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Graceful labeling of arrow graphs and double arrow graphs

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Abstract

In this paper we define arrow graph and double arrow graph. We also prove that arrow graphs A_n^2, A_n^3,A_n^5 are graceful and double arrow graphs DA_n^2 and DA_n^3 are graceful.

Keywords: Graceful labeling, arrow graph, double arrow graph.

2010 MSC: 05C78.

1 Introduction

The graceful labeling was introduced by A.Rosa[1] during 1967. Golomb[2] named such labeling as graceful labeling which was called earlier as β-valuation. We introduce new graphs which are called arrow graph(A_n^k) and double arrow graph(DA_n^k). Kaneria and Makadia[4] introduced step grid graph(St_n) and double step grid graph(DSt_n). These two graphs are graceful. They also proved that path union of finite copies of above graphs, star graph of above graphs, cycle graph of above graphs are graceful.

We begin with a simple, undirected finite graph G=(V,E), with $|V| = p$ vertices and $|E| = q$ edges. For all terminology and notations we follows Harary[3]. Here are some of the definitions, which are used in this paper.

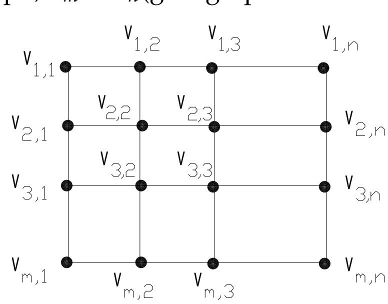
Definition 1.1 Graceful labeling

A function f is called graceful labeling of a graph G, if $f : V(G) \longrightarrow \{0, 1, ..., q\}$ is injective and the induced function $f^* : E(G) \longrightarrow \{1, 2, ..., q\}$ defined as $f^*(e) = |f(u) - f(v)|$ is bijective for every edge e=(u,v)∈ $E(G)$, where $q = |E(G)|$.

1A graph G is called graceful if it admits a graceful labeling.

Definition 1.2 superior vertices

In Graph, $P_m \times P_n$(grid graph on mn vertices)



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vertices $v_{1,1}, v_{2,1}, v_{3,1}, ..., v_{m,1}$ and vertices $v_{1,n}, v_{2,n}, v_{3,n}, ..., v_{m,n}$ are known as superior vertices from both the

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ends .

Definition 1.3 Arrow Graph

An arrow graph A_n^t with width t and length n is obtained by joining a vertex v with superior vertices of $P_m \times P_n$ by m new edges from one end.

Definition 1.4 Double Arrow Graph

A Double arrow graph DA_n^t with width t and length n is obtained by joining two vertices v and w with superior vertices of $P_m \times P_n$ by $m + m$ new edges from both the ends.

In this paper we introduce gracefulness of arrow graph and double arrow graph. For detail survey of grpah labeling we refer Gallian[5].

2 Main results

Theorem−2.1 **:** A_n^2 is a graceful graph, where $n \in N$

Proof : Let G = A_n^2 be an arrow graph obtained by joining a vertex v with superior vertices of $P_2 \times P_n$ by 2 new edges.

Let $u_{i,j}$ (i= 1,2; j= 1,2,. . . ,n) be vertices of $P_2 \times P_n$.

Join v with $u_{i,1}$ (i= 1,2) by two new edges to obtain G. Obviously $p = |V(G)| = 2n + 1$ and $q = |E(G)| = 3n$.

We define labeling function $f : V(G) \longrightarrow \{0, 1, 2, \ldots, q\}$ as follows,

f(v) = 0, f($u_{2,1}$) = 1

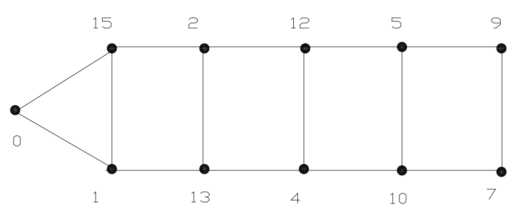
f($u_{1,j}$) = q-($\frac{j-1}{2}$)3 = q-$\frac{3}{2}$(j-1) ;when j $\equiv$ 1(mod 2)

= $\frac{3j}{2} - 1$;when j $\equiv$ 0(mod 2) $\forall j = 1, 2, \ldots, n;$

f($u_{2,j}$) = f($u_{1,j-1}$)-$(-1)^j$2, $\forall j = 2, 3, \ldots, n.$

ıAbove labeling pattern give rise a graceful labeling to given graph G.

Illustration−2.2**:** Arrow graph A_5^2 and its graceful labeling shown in figure−1



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Figure−1 Arrow Graph A_5^2 and its Graceful Labeling

Theorem−2.3 **:** A_n^3 is a graceful graph, where $n \geq 2$.

Proof : Let G = A_n^3 be an arrow graph obtained by joining a vertex v with superior vertices of $P_3 \times P_n$ by 3 new edges.

Let $u_{i,j}$ $(i = 1, 2, 3;\ t = 1, 2, \ldots, n)$ be vertices of $P_3 \times P_n$.

Join v with $u_{i,1}$ (i= 1,2,3) by three new edges to obtain G. Obviously $p = |V(G)| = 3n + 1$ and $q = |E(G)| = 5n$. We define labeling function $f : V(G) \longrightarrow \{0, 1, 2, \ldots, q\}$ as follows,

f(v) = 0 f($u_{2,1}$) = 2;

f($u_{1,j}$) = q-$\frac{5}{2}$(j-1) ;when j $\equiv$ 1(mod 2)

= $\frac{5j}{2} - 1$;when j $\equiv$ 0(mod 2) $\forall j = 1, 2, \ldots, n - 1;$

f($u_{2,j}$) = f($u_{1,j-1}$)-$(-1)^j \cdot 3$, $\forall j = 1, 2, \ldots, n - 1;$

f($u_{3,j}$) = f($u_{1,j}$)+$(-1)^j$, $\forall j = 1, 2, \ldots, n - 1;$

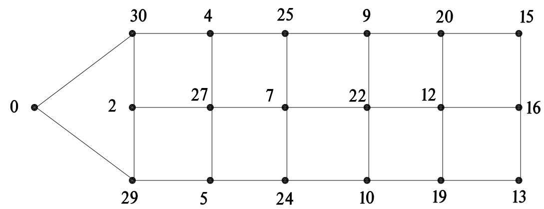
f($u_{1,n}$) = f($u_{1,n-1}$)-$(-1)^n \cdot 5$

f($u_{2,n}$) = f($u_{2,n-1}$)+$(-1)^n \cdot 4$

f($u_{3,n}$) = f($u_{3,n-1}$)-$(-1)^n \cdot 6$.

ıAbove labeling pattern give rise a graceful labeling to given graph G.

Illustration−2.4**:** Arrow graph A_6^3 and its graceful labeling shown in figure−2.



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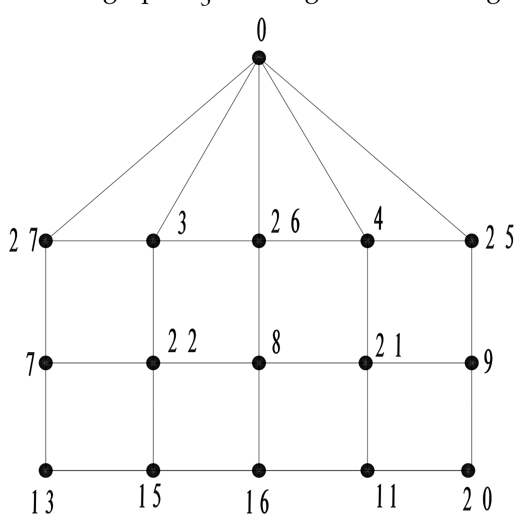
Figure−2 Arrow graph A_6^3 and its graceful labeling

Theorem−2.5 **:** A_n^5 is a graceful graph, where $n \geq 2$.
Proof : Let G = A_n^5 be an arrow graph obtained by joining a vertex v with superior vertices of $P_5 \times P_n$ by 5 new edges.
Let $u_{i,j}$ (i= 1,2,3,4,5; t= 1,2,. . . ,n) be vertices of $P_5 \times P_n$.
Join v with $u_{i,1}$ (i= 1,2,3,4,5) by five new edges to obtain G. Obviously $p = |V(G)| = 5n + 1$ and $q = |E(G)| = 9n$. We define labeling function $f : V(G) \longrightarrow \{0, 1, 2, \ldots, q\}$ as follows,

f(v) = 0 f($u_{2,1}$) = 3 f($u_{4,1}$)=4;
f($u_{1,j}$) = q-$\frac{9}{2}$(j-1) ;when j $\equiv$ 1(mod 2)
= $\frac{9j}{2} - 2$;when j $\equiv$ 0(mod 2), $\forall j = 1, 2, \ldots, n-1$;
f($u_{i,j}$) = f($u_{1,j}$)+$(-1)^j \frac{(i-1)}{2}$, $\forall i = 3, 5; \forall j = 1, 2, \ldots, n-1$;
f($u_{i,j}$) = f($u_{i-1,j-1}$)-$(-1)^j$ $\forall i = 2, 4; \forall j = 1, 2, \ldots, n-1$;
f($u_{1,n}$) = f($u_{1,n-1}$)-$(-1)^n \cdot 6$
f($u_{2,n}$) = f($u_{2,n-1}$)+$(-1)^n \cdot 7$
f($u_{3,n}$) = f($u_{3,n-1}$)-$(-1)^n \cdot 8$
f($u_{4,n}$) = f($u_{4,n-1}$)+$(-1)^n \cdot 10$
f($u_{5,n}$) = f($u_{5,n-1}$)-$(-1)^n \cdot 11$.

1Above labeling pattern give rise a graceful labeling to given graph G.
Illustration−2.6**:** Arrow graph A_3^5 and its graceful labeling shown in figure−3.

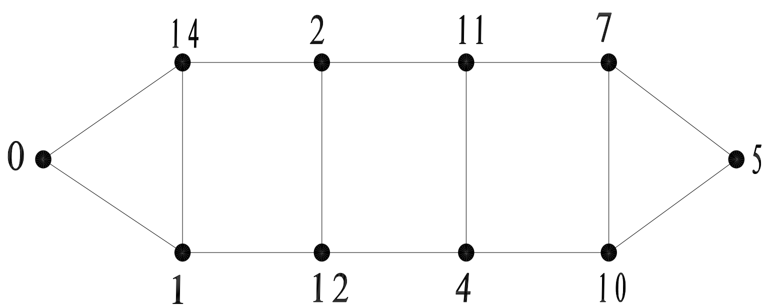


Figure−3 Arrow graph A_3^5 and its graceful labeling

Theorem−2.7 **:** DA_n^2 is a graceful graph, where $n \geq 2$
Proof : Let G = DA_n^2 be a double arrow graph obtained by joining two vertices v, w with $P_2 \times P_n$ by 2+2 new edges both sides.
Let $u_{i,j}$ (i= 1,2; j= 1,2,. . . ,n) be vertices of $P_2 \times P_n$.
Join v with $u_{i,1}$ (i= 1,2) and w with $u_{i,n}$ (i= 1,2) by 2+2 new edges to obtain G. Obviously $p = |V(G)| = 2n + 2$ and $q = |E(G)| = 3n + 2$. We define labeling function $f : V(G) \longrightarrow \{0, 1, 2, \ldots, q\}$ as follows,

f(v) = 0 f($u_{2,1}$) = 1
f($u_{1,j}$) = q-($\frac{j-1}{2}$)3 = q-$\frac{3}{2}$(j-1) ;when j $\equiv$ 1(mod 2)
= $\frac{3j}{2} - 1$;when j $\equiv$ 0(mod 2), $\forall j = 1, 2, \ldots, n$;
f($u_{2,j}$) = f($u_{1,j-1}$)-$(-1)^j \cdot 2$, $\forall j = 2, 3, \ldots, n$.

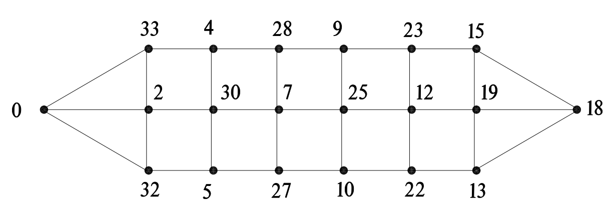
f($u_{1,n}$) = f($u_{1,n-1}$)-$(-1)^n \cdot 4$
f($u_{2,n}$) = f($u_{2,n-1}$)-$(-1)^n \cdot 6$
f(w) = f($u_{1,n}$)-$(-1)^n 2$.
ıAbove labeling patten give rise a graceful labeling to given graph G.
Illustration−2.8**:** Double arrow graph DA_4^2 and its graceful labeling shown in figure−4.



Figure−4 Double arrow graph DA_4^2 and its graceful labeling

Theorem−2.9 **:** DA_n^3 is a graceful graph, where $n \geq 2$
Proof : Let G = DA_n^3 be an arrow graph obtained by joining two vertices v and w with $P_3 \times P_n$ by 3+3 new edges both the sides.
Let $u_{i,j}$ (i= 1,2,3; t= 1,2,...,n) be vertices of $P_3 \times P_n$.
Join v with $u_{i,1}$ (i= 1,2,3) and w with $u_{i,n}$ by 3+3 new edges to obtain G. Obviously $p = |V(G)| = 3n + 2$ and $q = |E(G)| = 5n + 3$. We shall define labeling function $f : V(G) \longrightarrow \{0, 1, 2, \ldots, q\}$ as follows,
f(v) = 0 f($u_{2,1}$) = 2;
f($u_{1,j}$) = q-$\frac{5}{2}$(j-1) ;when j $\equiv$ 1(mod 2)
= $\frac{5j}{2} - 1$;when j $\equiv$ 0(mod 2), $\forall j = 1, 2, \ldots, n-1$;
f($u_{2,j}$) = f($u_{1,j-1}$)-$(-1)^j \cdot 3$, $\forall j = 1, 2, \ldots, n-1$;
f($u_{3,j}$) = f($u_{1,j}$)+$(-1)^j$, $\forall j = 1, 2, \ldots, n-1$;
f($u_{1,n}$) = f($u_{1,n-1}$)-$(-1)^n \cdot 8$
f($u_{2,n}$) = f($u_{2,n-1}$)+$(-1)^n \cdot 7$
f($v_{3,n}$) = f($u_{3,n-1}$)-$(-1)^n \cdot 9$
f(w) = f($u_{2,n}$)-$(-1)^n$
ıAbove labeling patten give rise a graceful labeling to given graph G.

Illustration−2.10**:** Double arrow graph DA_6^3 and its graceful labeling shown in figure−5.



Figure−5 Double arrow graph DA_6^3 and its graceful labeling

3 Concluding Remarks

ııWe have introduced new graceful graphs namely arrow graph and double arrow graph. We also proved that arrow graphs A_n^2 where $n \in N$, A_n^3 ,A_n^5 where $n \geq 2$ are graceful and double arrow graphs DA_n^2 ,DA_n^3 where $n \geq 2$ are graceful. Present work contribute some new results to the theory of graceful graphs. The labelling pattern is demonstrated by suitable illustration.

References

[1] A.Rosa, on certain valuation of graph, theory of graphs(Rome,july1966), Golden and Breach, N.Y. and paris, (1967), 349-355.

[2] S.W.Golomb, How to number a graph. In: Graph Theory and computing (R.C.Read.Ed.) Academic press. Newyork,(1972), 23-37.

[3] F.Harary, Graph theory, *Addition Wesley*, mass achusetts,(1972).

[4] V.J.Kaneria, H.M.Makadia, Graceful labelling for Double step grid graph, *Int. J. Math. And its App.*3(1)(2015), 33-38.

[5] J.A.Gallian, A Dynamic survey of Graph Labelling, *The Electronics Journal of Combinatorics*, 17(2014),♯DS6.

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