases and design and development of improvised artificial organs.

In view of this, a number of researchers have studied different aspects of blood flow analysis in arteries. Young [1], Lee and Fung [2],Padmanabhan [3] have studied the flow of blood in stenosed artery by considering blood as a Newtonian fluid. Blood behaves cerebral circulation can result in a stroke. There are number of differently when flowing in large vessels, in which Newtonian behavior is expected and in small vessels where non-Newtonian effects appear Buchanan *et al.*[4], Mandal[5],Ismail *et al.* [6], Radhakrishnamacharya [7]. In small vessels blood behaves like a Herschel-Bulkley fluid rather than Power law and Bingham fluids

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Chaturani and Samy [8]. But the blood can be regarded as magnetic fluid in which red blood cells are magnetic in nature. Liquid carries in the blood contain magnetic suspension of the particle Tzirtzilakis [9].

The MHD principles may be used to de accelerate the flow of blood in a human arterial system and is useful in the treatment of certain cardiovascular disorders and in the diseases which accelerate blood circulation like hemorrhage and hypertension etc. Das and Saha [10].

The effect of magnetic field on blood flow has been analyzed by treating blood as an electrically conductive fluid Chen [11]. Ogulua and Abbey [12] studied the effects of heat and magnetic field on blood through constricted artery. Shaw *et al.* [13] have shown the influence of the externally imposed body acceleration on the flow of blood through an asymmetric stenosed artery by considering blood as Casson fluid. Bali and Awasti [14] Studied the effect of an externally applied uniform magnetic field on the multi-stenosed artery. Sankar and Lee [15] have shown the effect of magnetic field in the pulsatile flow of blood through narrow arteries treating blood as Casson fluid. Recently Lokendra Pramar *et al.* [16] studied the role of magnetic field intensity through overlapping stenosis. Bhargva *et al.* [17] Showed that the magnetic field can be used as a flow control mechanism in medical applications.

With this motivation, a mathematical model on the effect of magnetic field on Herschel-Bulkley fluid through a uniform tube with two stenoses is developed. Expressions for the velocity, resistance to the flow and wall shear stress have been calculated by assuming that the stenosis to be mild. The effects of various parameters on these variables have been investigated

2 Mathematical Formulation

Consider the steady flow of an electrically conducting Herschel Bulkley fluid through a tube of uniform cross section with two stenoses. Assuming that the flow is axi-symmetric and the stenosis over a length of the artery have been developed in axi-symmetric manner. Let the length of the tube is L, the magnitude of the distance along the artery over which the stenosis is spread out be L_i, the locations of the stenosis be indicated by d_i and the maximum heights of the stenosis δ_i (where i=1,2). Here we consider the transverse magnetic field since the bio-magnetic fluid (blood) is subjected to a magnetic field. The schematic diagram is shown in Figure -1. The cylindrical polar coordinates (z,r,θ) is chosen so that the z -axis coincides with the axis of the tube

The radius of the cylindrical tube is given as

$$\overline{h} = \frac{\overline{R}(z)}{\overline{R}_0} = \begin{cases} 1 & 0 \le \overline{z} \le d_1 \\ 1 - \frac{\overline{\delta}_1}{2}\left(1 + Cos\frac{2\pi}{L_1}\left(\overline{z} - d_1 - \frac{L_1}{2}\right)\right) & d_1 \le \overline{z} \le d_1 + L_1 \\ 1 & d_1 + L_1 \le \overline{z} \le d_2 \\ 1 - \frac{\overline{\delta}_2}{2}\left(1 + Cos\frac{2\pi}{L_2}\left(\overline{z} - d_2 - \frac{L_2}{2}\right)\right) & d_2 \le \overline{z} \le d_2 + L_2 \\ 1 & d_2 + L_2 \le \overline{z} \le L \end{cases} \tag{2.1}$$

Where R(z) is the radius of the tube with stenosis ,$R_0(z)$ is the radius of the tube without stenosis, r_o is the radius of the plug flow region.
The basic momentum equation governing the flow is (Rekha Bali *et al.* [13])

$$-\frac{\partial \overline{p}}{\partial \overline{z}} + \frac{1}{\overline{r}}\frac{\partial}{\partial \overline{r}}(\overline{r}\overline{\tau}_{rz}) + \overline{\mu_0}\overline{M}(\frac{\partial \overline{H}}{\partial \overline{z}}) = 0 \tag{2.2}$$

Where $\overline{\tau}_{rz}$ is the shear stress for Herschel Bulkley fluid, is given by

$$\overline{\tau}_{rz} = (\frac{-\partial \overline{u}}{\partial \overline{r}})^n + \overline{\tau}_0, \text{ if } \overline{\tau}_{rz} \ge \overline{\tau}_0 \tag{2.3}$$

$$\frac{\partial \overline{u}}{\partial \overline{r}} = 0, \, if \, \overline{\tau}_{rz} < \overline{\tau}_0 \tag{2.4}$$

where $\overline{r}, \overline{z}$ denote the radial and axial coordinates respectively, $\overline{\mu_0}$ magnetic permeability, $\overline{M}$ magnetization,

$\overline{H}$ magnetic field intensity, $\overline{p}$ pressure,$\overline{u}$ is the velocity of the fluid, $\overline{\tau}_{rz}$ stress,$\overline{\tau}_0$ yield stress.

When $\overline{\tau}_{rz} < \overline{\tau}_0$ i.e. the shear stress is less than yield stress, there is a core region which flows as a plug (FIG.1), and Eq. (2.4) corresponds to vanishing velocity gradient in that region. However the fluid behavior is indicated whenever $\overline{\tau}_{rz} > \overline{\tau}_0$.
The boundary conditions are

$$\overline{\tau}_{rz}\, is\, finite\, at\, \overline{r} = 0 \tag{2.5}$$

$$\overline{u} = 0,\, at\, \overline{r} = \overline{h}(z) \tag{2.6}$$

Introducing the following non-dimensional quantities

$$\overline{z} = \frac{z}{L}, \overline{\delta} = \frac{\delta}{R_0}, \overline{R}(z) = \frac{R(z)}{R_0}, \overline{P} = \frac{P}{\mu UL/R_0^2}, \overline{\tau_0} = \frac{\tau_0}{\mu(U/R_0)}, \overline{\tau}_{rz} = \frac{\tau_{rz}}{\mu(U/R_0)}, \overline{Q} = \frac{Q}{(\pi R_0^2 U)}, \overline{H} = \frac{H}{H_0} \tag{2.1}$$

Where H_0 is external transerverse uniform constant magnetic field.
Using the non-dimensional scheme the governing equations from (2.1)-(2.6) can be written as
The radius of the cylindrical tube is given as

$$\overline{h} = \frac{R(z)}{R_0} = \begin{cases} 1 & 0 \le z \ \le d_1 \\ 1 - \frac{\delta_1}{2}\left(1 + Cos\frac{2\pi}{L_1}\left(z - d_1 - \frac{L_1}{2}\right)\right) & d_1 \le z \le d_1 + L_1 \\ 1 & d_1 + L_1 \le z \le d_2 \\ 1 - \frac{\delta_2}{2}\left(1 + Cos\frac{2\pi}{L_2}\left(z - d_2 - \frac{L_2}{2}\right)\right) & d_2 \le z \le d_2 + L_2 \\ 1 & d_2 + L_2 \le z \le L \end{cases} \tag{2.7}$$

$$-\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \mu_0 M(\frac{\partial H}{\partial z}) = 0 \tag{2.8}$$

Where

$$\tau_{rz} = (\frac{-\partial u}{\partial r})^n + \tau_0,\ if\ \tau_{rz} \ge \tau_0$$

$$\frac{\partial u}{\partial r} = 0,\ if\ \tau_{rz} < \tau_0 \tag{2.9}$$

The following restrictions for mild stenoses (MARUTHI PRASAD et al. [7]), are supposed to be satisfied.

$$\delta_i \ll min(R_0, R_out)$$

$$\delta_i \ll L_i,$$

where R_out=R(z) at $z = L$
Here L_i,δ_i (i=1, 2) are the lengths and maximum heights of the two stenoses. (The suffixes 1 and 2 refer to the first and second stenosis respectively).
The boundary conditions (2.5) and (2.6) becomes

$$\tau_{rz} is finite at r = 0 \tag{2.10}$$

$$u = 0, at r = h(z) \tag{2.11}$$

3 Solution

The solution of equation (2.8) under the boundary conditions (2.10) and (2.11), the velocity is obtained as

$$U = \frac{h^{k+1}(p-F)^k}{2^k(k+1)}\{(1-\frac{2\tau_0}{h(p-f)})^{k+1} - (\frac{r}{h} - \frac{2\tau_0}{h(p-f)})^{k+1}\},\ for\ \ r_0 \le r \le h \tag{3.1}$$

Where $\frac{\partial p}{\partial z} = P,\ f_1(\frac{\partial H}{\partial z}) = F,\ k = \frac{1}{n}$ Using the boundary condition (2.9), the upper limit of the plug flow region (i.e. the region $0 \le r \le r_0$) for which $\tau_r z < \tau_0$ is obtained as

$$r_0 = \frac{2\tau_0}{(P-F)} \tag{3.2}$$

Using the condition that $\tau_r z = \tau_h$, at r=h ,

$$\frac{r_0}{h} = \frac{\tau_0}{\tau_h} = \tau,\ \ for\ 0 < \tau < 1 \tag{3.3}$$

Taking r=r_0 in Eq. (3.1), the plug core velocity

$$u_p = \frac{h^{k+1}(P-F)^k}{2^k(k+1)}(1-\frac{r_0}{h})^{k+1},\ for\, 0 \le r \le r_0 \tag{3.4}$$

The volume flow rate is defined by

$$Q = 2[\int_0^{r_0} u_p r d_r + \int_{r_0}^{h} u r d_r] \tag{3.5}$$

On integrating,

$$Q = A((k+2)(k+3)(1-\frac{r_0}{h})^{k+1} - 2(k+3)(1-\frac{r_0}{h})^{k+2} + 2(1-\frac{r_0}{h})^{k+3}) \tag{3.6}$$

where $A = \frac{h^{(k+3)}(P-F)^k}{2^k(k+1)(k+2)(k+3)}$

$$From Eq.(3.6), P - F = \frac{2Q^{\frac{1}{k}}[(k+1)(k+2)(k+3)]^{\frac{1}{k}}}{h^{1+\frac{3}{k}}\{(k+2)(k+3)(1-\frac{r_0}{h})^{k+1} - 2(k+3)(1-\frac{r_0}{h})^{k+2} + 2(1-\frac{r_0}{h})^{k+3}\}^{\frac{1}{k}}} \tag{3.7}$$

$$\frac{dp}{dz} = \frac{2Q^{\frac{1}{k}}[(k+1)(k+2)(k+3)]^{\frac{1}{k}}}{h^{1+\frac{3}{k}}\{(k+2)(k+3)(1-\frac{r_0}{h})^{k+1} - 2(k+3)(1-\frac{r_0}{h})^{k+2} + 2(1-\frac{r_0}{h})^{k+3}\}^{\frac{1}{k}}} + F \tag{3.8}$$

When k=1,H=0 and $\tau_0 \to 0$ Eq. (3.8) reduces to the results of YOUNG [1] .
The pressure drop Δp across the stenosis between z=0 to z=1 is obtained by integrating Eq. (3.8), as

$$\Delta p = \int_0^1 (\frac{2Q^{\frac{1}{k}}[(k+1)(k+2)(k+3)]^{\frac{1}{k}}}{h^{1+\frac{3}{k}}\{(k+2)(k+3)(1-\frac{r_0}{h})^{k+1} - 2(k+3)(1-\frac{r_0}{h})^{k+2} + 2(1-\frac{r_0}{h})^{k+3}\}^{\frac{1}{k}}} + F)dz \tag{3.9}$$

The resistance to the flow,λ, is defined by

$$\lambda = \frac{\Delta p}{Q} = \frac{1}{Q}\int_0^1 (\frac{2Q^{\frac{1}{k}}[(k+1)(k+2)(k+3)]^{\frac{1}{k}}}{h^{1+\frac{3}{k}}\{(k+2)(k+3)(1-\frac{r_0}{h})^{k+1} - 2(k+3)(1-\frac{r_0}{h})^{k+2} + 2(1-\frac{r_0}{h})^{k+3}\}^{\frac{1}{k}}} + F)dz \tag{3.10}$$

the pressure drop in the absence of stenosis (h=1) is denoted by ΔP_N, is obtained from Eq. (3.9).

$$\Delta P_N = \int_0^1 (\frac{2Q^{\frac{1}{k}}[(k+1)(k+2)(k+3)]^{\frac{1}{k}}}{\{(k+2)(k+3)(1-r_0)^{k+1} - 2(k+3)(1-r_0)^{k+2} + 2(1-r_0)^{k+3}\}^{\frac{1}{k}}} + F)dz \tag{3.11}$$

The resistance to the flow in the absence of stenosis is denoted by λ_N is obtained from Eq. (3.10) as

$$\lambda_N = \frac{\Delta P_N}{Q} = \frac{1}{Q}\int_0^1 \Big(\frac{2Q^{\frac{1}{k}}\left[(k+1)(k+2)(k+3)\right]^{\frac{1}{k}}}{\left\{(k+2)(k+3)(1-r_0)^{k+1} - 2(k+3)(1-r_0)^{k+2} + 2(1-r_0)^{k+3}\right\}^{\frac{1}{k}}} + F\Big) dz \qquad (3.12)$$

The normalized resistance to the flow denoted by

$$\overline{\lambda} = \frac{\lambda}{\lambda_N} \qquad (3.13)$$

And the wall shear stress

$$\tau_h = \frac{h}{2}\frac{dp}{dz} \qquad (3.14)$$

4 Results

The expressions for velocity (u), core velocity(u_p), volumetric flow rate (Q) ,resistance to the flow $(\overline{\lambda})$ and wall shear stress$(\tau_h)$are given by the equations (3.1,3.4,3.6,3.13,3.14). The effects of various parameters on the resistance to the flow $((\overline{\lambda}))$, wall shear stress (τ_h) have been computed numerically by using mathematica 8.1 and results are shown graphically in Fig.2-14, by taking d_1=0.2,d_2=0.6,L_1=L_2=0.2,L=1.

It is observed that the resistance to the flow increases with the heights of the stenosis (δ_1,δ_2) (fig.2-10).It can be seen from the fig 2-3 that, the resistance to the flow increases with the power law index (k=1/n) along with the heights of the primary and secondary stenosis (δ_1,δ_2).It is interesting to note that the increase in resistance is significant only when the height of the second stenosis exceeds the value 0.02.

From, Fig.7 & 8 it is observed that the resistance to the flow increases with volumetric flow rate (Q), radius of plug core region (r_0) (Fig.9 & 10) and yield stress (τ_0) (Fig.11).

It is interesting to observed that the resistance to the flow decreases with the increase of the magnetic field (**H**) (Figs.4 & 5), and it is also seen that resistance to the flow is more in non-Newtonian fluid than the Newtonian fluid (Fig.6).

The effects various parameters on shear stress are shown in (Figs. 12-14). It is noted that the wall shear stress is increasing with the heights of the stenoses and the radius of the plug-core region.

5 Conclusion

A mathematical model for electrically conducting flow of Herschel-Bulkley fluid through a uniform tube of multiple stenoses has been studied. It is observed that the resistance to the flow increases with the heights of the stenoses, power law index, volumetric flow rate, radius of the plug core-region and yield stress, but decreases with induced magnetic field and shear stress. It is also observed that the wall shear stress is increasing with the heights of the stenoses and radius of the plug core-region.

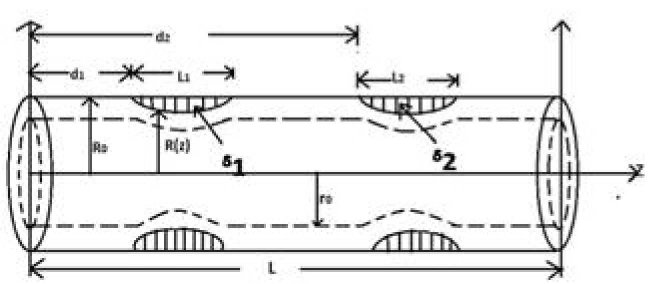

Figure 1: Schematic diagram of multiple stenosed artery

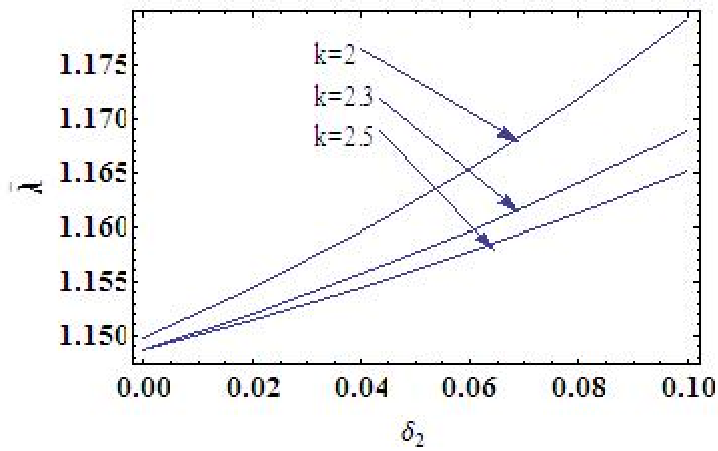

Figure 2: Variation of impedance $\overline{\lambda}$ with δ_2 for different k (d_1=0.2,d_2=0.6,H=0.2,L_1=L_2=0.2,L=1,Q=0.1,δ_1=0.0,r_0=0.2)

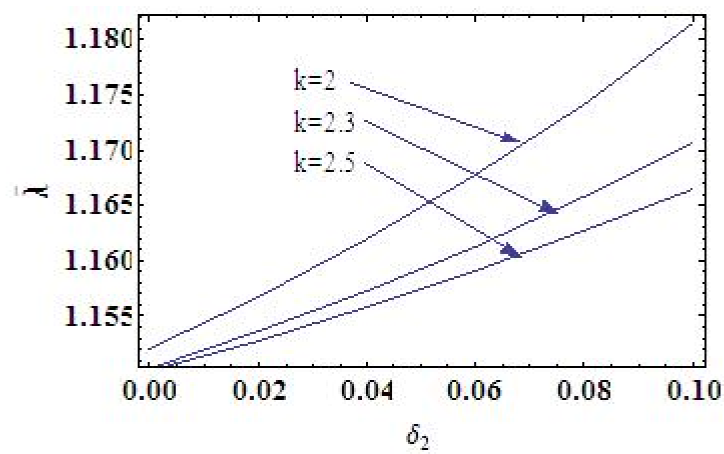

Figure 3: Variation of impedance $\overline{\lambda}$ with δ_2 for different k (d_1=0.2,d_2=0.6,H=0.2,L_1=L_2=0.2,L=1,Q=0.1,δ_1=0.01,r_0=0.2)

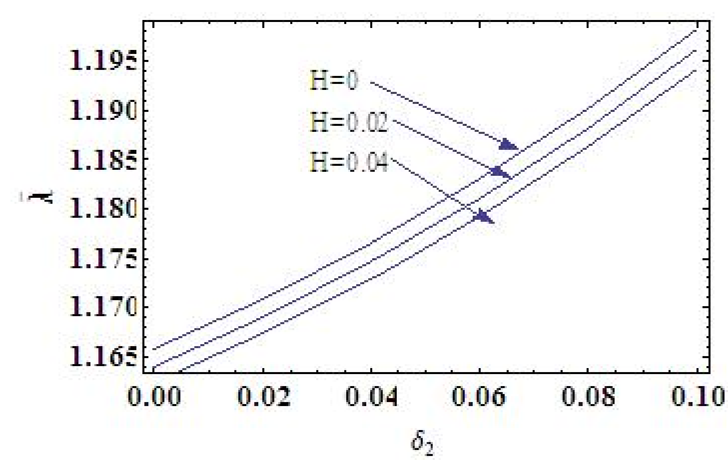

Figure 4: Variation of impedance $\overline{\lambda}$ with δ_2 for different H (d_1=0.2,d_2=0.6,H=0.2,L_1=L_2=0.2,L=1,Q=0.1,k=2,δ_1=0.0,r_0=0.2)

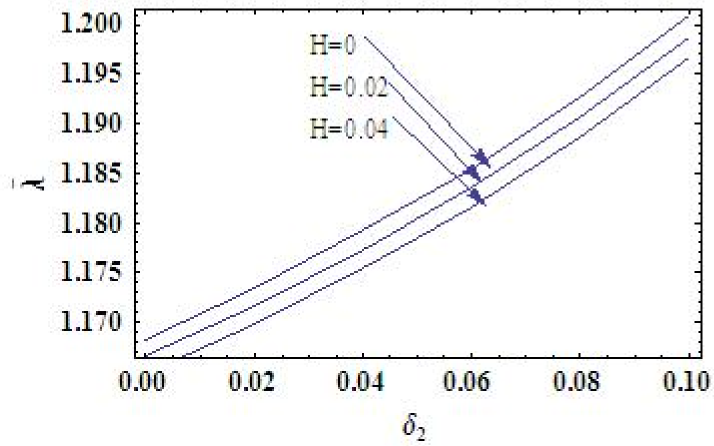

Figure 5: Variation of impedance $\overline{\lambda}$ with δ_2 for different H (d_1=0.2,d_2=0.6,H=0.2,L_1=L_2=0.2,L=1,Q=0.1,k=2,δ_1=0.01,r_0=0.2)

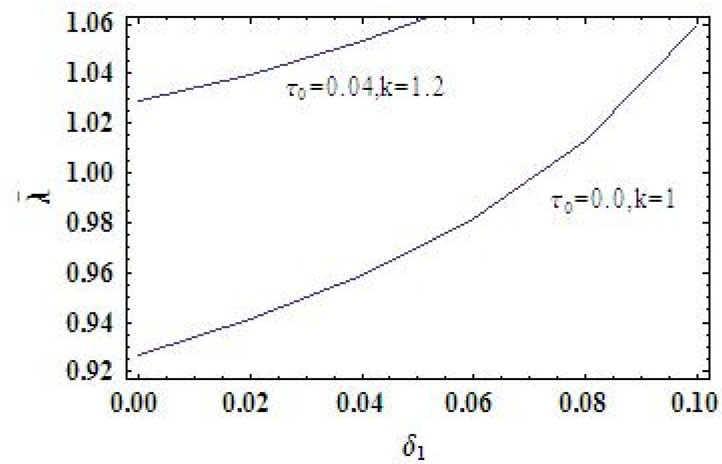

Figure 6: Comparison of magnetic field effect on Newtonian and non-Newtonian fluids. (d_1=0.2,d_2=0.6,L_1=L_2=0.2,L=1,Q=0.1,H=0.2,δ_1=0.01)

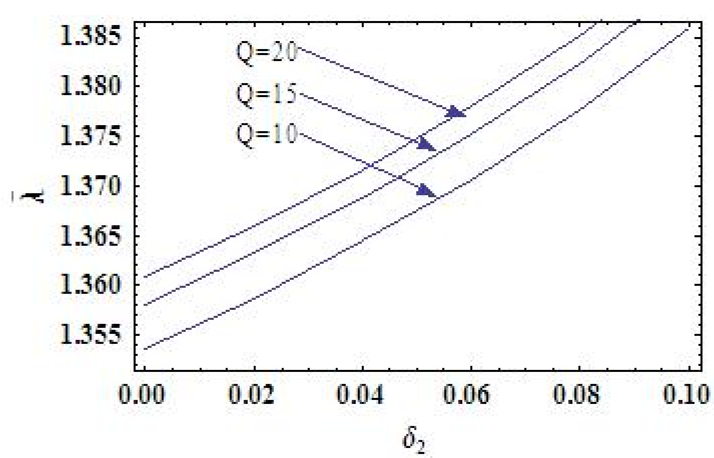

Figure 7: Variation of impedance $\overline{\lambda}$ with δ_2 for different Q (d_1=0.2,d_2=0.6,H=0.2,L_1=L_2=0.2,L=1,k=2,r_0=0.2,δ_1=0.0)

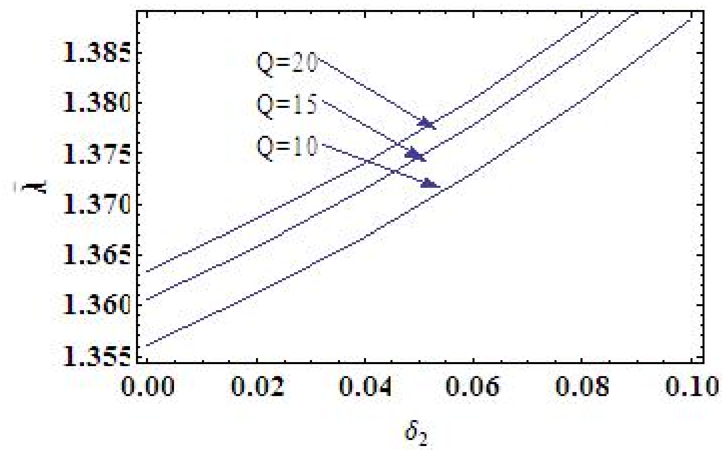

Figure 8: Variation of impedance $\overline{\lambda}$ with δ_2 for different Q (d_1=0.2,d_2=0.6,H=0.2,L_1=L_2=0.2,L=1,k=2,r_0=0.2,δ_1=0.01)

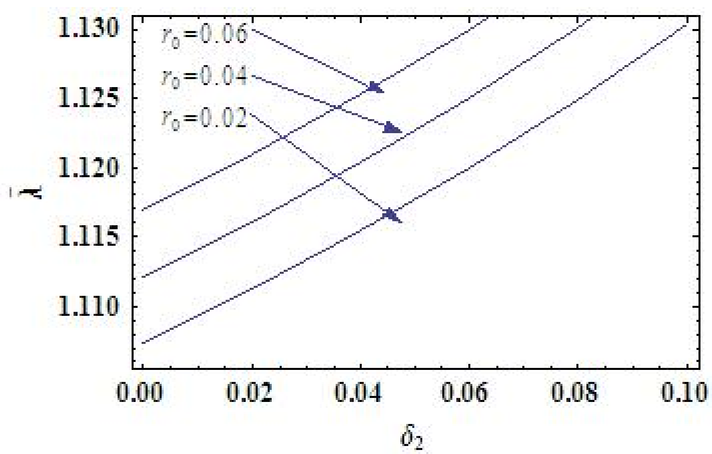

Figure 9: Variation of impedance $\overline{\lambda}$ with δ_2 for different r_0 (d_1=0.2,d_2=0.6,H=0.2,L_1=L_2=0.2,L=1,k=2,δ_1=0.0,Q=0.1)

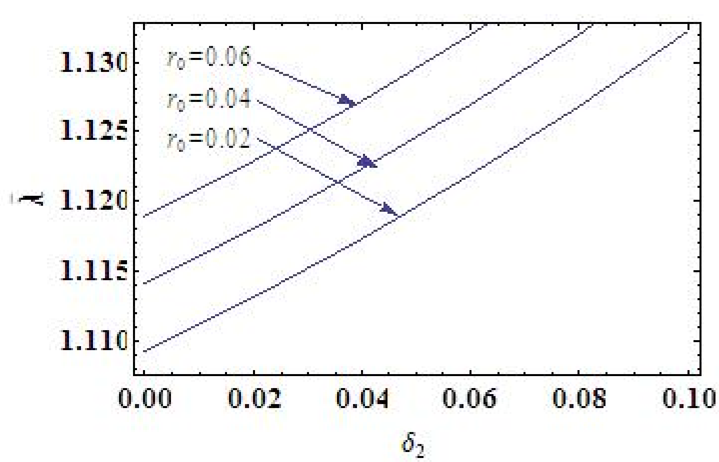

Figure 10: Variation of impedance $\overline{\lambda}$ with δ_2 for different r_0 (d_1=0.2,d_2=0.6,H=0.2,L_1=L_2=0.2,L=1,k=2,δ_1=0.01,Q=0.1)

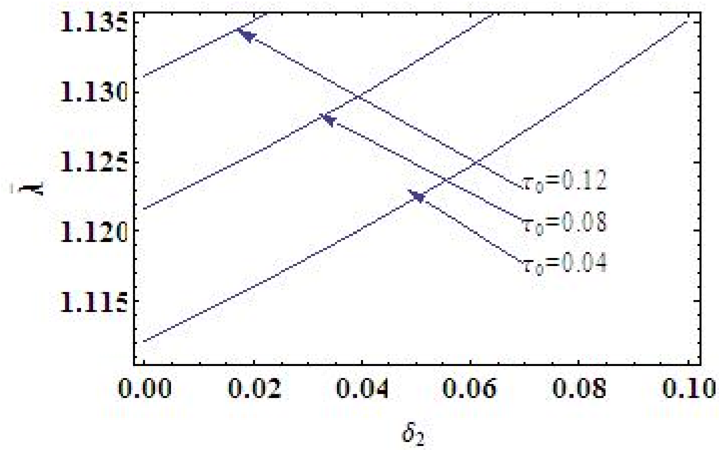

Figure 11: Variation of impedance $\overline{\lambda}$ with δ_2 for different τ_0 (d_1=0.2,d_2=0.6,H=0.2,L_1=L_2=0.2,L=1,k=2,δ_1=0.0,Q=0.1,τ_h=1)

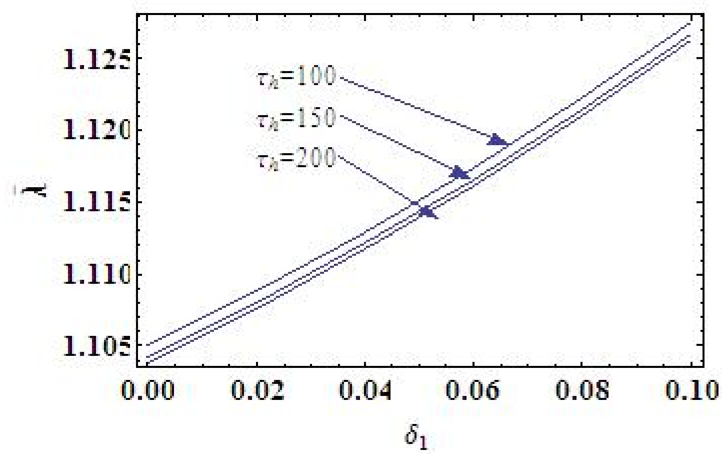

Figure 12: Variation of impedance $\overline{\lambda}$ with δ_1 for different τ_h (d_1=0.2,d_2=0.6,H=0.2,L_1=L_2=0.2,L=1,k=2,δ_2=0.0,Q=0.1,τ_0=1)

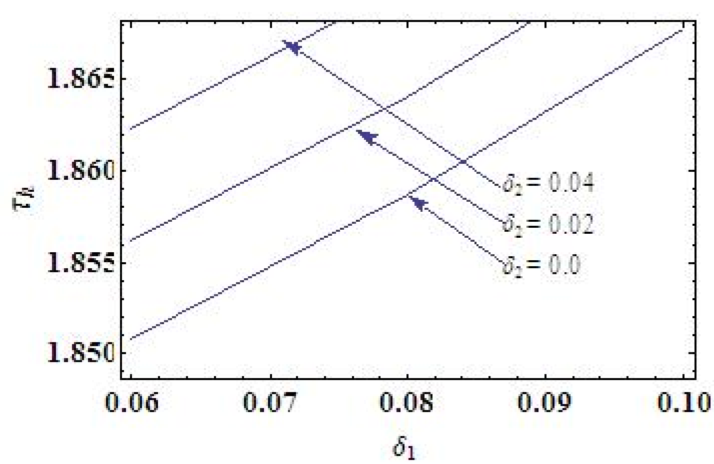

Figure 13: Variation of wall shear stress τ_h with δ_1 for different δ_2 (r_0=0.02,k=2,Q=0.01)

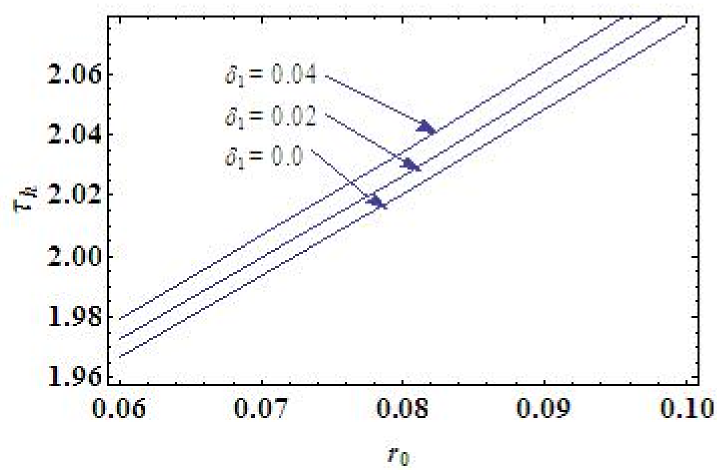

Figure 14: Variation of wall shear stress τ_h with r_0 for different δ_1 (δ_2=0.1,k=2,Q=0.01)

References

[1] D.F. Young, Effects of a time-dependent stenosis on flow through a tube, J. Engrg. Ind., Trans ASME, 90, 248-254, 1968.

[2] J.S. Lee, Y.C. Fung, Flow in Locally- constricted Tubes and Low Renolds Numbers, J. Appl. Mech., Trans ASME. 37, 9-16, 1970

[3] Padmanabhan, Mathematical model of arterial stenosis. Med. Biol. Eng. Comput., 18, 281-286, 1980.

[4] J.R. Buchanan, C. Kleinstreuer, and J.K. Corner, Rheological effects on pulsatile hemodynamics in a stenosed tube. Computers and Fluids, 29, 695-724, 2000.

[5] P.K. Mandal, An unsteady analysis of non-Newtonian blood flow through tapered arteries with a stenosis . International journal of Non Linear Mechanics, 40, 151-164, 2005.

[6] Z. Ismail, I. Abdullah, N. Mustapha and N. Amin, A power-law model of blood flow through a tapered overlapping stenosed artery. Applied Mathematics and Computation, 195, 669-680, 2007.

[7] K. Maruthi prasad, G. Radhakrishnamacharya, Effect of multiple stenoses on Herschel-Bulkley fluid through a tube with non-uniform cross-section, International e-journal of engineering mathematics: Theory and Application, 1, 69-76, 2007.

[8] P. Chaturani, R. Ponnalagar samy, A study of non-Newtonian aspects of blood flow through stenosed arteries and its applications in arterial diseases, Biorheol., 22, 521-531, 1985.

[9] E.E. Tzritzilakis, A mathematical model for blood flow in magnetic field. Physics of Fluids, 17: 077103, 1-15, 2005.

[10] K. Das and G.C. Saha, Arterial MHD pulsatile flow of blood under periodic body acceleration. Bulletin of Mathematical society, 16, 68-81, 2009

[11] I.I.H. Chen, Analysis of an intensive magnetic field on blood flow: part 2, Electromagnetic biology and Medicine, Vol 4, No1, 55-61, 1985.

[12] A.T. Ogulu and T.M. Abbey, Simulation of Heat Transfer on an Oscillatory Blood Flow in an Indented Porous artery, International Communication in Heat and Mass Transfer, Vol. 32, No. 7, 983-989, 2005.

[13] S.Shaw, P.V.S. N Murthy, S.C. Pradhan, The effect of body acceleration on two dimensional flow of casson fluid through an artery with Asymmetric stenosis, The Open Transport Phenomena Journal, 2 55-68,2010.

[14] Rekha Bali and Usha Awasthi, A casson fluid model for multiple stenosed arteries in the presence of magnetic field. Applied Mathematics, 3, 436-441, 2012.

[15] D.S, Sankar, U. Lee, FDM Analysis for MHD flow of a non-Newtonian fluid in stenosed arteries, Journal of Mechanical Science and Technology, 25, 2573-2581, 2011.

[16] Lokendra Parmar, S.B. Kulshreshthaand D.P. Singh, The role of magnetic field intensity in blood flow through overlapping stenosed artery: A Herschel-Bulkley fluid model. Advances in Applied Science Research, 4 (6), 318-328, 2013.

[17] R. Bhargava, S. Rawat, H.S. Takhar, O.A.Beg, Pulsatile magneto-biofluid flow and mass transfer in a non-Darcian porous medium channel, Meccanica, 42,247-262,2007