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Some new results on intuitionistic fuzzy H-ideal in BCI-algebra

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Abstract

Intuitionistic fuzzy set, involving membership, non-membership and hesitancy consideration present mathematically a very general structure. Because of these considerations it is possible to define several operations / compositions of these sets. In the existing literature ten different operations on such sets are defined. These ten operations on intuitionistic fuzzy sets bear interesting properties. In this paper we have identified and proved several of these properties, particularly those involving the operation $A \to B$ defined as standard intuitionistic fuzzy implicational with other operations.

Keywords: Intuitionistic fuzzy sets, equality, intuitionistic fuzzy implication, operation intuitionistic fuzzy H-ideal.

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1 Introduction

Intuitionistic fuzzy sets (IFS) as a generalization of fuzzy sets [7], was introduced by atanassov [1, 2], it assigns to each element degrees of membership, non-membership hesitancy. Some results x, with intuitionistic fuzzy sets based on operations(denoted by $\cup, \cap$, $\oplus$, $\otimes$, $\odot$, Θ , $*$, @, #, \$) have been established in [1, 2, 3, 4, 5 and 6]. The paper is organized as follows: In section 2 some basic definitions related to intuitionistic fuzzy set theory are presented. In section 3 results associated with standard intuitionistic fuzzy implication are proved.

2 Preliminaries

An algebra $(X; *, 0)$ of type $(2, 0)$ is called a BCI-algebra if it satisfies the following conditions:

(I) $(\forall x, y, z \in X)(((x * y) * (x * z)) * (z * y) = 0)$,

(II) $(\forall x, y \in X)((x * (x * y)) * y = 0)$,

(III) $(\forall x \in X)(x * x = 0)$,

(IV) $(\forall x, y \in X)(x * y = 0, y * x = 0 \Rightarrow x = y)$.

We can define a partial order $' \leq '$ on X by $x \leq y$ if and only if $x * y = 0$. Any BCI-algebra X has the following properties:

(a1) $(\forall x \in X)(x * 0 = x)$.

(a2) $(\forall x, y, z \in X)((x * y) * z = (x * z) * y)$.

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(a3) $(\forall x, y, z \in X)(x \leq y \Rightarrow x * z \leq y * z, z * y \leq z * x)$.

A mapping $\mu : X \rightarrow [0, 1]$, where X is an arbitrary nonempty set, is called a fuzzy set in X. For any fuzzy set μ in X and any $t \in [0, 1]$ we define two sets $U(\mu; t) = \{x \in X | \mu(x) \geq t\}$ and $L(\mu; t) = \{x \in X | \mu(x) \leq t\}$, which are called an upper and lower t-level cut of μ and can be used to the characterization of μ. As an important generalization of the notion of fuzzy sets in X, Atanassov[1,2] introduced the concept of an intuitionstic fuzzy set (IFS for short) defined on a nonempty set X as objects having the form $A = \{< x, \mu_A(x), \lambda_A(x) > | x \in X\}$, where the functions μ_A: $X \rightarrow [0, 1]$ and $\lambda_A : X \rightarrow [0, 1]$ denote the degree of membership (namely μ_A(x)) and the degree of non-membership (namely λ_A(x)) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$. Such defined objects are studied by many authors (see for Example two journals: 1. Fuzzy Sets and Systems and 2. Notes on Intuitionistic Fuzzy Sets) and have many interesting applications not only in mathematics (see Chapter 5 in the book [3]). For the sake of simplicity, we shall use the symbol $A = \{X, \mu_A, \lambda_A\}$ for the intuitionistic fuzzy set $A = \{< x, \mu_A(x), \lambda_A(x) >: x \in X\}$.

A nonempty subset A of a BCI-algebra X is called an ideal of X if it satisfies:

(11) $0 \in A$,

(12) $(\forall x, y \in X)(\forall y \in A)(x * y \in A \Rightarrow x \in A)$.

A nonempty subset A of a BCI-algebra X is called H-ideal of X if it satisfies (11) and (12) $(\forall x, y \in X)(\forall z \in A)((x * (y * z)), y \in A \Rightarrow x * z \in A)$

Definition:2.1

An IFS $A = \{< x, \mu_A(x), \lambda_A(x) >: x \in X\}$ in a BCI-algebra X is called an intuitionistic fuzzy ideal of X if it satisfies:$(\forall x \in X)(\mu_A(0) \geq \mu_A(x), \lambda_A(0) \leq \lambda_A(x))$ and

$(\forall x, y \in X)(\mu_A(x) \geq \min\{\mu_A(x * y), \mu_A(y)\})$,

$(\forall x, y \in X)(\lambda_A(x) \leq \max\{\lambda_A(x * y), \lambda_A(y)\}$.

3 Intuitionistic fuzzy H-ideals

In what follows, let X denotes a BCI-algebra unless otherwise specified. We first consider the intuitionistic fuzzification of the notion of H-ideals in a BCI-algebra as follows.

Definition:3.1. $[1, 2]$:

An intuitionistic fuzzy set A in a finite universe of discourse $X = \{x_1, x_2, x_3, x_4, \cdots x_n\}$ is given by $A = \{< \mu_A(x), \lambda_A(x) >: x \in X\}$, Where $\mu_A : X \rightarrow [0, 1]$ and $\lambda_A : X \rightarrow [0, 1]$ such that $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$. The number $\mu_A(x)$ and $\lambda_A(x)$ denote the degree of membership and non-membership of $x \in X$ to A, respectively. For each IFS A in X, if $\pi_A(x) = 1 - \mu_A(x) - \lambda_A(x), x \in X$, then $\pi_A(x)$ represent the degree of hesitance of x to A, $\Pi_A(x)$ is called Intuitionistic index. Obviously, when $\pi_A(x) = 0$, i.e, $\lambda_A(x) = 1 - \mu_A(x)$ for each x in X, then the IFS set A becomes fuzzy set. Thus, fuzzy sets are the special cases of IFSs. For studying sets, there is need to consider relations and operations, which in the study of Intuitionistic fuzzy sets are defined as follows.

Definition:3.2

An IFS A = $\{< x, \mu_A(x), \lambda_A(x) >: x \in X\}$ in X is called an intuitionistic fuzzy H-ideal of X if it satisfies $(\forall x \in X)(\mu_A(0) \geq \mu_A(x), \lambda_A(0) \leq \lambda_A(x))$ and

$(\forall x, y, z \in X)(\mu_A(x * z) \geq \min\{\mu_A(x * (y * z)), \mu_A(y)\})$,

$(\forall x, y, z \in X)(\lambda_A(y * x) \leq \max\{\lambda_A(x * (y * z)), \lambda_A(y)\})$

Definition:3.3 Set Operators on Intuitionistic Fuzzy Set

Let IFSs(X) denotes the family of all IFSs (X) on the universe, $_B(x)$: $x \in X\}$

1. $A \cup B = \{< X, \max\{\mu_A(x), \mu_B(x)\}, \min\{\lambda_A(x), \lambda_B(x)\} >: x \in X\}$
2. $A \cap B = \{< X, \min\{\mu_A(x), \mu_B(x)\}, \max\{\lambda_A(x), \lambda_B(x)\} >: x \in X\}$

3. $A^C = \{< X, \mu_A(x), \lambda_A(x) >: x \in X\}$

4. $A@B = \{< X, \frac{1}{2}[\mu_A(x) + \mu_B(x)], \frac{1}{2}[\lambda_A(x) + \lambda_B(x)] >: x \in X\}$

5. $A\$B = \left\{ < X, \sqrt{\mu_A(x)\mu_B(x)}, \sqrt{\lambda_A(x)\lambda_B(x)} >: x \in X \right\}$

6. $A\#B = \left\{ < X, \frac{2\mu_A(x).\mu_B(x)}{\mu_A(x)+\mu_B(x)}, \frac{2\lambda_A(x)\lambda_B(x)}{\lambda_A(x)+\lambda_B(x)} >: x \in X \right\}$

7. $A * B = \left\{ < X, \frac{\mu_A(x)+\mu_B(x)}{2[\mu_A(x).\mu_B(x)+1]}, \frac{\lambda_A(x)+\lambda_B(x)}{2[\lambda_A(x).\lambda_B(x)+1]} >: x \in X \right\}$

8. $A \oplus B = \left\{ < X, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), \lambda_A(x).\lambda_B(x) >: x \in X \right\}$

9. $A \otimes B = \{< X, \mu_A(x) + \mu_B(x), \lambda_A(x) + \lambda_B(x) - \lambda_A(x)\lambda_B(x) >: x \in X\}$

10. Hamacher Union Function
 $\mu_{A\cup B}(x) = \frac{\mu_A(x)+\mu_B(x)-(2-\gamma)\mu_A(x)\mu_B(x)}{1-(1-\gamma)\mu_A(x).\mu_B(x)}$, Where $\gamma \geq 0$

11. Hamacher Intersection Function
 $\mu_{A\cap B}(x) = \frac{\mu_A(x).\mu_B(x)}{\gamma+(1-\gamma)[\mu_A(x)+\mu_B(x)-\mu_A(x).\mu_B(x)]}$, Where $\gamma \geq 0$

12. Bounded Difference: $\mu_{A\ominus B}(x) = Max\{0, \mu_A(x) - \mu_B(x) : x \in X\}$

13. Bounded Product: $\mu_{A\odot B}(x) = Max\{0, \mu_A(x) + \mu_B(x) - 1 : x \in X\}$

14. Bounded Sum: $\mu_{A\oplus B}(x) = Min\{1, \mu_A(x) + \mu_B(x) : x \in X\}$

15. Simple Disjunctive Sum:
 $\mu_{A\otimes B}(x) = Max\{Min\{\mu_A(x), 1 - \mu_B(x)\}, Min\{1 - \mu_A(x), \mu_B(x) : x \in X\}$

16. Disjoint Sum: $\mu_{A\Delta B}(x) = |\mu_A(x) - \mu_B(x)|$

Example 3.1. *Let $X = \{0, 1, 2, 3, 4\}$ be a BCI-algebra with the following Cayley table:*

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

We define an intuitionistic fuzzy set $A = < X, \mu_A, \lambda_A >$ in X by A

X	0	1	2	3	4
μ_A	.77	.66	.43	.43	.66
λ_A	.22	.33	.54	.54	.33

By routine calculation, we know that $A = < X, \mu_A, \lambda_A >$ is an intuitionistic fuzzy H-ideal of X.
We define an intuitionistic fuzzy set $B = < X, \mu_B, \lambda_B >$ in X .

X	0	1	2	3	4
μ_B	.78	.51	.57	.51	.51
λ_B	.21	.45	.41	.45	.45

By routine calculation, we know that $B = < X, \mu_B, \lambda_B >$ is an intuitionistic fuzzy H-ideal of X. Then A and B are intuitionistic fuzzy H-ideal Of X.Obviously, the union $(A \cup B)$ is not an intuitionistic fuzzy H- ideal of X,

Since $x = 3, y = 4, z = 0$.
$\Rightarrow \mu_A(x * z) \geq \min\{\mu_A(x * (y * z)), \mu_A(y)\}$
$\Rightarrow \mu_A(3 * 0) \geq \min\{\mu_A(3 * (4 * 0)), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(3 * 4), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(2), \mu_A(4)\}$
$\Rightarrow 0.43 \geq Min\{0.43, 0.66\}$
$\Rightarrow 0.43 \geq 0.43$
And $\mu_B(3) \geq \min\{\mu_B(2), \mu_B(4)\}$
$\Rightarrow 0.51 \geq Min\{0.57, 0.51\}$
$\Rightarrow 0.51 \geq 0.51$
Now $\mu_{A\cup B}(3) \geq Min\{\mu_{A\cup B}(2), \mu_{A\cup B}(4)\}$
$\Rightarrow Max\{0.43, 0.51\} \geq Min\{Max\{0.43, 0.57\}, Max\{0.66, 0.51\}\}$
$\Rightarrow 0.51 \geq Min\{0.57, 0.66\}$
$\Rightarrow 0.51 \not\geq 0.57$
$\Rightarrow \lambda_A(x) \leq Max\{\lambda_A((x * z) * (y * z)), \lambda_A(y)\}$
$\Rightarrow \lambda_A(3) \leq Max\{\lambda_A(2), \lambda_A(4)\}$
$\Rightarrow 0.54 \leq Max\{0.54, 0.33\}$
$\Rightarrow 0.54 \leq 0.54$
And $\lambda_B(x) \leq Max\{\lambda_B((x * z) * (y * z)), \lambda_B(y)\}$
$\Rightarrow \lambda_B(3) \leq Max\{\lambda_B(2), \lambda_B(4)\}$
$\Rightarrow 0.45 \leq Max\{0.41, 0.45\}$
$\Rightarrow 0.45 \leq 0.45$
Now $\lambda_{A\cup B}(3) \leq Max\{\lambda_{A\cup B}(2), \lambda_{A\cup B}(4)\}$
$\Rightarrow Min\{0.54, 0.45\} \leq Max\{Min\{0.54, 0.41\}, Min\{0.33, 0.45\}\}$
$\Rightarrow 0.45 \leq Max\{0.41, 0.33\}$
$\Rightarrow 0.45 \not\leq 0.41$

Example 3.2. *Let $X = \{0, 1, 2, 3, 4\}$ be a BCI-algebra with the following Cayley table:*

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

We define an intuitionistic fuzzy set $A = < X, \mu_A, \lambda_A >$ in X.

X	0	1	2	3	4
μ_A	.77	.66	.43	.43	.66
λ_A	.22	.33	.54	.54	.33

By routine calculation, we know that $A = < X, \mu_A, \lambda_A >$ is an intuitionistic fuzzy H-ideal of X.
We define an intuitionistic fuzzy set $B = < X, \mu_B, \lambda_B >$ in X .

X	0	1	2	3	4
μ_B	.78	.51	.57	.51	.51
λ_B	.21	.45	.41	.45	.45

By routine calculation, we know that $B = < X, \mu_B, \lambda_B >$ is an intuitionistic fuzzy H-ideal of X. Then A and B are intuitionistic fuzzy H-ideal Of X. Obviously, the intersection $(A \cap B)$ is an intuitionistic fuzzy H- ideal of X,

Since $x = 3, y = 4, z = 0$.

$\Rightarrow \mu_A(x * z) \geq min\{\mu_A(x * (y * z)), \mu_A(y)\}$
$\Rightarrow \mu_A(3 * 0) \geq min\{\mu_A(3 * (4 * 0)), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq min\{\mu_A(3 * 4), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq min\{\mu_A(2), \mu_A(4)\}$
$\Rightarrow 0.43 \geq Min\{0.43, 0.66\}$
$\Rightarrow 0.43 \geq 0.43$
And $\mu_B(3) \geq min\{\mu_B(2), \mu_B(4)\}$
$\Rightarrow 0.51 \geq Min\{0.57, 0.51\}$
$\Rightarrow 0.51 \geq 0.51$
Now $\mu_{A\cap B}(3) \geq Min\{\mu_{A\cap B}(2), \mu_{A\cap B}(4)\}$
$\Rightarrow Max\{0.43, 0.51\} \geq Min\{Min\{0.43, 0.57\}, Min\{0.66, 0.51\}\}$
$\Rightarrow 0.43 \geq Min\{0.43, 0.51\}$
$\Rightarrow 0.43 \geq 0.43$
$\Rightarrow \lambda_A(x) \leq Max\{\lambda_A((x * z) * (y * z)), \lambda_A(y)\}$
$\Rightarrow \lambda_A(3) \leq Max\{\lambda_A(2), \lambda_A(4)\}$
$\Rightarrow 0.54 \leq Max\{0.54, 0.33\}$
$\Rightarrow 0.54 \leq 0.54$
And $\Rightarrow \lambda_B(x) \leq Max\{\lambda_B((x * z) * (y * z)), \lambda_B(y)\}$
$\Rightarrow \lambda_B(3) \leq Max\{\lambda_B(2), \lambda_B(4)\}$
$\Rightarrow 0.45 \leq Max\{0.41, 0.45\}$
$\Rightarrow 0.45 \leq 0.45$
Now $\lambda_{A\cap B}(3) \leq Max\{\lambda_{A\cap B}(2), \lambda_{A\cap B}(4)\}$
$\Rightarrow Max\{0.54, 0.45\} \leq Max\{Max\{0.55, 0.51\}, Max\{0.33, 0.45\}\}$
$\Rightarrow 0.54 \leq Max\{0.54, 0.45\}$
$\Rightarrow 0.54 \leq 0.54$

Example 3.3. *Let $X = \{0, 1, 2, 3, 4\}$ be a BCI-algebra with the following Cayley table:*

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

We define an intuitionistic fuzzy set $A = < X, \mu_A, \lambda_A >$ in X by A

X	0	1	2	3	4
μ_A	.77	.66	.43	.43	.66
λ_A	.22	.33	.54	.54	.33

By routine calculation, we know that $A = < X, \mu_A, \lambda_A >$ is an intuitionistic fuzzy H-ideal of X.
We define an intuitionistic fuzzy set $B = < X, \mu_B, \lambda_B >$ in X .

X	0	1	2	3	4
μ_B	.78	.51	.57	.51	.51
λ_B	.21	.45	.41	.45	.45

By routine calculation, we know that $B = < X, \mu_B, \lambda_B >$ is an intuitionistic fuzzy H-ideal of X. Then A and B are intuitionistic fuzzy H-ideal Of X. Obviously, then $A \oplus B$ is not an intuitionistic fuzzy P- ideal of X,
Since $x = 3$, $y = 4$, $z = 0$.
$\Rightarrow \mu_A(x * z) \geq \min\{\mu_A(x * (y * z)), \mu_A(y)\}$
$\Rightarrow \mu_A(3 * 0) \geq \min\{\mu_A(3 * (4 * 0)), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(3 * 4), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(2), \mu_A(4)\}$
$\Rightarrow 0.43 \geq Min\{0.43, 0.66\}$

$\Rightarrow 0.43 \geq 0.43$
And $\mu_B(3) \geq \min\{\mu_B(2), \mu_B(4)\}$
$\Rightarrow 0.51 \geq Min\{0.57, 0.51\}$
$\Rightarrow 0.51 \geq 0.51$
Now $\mu_{A\oplus B}(3) \geq Min\{\mu_{A\oplus B}(2), \mu_{A\oplus B}(4)\}$
$\Rightarrow \mu_A(3) + \mu_B(3) - \mu_A(3).\mu_B(3) \geq Min\{\mu_A(2) + \mu_B(2) - \mu_A(2).\mu_B(2), \mu_A(4) + \mu_B(4) - \mu_A(4).\mu_B(4)\}$
$\Rightarrow 0.7207 \geq Min\{0.7549, 0.8334\}$
$\Rightarrow 0.7207 \ngeq 0.7549$
$\Rightarrow \lambda_A(x) \leq Max\{\lambda_A((x*z)*(y*z)), \lambda_A(y)\}$
$\Rightarrow \lambda_A(3) \leq Max\{\lambda_A(2), \lambda_A(4)\}$
$\Rightarrow 0.54 \leq Max\{0.54, 0.33\}$
$\Rightarrow 0.54 \leq 0.54$
And $\lambda_B(x) \leq Max\{\lambda_B((x*z)*(y*z)), \lambda_B(y)\}$
$\Rightarrow \lambda_B(3) \leq Max\{\lambda_B(2), \lambda_B(4)\}$
$\Rightarrow 0.45 \leq Max\{0.41, 0.45\}$
$\Rightarrow 0.45 \leq 0.45$
Now $\lambda_{A\oplus B}(3) \leq Max\{\lambda_{A\oplus B}(2), \lambda_{A\oplus B}(4)\}$
$\Rightarrow \lambda_A(3)\lambda_B(3) \leq Max\{\lambda_A(2)\lambda_B(2), \lambda_A(4)\lambda_B(4)\}$
$\Rightarrow 0.243 \leq max\{0.2214, .1485\}$
$0.243 \nleq 0.2214$

Example 3.4. *Let* $X = \{0, 1, 2, 3, 4\}$ *be a BCI-algebra with the following Cayley table:*

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

We define an intuitionistic fuzzy set $A = < X, \mu_A, \lambda_A >$ in X

X	0	1	2	3	4
μ_A	.77	.66	.43	.43	.66
λ_A	.22	.33	.54	.54	.33

By routine calculation, we know that $A = < X, \mu_A, \lambda_A >$ is an intuitionistic fuzzy H-ideal of X.
We define an intuitionistic fuzzy set $B = < X, \mu_B, \lambda_B >$ in X .

X	0	1	2	3	4
μ_B	.78	.51	.57	.51	.51
λ_B	.21	.45	.41	.45	.45

By routine calculation, we know that $B = < X, \mu_B, \lambda_B >$ is an intuitionistic fuzzy H-ideal of X. Then A and B are intuitionistic fuzzy H-ideal Of X. Obviously, then $A \otimes B$ is not an intuitionistic fuzzy H- ideal of X,
Since $x = 3, y = 4, z = 0$.
$\Rightarrow \mu_A(x*z) \geq \min\{\mu_A(x*(y*z)), \mu_A(y)\}$
$\Rightarrow \mu_A(3*0) \geq \min\{\mu_A(3*(4*0)), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(3*4), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(2), \mu_A(4)\}$
$\Rightarrow 0.43 \geq Min\{0.43, 0.66\}$
$\Rightarrow 0.43 \geq 0.43$
And $\mu_B(3) \geq \min\{\mu_B(2), \mu_B(4)\}$
$\Rightarrow 0.51 \geq Min\{0.57, 0.51\}$
$\Rightarrow 0.51 \geq 0.51$
Now $\mu_{A\otimes B}(3) \geq Min\{\mu_{A\otimes B}(2), \mu_{A\otimes B}(4)\}$

$\Rightarrow \mu_A(3)\mu_B(3) \geq Min\{\mu_A(2)\mu_B(2), \mu_A(4), \mu_B(4)\}$
$\Rightarrow 0.2193 \geq Min\{0.2551, 0.3366\}$
$\Rightarrow 0.2193 \ngeq 0.2451$
$\Rightarrow \lambda_A(x) \leq Max\{\lambda_A((x*z)*(y*z)), \lambda_A(y)\}$
$\Rightarrow \lambda_A(3) \leq Max\{\lambda_A(2), \lambda_A(4)\}$
$\Rightarrow 0.54 \leq Max\{0.54, 0.33\}$
$\Rightarrow 0.54 \leq 0.54$
And $\lambda_B(x) \leq Max\{\lambda_B((x*z)*(y*z)), \lambda_B(y)\}$
$\Rightarrow \lambda_B(3) \leq Max\{\lambda_B(2), \lambda_B(4)\}$
$\Rightarrow 0.45 \leq Max\{0.41, 0.45\}$
$\Rightarrow 0.45 \leq 0.45$
Now $\lambda_{A\otimes B}(3) \leq Max\{\lambda_{A\otimes B}(2), \lambda_{A\otimes B}(4)\}$
$\Rightarrow \lambda_A(3) + \lambda_B(3) - \lambda_A(3)\lambda_B(3) \leq Max\{\lambda_A(2) + \lambda_B(2) - \lambda_A(2)\lambda_B(2), \lambda_A(4) + \lambda_B(4) - \lambda_A(4)\lambda_B(4)\}$
$\Rightarrow 0.747 \leq Max\{0.7286, 0.6315\}$
$\Rightarrow 0.747 \nleq 0.6315$

Example 3.5. *Let* $X = \{0, 1, 2, 3, 4\}$ *be a BCI-algebra with the following Cayley table:*

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

We define an intuitionistic fuzzy set $A = < X, \mu_A, \lambda_A >$ in X.

X	0	1	2	3	4
μ_A	.77	.66	.43	.43	.66
λ_A	.22	.33	.54	.54	.33

By routine calculation, we know that $A = < X, \mu_A, \lambda_A >$ is an intuitionistic fuzzy H-ideal of X.
We define an intuitionistic fuzzy set $B = < X, \mu_B, \lambda_B >$ in X .

X	0	1	2	3	4
μ_B	.78	.51	.57	.51	.51
λ_B	.21	.45	.41	.45	.45

By routine calculation, we know that $B = < X, \mu_B, \lambda_B >$ is an intuitionistic fuzzy H-ideal of X. Then A and B are intuitionistic fuzzy H-ideal Of X. Obviously, the hamacher union is not an intuitionistic fuzzy H- ideal of X,

Since $x = 3, y = 4, z = 0$.
$\Rightarrow \mu_A(x*z) \geq \min\{\mu_A(x*(y*z)), \mu_A(y)\}$
$\Rightarrow \mu_A(3*0) \geq \min\{\mu_A(3*(4*0)), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(3*4), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(2), \mu_A(4)\}$
$\Rightarrow 0.43 \geq Min\{0.43, 0.66\}$
$\Rightarrow 0.43 \geq 0.43$
And $\mu_B(3) \geq \min\{\mu_B(2), \mu_B(4)\}$
$\Rightarrow 0.51 \geq Min\{0.57, 0.51\}$
$\Rightarrow 0.51 \geq 0.51$
Now $\mu_{A\cup B}(3) \geq Min\{\mu_{A\cup B}(2), \mu_{A\cup B}(4)\}$ and $\gamma = 0.46$
$\Rightarrow 0.6839 \geq Min\{0.7183, 0.7972\}$
$\Rightarrow 0.6839 \ngeq 0.7183$
$\Rightarrow \lambda_A(x) \leq Max\{\lambda_A((x*z)*(y*z)), \lambda_A(y)\}$
$\Rightarrow \lambda_A(3) \leq Max\{\lambda_A(2), \lambda_A(4)\}$

$\Rightarrow 0.54 \leq Max\{0.54, 0.33\}$
$\Rightarrow 0.54 \leq 0.54$
And $\lambda_B(x) \leq Max\{\lambda_B((x*z)*(y*z)), \lambda_B(y)\}$
$\Rightarrow \lambda_B(3) \leq Max\{\lambda_B(2), \lambda_B(4)\}$
$\Rightarrow 0.45 \leq Max\{0.41, 0.45\}$
$\Rightarrow 0.45 \leq 0.45$
Now $\lambda_{A\cup B}(3) \leq Max\{\lambda_{A\cup B}(2), \lambda_{A\cup B}(4)\}$ and $\gamma = 0.46$
$\Rightarrow 0.7095 \leq Max\{0.692, 0.6000\}$
$\Rightarrow 0.7095 \nleq 0.6925$

Example 3.6. *Let $X = \{0,1,2,3,4\}$ be a BCI-algebra with the following Cayley table:*

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

We define an intuitionistic fuzzy set $A = < X, \mu_A, \lambda_A >$ in X.

X	0	1	2	3	4
μ_A	.77	.66	.43	.43	.66
λ_A	.22	.33	.54	.54	.33

By routine calculation, we know that $A = < X, \mu_A, \lambda_A >$ is an intuitionistic fuzzy H-ideal of X.
We define an intuitionistic fuzzy set $B = < X, \mu_B, \lambda_B >$ in X .

X	0	1	2	3	4
μ_B	.78	.51	.57	.51	.51
λ_B	.21	.45	.41	.45	.45

By routine calculation, we know that $B = < X, \mu_B, \lambda_B >$ is an intuitionistic fuzzy H-ideal of X. Then A and B are intuitionistic fuzzy H-ideal Of X. Obviously, the hamacher intersection is an intuitionistic fuzzy H-ideal of X,
Since $x = 3, y = 4, z = 0$.
$\Rightarrow \mu_A(x*z) \geq \min\{\mu_A(x*(y*z)), \mu_A(y)\}$
$\Rightarrow \mu_A(3*0) \geq \min\{\mu_A(3*(4*0)), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(3*4), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(2), \mu_A(4)\}$
$\Rightarrow 0.43 \geq Min\{0.43, 0.66\}$
$\Rightarrow 0.43 \geq 0.43$
And $\mu_B(3) \geq \min\{\mu_B(2), \mu_B(4)\}$
$\Rightarrow 0.51 \geq Min\{0.57, 0.51\}$
$\Rightarrow 0.51 \geq 0.51$
Now $\mu_{A\cap B}(3) \geq Min\{\mu_{A\cap B}(2), \mu_{A\cap B}(4)\}$ and $\gamma = 0.46$
$\Rightarrow 0.2574 \geq Min\{0.2816, 0.3697\}$
$\Rightarrow 0.2574 \ngeq 0.2816$
$\Rightarrow \lambda_A(x) \leq Max\{\lambda_A((x*z)*(y*z)), \lambda_A(y)\}$
$\Rightarrow \lambda_A(3) \leq Max\{\lambda_A(2), \lambda_A(4)\}$
$\Rightarrow 0.54 \leq Max\{0.54, 0.33\}$
$\Rightarrow 0.54 \leq 0.54$
And $\lambda_B(x) \leq Max\{\lambda_B((x*z)*(y*z)), \lambda_B(y)\}$
$\Rightarrow \lambda_B(3) \leq Max\{\lambda_B(2), \lambda_B(4)\}$
$\Rightarrow 0.45 \leq Max\{0.41, 0.45\}$
$\Rightarrow 0.45 \leq 0.45$

Now $\lambda_{A\cap B}(3) \leq Max\{\lambda_{A\cap B}(2), \lambda_{A\cap B}(4)\}$ and $\gamma = 0.46$
$\Rightarrow 0.1845 \leq Max\{0.2585, 0.1845\}$
$\Rightarrow 0.1845 \leq 0.2585$

Example 3.7. *Let $X = \{0, 1, 2, 3, 4\}$ be a BCI-algebra with the following Cayley table:*

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

We define an intuitionistic fuzzy set $A = < X, \mu_A, \lambda_A >$ in X

X	0	1	2	3	4
μ_A	.77	.66	.43	.43	.66
λ_A	.22	.33	.54	.54	.33

By routine calculation, we know that $A = < X, \mu_A, \lambda_A >$ is an intuitionistic fuzzy H-ideal of X.
We define an intuitionistic fuzzy set $B = < X, \mu_B, \lambda_B >$ in X .

X	0	1	2	3	4
μ_B	.78	.51	.57	.51	.51
λ_B	.21	.45	.41	.45	.45

By routine calculation, we know that $B = < X, \mu_B, \lambda_B >$ is an intuitionistic fuzzy H-ideal of X. Then A and B are intuitionistic fuzzy H-ideal Of X. Obviously, the bounded difference is not an intuitionistic fuzzy H- ideal of X,

Since $x = 3, y = 4, z = 0$.
$\Rightarrow \mu_A(x * z) \geq \min\{\mu_A(x * (y * z)), \mu_A(y)\}$
$\Rightarrow \mu_A(3 * 0) \geq \min\{\mu_A(3 * (4 * 0)), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(3 * 4), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(2), \mu_A(4)\}$
$\Rightarrow 0.43 \geq Min\{0.43, 0.66\}$
$\Rightarrow 0.43 \geq 0.43$
And $\mu_B(3) \geq \min\{\mu_B(2), \mu_B(4)\}$
$\Rightarrow 0.51 \geq Min\{0.57, 0.51\}$
$\Rightarrow 0.51 \geq 0.51$
Now $\mu_{A\Theta B}(3) \geq Min\{\mu_{A\Theta B}(2), \mu_{A\Theta B}(4)\}$
$\Rightarrow 0 \geq Min\{0, 0.15\}$
$\Rightarrow 0 \geq 0$
$\Rightarrow \lambda_A(x) \leq Max\{\lambda_A((x * z) * (y * z)), \lambda_A(y)\}$
$\Rightarrow \lambda_A(3) \leq Max\{\lambda_A(2), \lambda_A(4)\}$
$\Rightarrow 0.54 \leq Max\{0.54, 0.33\}$
$\Rightarrow 0.54 \leq 0.54$
And $\lambda_B(x) \leq Max\{\lambda_B((x * z) * (y * z)), \lambda_B(y)\}$
$\Rightarrow \lambda_B(3) \leq Max\{\lambda_B(2), \lambda_B(4)\}$
$\Rightarrow 0.45 \leq Max\{0.41, 0.45\}$
$\Rightarrow 0.45 \leq 0.45$
Now $\lambda_{A\Theta B}(3) \leq Max\{\lambda_{A\Theta B}(2), \lambda_{A\Theta B}(4)\}$ and $\gamma = 0.46$
$\Rightarrow 0.45 \leq Max\{0.13, 0\}$
$\Rightarrow 0.45 \not\leq 0.13$

Example 3.8. *Let $X = \{0, 1, 2, 3, 4\}$ be a BCI-algebra with the following Cayley table:*

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

We define an intuitionistic fuzzy set $A = < X, \mu_A, \lambda_A >$ in X

X	0	1	2	3	4
μ_A	.77	.66	.43	.43	.66
λ_A	.22	.33	.54	.54	.33

By routine calculation, we know that $A = < X, \mu_A, \lambda_A >$ is an intuitionistic fuzzy H-ideal of X.
We define an intuitionistic fuzzy set $B = < X, \mu_B, \lambda_B >$ in X .

X	0	1	2	3	4
μ_B	.78	.51	.57	.51	.51
λ_B	.21	.45	.41	.45	.45

By routine calculation, we know that $B = < X, \mu_B, \lambda_B >$ is an intuitionistic fuzzy H-ideal of X. Then A and B are intuitionistic fuzzy H-ideal Of X. Obviously, the simple disjunctive sum is an intuitionistic fuzzy H- ideal of X,

Since $x = 3$, $y = 4$, $z = 0$.
$\Rightarrow \mu_A(x * z) \geq \min\{\mu_A(x * (y * z)), \mu_A(y)\}$
$\Rightarrow \mu_A(3 * 0) \geq \min\{\mu_A(3 * (4 * 0)), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(3 * 4), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(2), \mu_A(4)\}$
$\Rightarrow 0.43 \geq Min\{0.43, 0.66\}$
$\Rightarrow 0.43 \geq 0.43$
And $\mu_B(3) \geq \min\{\mu_B(2), \mu_B(4)\}$
$\Rightarrow 0.51 \geq Min\{0.57, 0.51\}$
$\Rightarrow 0.51 \geq 0.51$
Now $\mu_{A\oplus B}(3) \geq Min\{\mu_{A\oplus B}(2), \mu_{A\oplus B}(4)\}$
$Max\{Min\{0.43, 0.49\}, Min\{0.57, 0.51\}\} \geq$
$Min\{Max\{Min\{0.43, 0.43\}, Min\{0.57, 0.57\}, \{Max\{Min\{0.66, 0.49\}, Min\{0.34, 0.51\}\}$
$\Rightarrow Max\{0.43, 0.51\} \geq Min\{Max\{0.43, 0.57\}, Max\{0.49, 0.34\}\}$
$\Rightarrow 0.51 \geq Min\{0.57, 0.49\}$
$\Rightarrow 0.51 \geq 0.49$
$\Rightarrow \lambda_A(x) \leq Max\{\lambda_A((x * z) * (y * z)), \lambda_A(y)\}$
$\Rightarrow \lambda_A(3) \leq Max\{\lambda_A(2), \lambda_A(4)\}$
$\Rightarrow 0.54 \leq Max\{0.54, 0.33\}$
$\Rightarrow 0.54 \leq 0.54$
And $\lambda_B(x) \leq Max\{\lambda_B((x * z) * (y * z)), \lambda_B(y)\}$
$\Rightarrow \lambda_B(3) \leq Max\{\lambda_B(2), \lambda_B(4)\}$
$\Rightarrow 0.45 \leq Max\{0.41, 0.45\}$
$\Rightarrow 0.45 \leq 0.45$
Now $\lambda_{A\oplus B}(3) \leq Max\{\lambda_{A\oplus B}(2), \lambda_{A\oplus B}(4)\}$
$Max\{Min\{0.54, 0.55\}, Min\{0.46, 0.45\}\} \leq$
$Max\{Max\{Min\{0.54, 0.59\}, Min\{0.46, 0.41\}, \{Max\{Min\{0.33, 0.55\}, Min\{0.67, 0.45\}\}$
$\Rightarrow Max\{0.54, 0.45\} \leq Max\{Max\{0.54, 0.41\}, Max\{0.33, 0.45\}\}$
$\Rightarrow 0.54 \leq Max\{0.54, 0.45\}$
$\Rightarrow 0.54 \leq 0.54$

Example 3.9. *Let $X = \{0, 1, 2, 3, 4\}$ be a BCI-algebra with the following Cayley table:*

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

We define an intuitionistic fuzzy set $A = < X, \mu_A, \lambda_A >$ in X

X	0	1	2	3	4
μ_A	.77	.66	.43	.43	.66
λ_A	.22	.33	.54	.54	.33

By routine calculation, we know that $A = < X, \mu_A, \lambda_A >$ is an intuitionistic fuzzy H-ideal of X.
We define an intuitionistic fuzzy set $B = < X, \mu_B, \lambda_B >$ in X .

X	0	1	2	3	4
μ_B	.78	.51	.57	.51	.51
λ_B	.21	.45	.41	.45	.45

By routine calculation, we know that $B = < X, \mu_B, \lambda_B >$ is an intuitionistic fuzzy H-ideal of X. Then A and B are intuitionistic fuzzy H-ideal Of X. Obviously, the bounded product is an intuitionistic fuzzy H- ideal of X,

Since $x = 3, y = 4, z = 0$.
$\Rightarrow \mu_A(x * z) \geq \min\{\mu_A(x * (y * z)), \mu_A(y)\}$
$\Rightarrow \mu_A(3 * 0) \geq \min\{\mu_A(3 * (4 * 0)), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(3 * 4), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(2), \mu_A(4)\}$
$\Rightarrow 0.43 \geq Min\{0.43, 0.66\}$
$\Rightarrow 0.43 \geq 0.43$
And $\mu_B(3) \geq \min\{\mu_B(2), \mu_B(4)\}$
$\Rightarrow 0.51 \geq Min\{0.57, 0.51\}$
$\Rightarrow 0.51 \geq 0.51$
Now $\mu_{A\odot B}(3) \geq Min\{\mu_{A\odot B}(2), \mu_{A\odot B}(4)\}$
$\Rightarrow Max\{0, -0.06\} \geq Min\{Max\{0, 0\}, Max\{0, 0.17\}\}$
$\Rightarrow 0 \geq Min\{0, 0.17\}$
$\Rightarrow 0 \geq 0$
$\Rightarrow \lambda_A(x) \leq Max\{\lambda_A((x * z) * (y * z)), \lambda_A(y)\}$
$\Rightarrow \lambda_A(3) \leq Max\{\lambda_A(2), \lambda_A(4)\}$
$\Rightarrow 0.54 \leq Max\{0.54, 0.33\}$
$\Rightarrow 0.54 \leq 0.54$
And $\lambda_B(x) \leq Max\{\lambda_B((x * z) * (y * z)), \lambda_B(y)\}$
$\Rightarrow \lambda_B(3) \leq Max\{\lambda_B(2), \lambda_B(4)\}$
$\Rightarrow 0.45 \leq Max\{0.41, 0.45\}$
$\Rightarrow 0.45 \leq 0.45$
Now $\lambda_{A\odot B}(3) \leq Max\{\lambda_{A\odot B}(2), \lambda_{A\odot B}(4)\}$
$\Rightarrow Max\{0, -0.01\} \leq Max\{Max\{0, -0.05\}, Max\{0, -0.22\}\}$
$\Rightarrow 0 \leq Max\{0, 0\}$
$\Rightarrow 0 \leq 0$

Example 3.10. *Let $X = \{0, 1, 2, 3, 4\}$ be a BCI-algebra with the following Cayley table:*

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

We define an intuitionistic fuzzy set $A = < X, \mu_A, \lambda_A >$ in X

X	0	1	2	3	4
μ_A	.77	.66	.43	.43	.66
λ_A	.22	.33	.54	.54	.33

By routine calculation, we know that $A = < X, \mu_A, \lambda_A >$ is an intuitionistic fuzzy H-ideal of X.
We define an intuitionistic fuzzy set $B = < X, \mu_B, \lambda_B >$ in X .

X	0	1	2	3	4
μ_B	.78	.51	.57	.51	.51
λ_B	.21	.45	.41	.45	.45

By routine calculation, we know that $B = < X, \mu_B, \lambda_B >$ is an intuitionistic fuzzy H-ideal of X. Then A and B are intuitionistic fuzzy H-ideal Of X. Obviously, the bounded sum is not an intuitionistic fuzzy H- ideal of X,

Since $x = 3, y = 4, z = 0$.
$\Rightarrow \mu_A(x * z) \geq \min\{\mu_A(x * (y * z)), \mu_A(y)\}$
$\Rightarrow \mu_A(3 * 0) \geq \min\{\mu_A(3 * (4 * 0)), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(3 * 4), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(2), \mu_A(4)\}$
$\Rightarrow 0.43 \geq Min\{0.43, 0.66\}$
$\Rightarrow 0.43 \geq 0.43$
And $\mu_B(3) \geq \min\{\mu_B(2), \mu_B(4)\}$
$\Rightarrow 0.51 \geq Min\{0.57, 0.51\}$
$\Rightarrow 0.51 \geq 0.51$
Now $\mu_{A\oplus B}(3) \geq Min\{\mu_{A\oplus B}(2), \mu_{A\oplus B}(4)\}$
$\Rightarrow Min\{1, 0.94\} \geq Min\{Min\{1, 1\}, Min\{1, 1.17\}\}$
$\Rightarrow 0.94 \geq Min\{1, 1\}$
$\Rightarrow 0.94 \ngeq 1$
$\Rightarrow \lambda_A(x) \leq Max\{\lambda_A((x * z) * (y * z)), \lambda_A(y)\}$
$\Rightarrow \lambda_A(3) \leq Max\{\lambda_A(2), \lambda_A(4)\}$
$\Rightarrow 0.54 \leq Max\{0.54, 0.33\}$
$\Rightarrow 0.54 \leq 0.54$
And $\lambda_B(x) \leq Max\{\lambda_B((x * z) * (y * z)), \lambda_B(y)\}$
$\Rightarrow \lambda_B(3) \leq Max\{\lambda_B(2), \lambda_B(4)\}$
$\Rightarrow 0.45 \leq Max\{0.41, 0.45\}$
$\Rightarrow 0.45 \leq 0.45$
Now $\lambda_{A\oplus B}(3) \leq Max\{\lambda_{A\oplus B}(2), \lambda_{A\oplus B}(4)\}$
$\Rightarrow Min\{1, 0.99\} \leq Max\{Min\{1, 0.95\}, Min\{1, 0.78\}\}$
$\Rightarrow .99 \leq Max\{.95, 0.78\}$
$\Rightarrow .99 \nleq .95$

Example 3.11. *Let $X = \{0, 1, 2, 3, 4\}$ be a BCI-algebra with the following Cayley table:*

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

We define an intuitionistic fuzzy set $A = < X, \mu_A, \lambda_A >$ in X

X	0	1	2	3	4
μ_A	.77	.66	.43	.43	.66
λ_A	.22	.33	.54	.54	.33

By routine calculation, we know that $A = < X, \mu_A, \lambda_A >$ is an intuitionistic fuzzy H-ideal of X.
We define an intuitionistic fuzzy set $B = < X, \mu_B, \lambda_B >$ in X .

X	0	1	2	3	4
μ_B	.78	.51	.57	.51	.51
λ_B	.21	.45	.41	.45	.45

By routine calculation, we know that $B = < X, \mu_B, \lambda_B >$ is an intuitionistic fuzzy H-ideal of X. Then A and B are intuitionistic fuzzy H-ideal Of X. Obviously, then $A@B$ is not an intuitionistic fuzzy H- ideal of X,
Since $x = 3, y = 4, z = 0$.
$\Rightarrow \mu_A(x * z) \geq \min\{\mu_A(x * (y * z)), \mu_A(y)\}$
$\Rightarrow \mu_A(3 * 0) \geq \min\{\mu_A(3 * (4 * 0)), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(3 * 4), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(2), \mu_A(4)\}$
$\Rightarrow 0.43 \geq Min\{0.43, 0.66\}$
$\Rightarrow 0.43 \geq 0.43$
And $\mu_B(3) \geq \min\{\mu_B(2), \mu_B(4)\}$
$\Rightarrow 0.51 \geq Min\{0.57, 0.51\}$
$\Rightarrow 0.51 \geq 0.51$
Now $\mu_{A@B}(3) \geq Min\{\mu_{A@B}(2), \mu_{A@B}(4)\}$
$\Rightarrow 0.47 \geq Min\{0.5, 0.585\}$
$\Rightarrow 0.47 \ngeq 0.5$
$\Rightarrow \lambda_A(x) \leq Max\{\lambda_A((x * z) * (y * z)), \lambda_A(y)\}$
$\Rightarrow \lambda_A(3) \leq Max\{\lambda_A(2), \lambda_A(4)\}$
$\Rightarrow 0.54 \leq Max\{0.54, 0.33\}$
$\Rightarrow 0.54 \leq 0.54$
And $\lambda_B(x) \leq Max\{\lambda_B((x * z) * (y * z)), \lambda_B(y)\}$
$\Rightarrow \lambda_B(3) \leq Max\{\lambda_B(2), \lambda_B(4)\}$
$\Rightarrow 0.45 \leq Max\{0.41, 0.45\}$
$\Rightarrow 0.45 \leq 0.45$
Now $\lambda_{A@B}(3) \leq Max\{\lambda_{A@B}(2), \lambda_{A@B}(4)\}$
$\Rightarrow 0.495 \leq Max\{0.475, 0.393\}$
$\Rightarrow 0.495 \nleq 0.475$

Example 3.12. *Let $X = \{0, 1, 2, 3, 4\}$ be a BCI-algebra with the following Cayley table:*

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

We define an intuitionistic fuzzy set $A = < X, \mu_A, \lambda_A >$ in X

X	0	1	2	3	4
μ_A	.77	.66	.43	.43	.66
λ_A	.22	.33	.54	.54	.33

By routine calculation, we know that $A = < X, \mu_A, \lambda_A >$ is an intuitionistic fuzzy H-ideal of X.
We define an intuitionistic fuzzy set $B = < X, \mu_B, \lambda_B >$ in X .

X	0	1	2	3	4
μ_B	.78	.51	.57	.51	.51
λ_B	.21	.45	.41	.45	.45

By routine calculation, we know that $B = < X, \mu_B, \lambda_B >$ is an intuitionistic fuzzy H-ideal of X. Then A and B are intuitionistic fuzzy H-ideal Of X. Obviously, then $A\$B$ is not an intuitionistic fuzzy H- ideal of X,

Since $x = 3, y = 4, z = 0$.
$\Rightarrow \mu_A(x * z) \geq \min\{\mu_A(x * (y * z)), \mu_A(y)\}$
$\Rightarrow \mu_A(3 * 0) \geq \min\{\mu_A(3 * (4 * 0)), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(3 * 4), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(2), \mu_A(4)\}$
$\Rightarrow 0.43 \geq Min\{0.43, 0.66\}$
$\Rightarrow 0.43 \geq 0.43$
And $\mu_B(3) \geq \min\{\mu_B(2), \mu_B(4)\}$
$\Rightarrow 0.51 \geq Min\{0.57, 0.51\}$
$\Rightarrow 0.51 \geq 0.51$
Now $\mu_{A\$B}(3) \geq Min\{\mu_{A\$B}(2), \mu_{A\$B}(4)\}$
$\Rightarrow 0.4682 \geq Min\{0.4950, 0.5801\}$
$\Rightarrow 0.4682 \ngeq 0.4950$
$\Rightarrow \lambda_A(x) \leq Max\{\lambda_A((x * z) * (y * z)), \lambda_A(y)\}$
$\Rightarrow \lambda_A(3) \leq Max\{\lambda_A(2), \lambda_A(4)\}$
$\Rightarrow 0.54 \leq Max\{0.54, 0.33\}$
$\Rightarrow 0.54 \leq 0.54$
And $\lambda_B(x) \leq Max\{\lambda_B((x * z) * (y * z)), \lambda_B(y)\}$
$\Rightarrow \lambda_B(3) \leq Max\{\lambda_B(2), \lambda_B(4)\}$
$\Rightarrow 0.45 \leq Max\{0.41, 0.45\}$
$\Rightarrow 0.45 \nleq 0.45$
Now $\lambda_{A\$B}(3) \leq Max\{\lambda_{A\$B}(2), \lambda_{A\$B}(4)\}$
$\Rightarrow 0.4929 \leq Max\{0.4705, 0.3853\}$
$\Rightarrow 0.4929 \nleq 0.4705$

Example 3.13. *Let $X = \{0, 1, 2, 3, 4\}$ be a BCI-algebra with the following Cayley table:*

$*$	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

We define an intuitionistic fuzzy set $A = < X, \mu_A, \lambda_A >$ in X

X	0	1	2	3	4
μ_A	.77	.66	.43	.43	.66
λ_A	.22	.33	.54	.54	.33

By routine calculation, we know that $A = < X, \mu_A, \lambda_A >$ is an intuitionistic fuzzy H-ideal of X.

We define an intuitionistic fuzzy set $B = < X, \mu_B, \lambda_B >$ in X .

X	0	1	2	3	4
μ_B	.78	.51	.57	.51	.51
λ_B	.21	.45	.41	.45	.45

By routine calculation, we know that $B = < X, \mu_B, \lambda_B >$ is an intuitionistic fuzzy H-ideal of X. Then A and B are intuitionistic fuzzy H-ideal Of X. Obviously, then $A\#B$ is not an intuitionistic fuzzy H- ideal of X,

Since $x = 3, y = 4, z = 0$.
$\Rightarrow \mu_A(x * z) \geq \min\{\mu_A(x * (y * z)), \mu_A(y)\}$
$\Rightarrow \mu_A(3 * 0) \geq \min\{\mu_A(3 * (4 * 0)), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(3 * 4), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(2), \mu_A(4)\}$

$\Rightarrow 0.43 \geq Min\{0.43, 0.66\}$
$\Rightarrow 0.43 \geq 0.43$
And $\mu_B(3) \geq \min\{\mu_B(2), \mu_B(4)\}$
$\Rightarrow 0.51 \geq Min\{0.57, 0.51\}$
$\Rightarrow 0.51 \geq 0.51$
Now $\mu_{A\#B}(3) \geq Min\{\mu_{A\#B}(2), \mu_{A\#B}(4)\}$
$\Rightarrow 0.4665 \geq Min\{0.4902, 0.5753\}$
$\Rightarrow 0.4665 \not\geq 0.4902$
$\Rightarrow \lambda_A(x) \leq Max\{\lambda_A((x*z)*(y*z)), \lambda_A(y)\}$
$\Rightarrow \lambda_A(3) \leq Max\{\lambda_A(2), \lambda_A(4)\}$
$\Rightarrow 0.54 \leq Max\{0.54, 0.33\}$
$\Rightarrow 0.54 \leq 0.54$
And $\lambda_B(x) \leq Max\{\lambda_B((x*z)*(y*z)), \lambda_B(y)\}$
$\Rightarrow \lambda_B(3) \leq Max\{\lambda_B(2), \lambda_B(4)\}$
$\Rightarrow 0.45 \leq Max\{0.41, 0.45\}$
$\Rightarrow 0.45 \leq 0.45$
Now $\lambda_{A\#B}(3) \leq Max\{\lambda_{A\#B}(2), \lambda_{A\#B}(4)\}$
$\Rightarrow 0.4909 \leq Max\{0.4661, 0.3807\}$
$\Rightarrow 0.4909 \not\leq 0.4661$

Example 3.14. *Let $X = \{0, 1, 2, 3, 4\}$ be a BCI-algebra with the following Cayley table:*

*	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	0	0
2	2	2	0	0	2
3	3	2	1	0	2
4	4	1	4	1	0

We define an intuitionistic fuzzy set $A = < X, \mu_A, \lambda_A >$ in X

X	0	1	2	3	4
μ_A	.77	.66	.43	.43	.66
λ_A	.22	.33	.54	.54	.33

By routine calculation, we know that $A = < X, \mu_A, \lambda_A >$ is an intuitionistic fuzzy H-ideal of X.
We define an intuitionistic fuzzy set $B = < X, \mu_B, \lambda_B >$ in X .

X	0	1	2	3	4
μ_B	.78	.51	.57	.51	.51
λ_B	.21	.45	.41	.45	.45

By routine calculation, we know that $B = < X, \mu_B, \lambda_B >$ is an intuitionistic fuzzy H-ideal of X. Then A and B are intuitionistic fuzzy H-ideal Of X. Obviously, then $A * B$ is not an intuitionistic fuzzy H- ideal of X,
Since $x = 3, y = 4, z = 0$.
$\Rightarrow \mu_A(x*z) \geq \min\{\mu_A(x*(y*z)), \mu_A(y)\}$
$\Rightarrow \mu_A(3*0) \geq \min\{\mu_A(3*(4*0)), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(3*4), \mu_A(4)\}$
$\Rightarrow \mu_A(3) \geq \min\{\mu_A(2), \mu_A(4)\}$
$\Rightarrow 0.43 \geq Min\{0.43, 0.66\}$
$\Rightarrow 0.43 \geq 0.43$
And $\mu_B(3) \geq \min\{\mu_B(2), \mu_B(4)\}$
$\Rightarrow 0.51 \geq Min\{0.57, 0.51\}$
$\Rightarrow 0.51 \geq 0.51$
Now $\mu_{A*B}(3) \geq Min\{\mu_{A*B}(2), \mu_{A*B}(4)\}$
$\Rightarrow 0.3854 \geq Min\{0.4015, 0.4376\}$

$\Rightarrow 0.3854 \ngeq 0.4015$
$\Rightarrow \lambda_A(x) \leq Max\{\lambda_A((x*z)*(y*z)), \lambda_A(y)\}$
$\Rightarrow \lambda_A(3) \leq Max\{\lambda_A(2), \lambda_A(4)\}$
$\Rightarrow 0.54 \leq Max\{0.54, 0.33\}$
$\Rightarrow 0.54 \leq 0.54$
And $\lambda_B(x) \leq Max\{\lambda_B((x*z)*(y*z)), \lambda_B(y)\}$
$\Rightarrow \lambda_B(3) \leq Max\{\lambda_B(2), \lambda_B(4)\}$
$\Rightarrow 0.45 \leq Max\{0.41, 0.45\}$
$\Rightarrow 0.45 \leq 0.45$
Now $\lambda_{A*B}(3) \leq Max\{\lambda_{A*B}(2), \lambda_{A*B}(4)\}$
$\Rightarrow 0.3982 \leq Max\{0.3888, 0.3395\}$
$\Rightarrow 0.3982 \nleq 0.3888$.

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