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Further results on nonsplit dom strong domination number

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Abstract

A subset S of V is called a dom strong dominating set if for every vertex $v \in V - S$, there exists $u_1, u_2 \in S$ such that u_1v, $u_2v \in E(G)$ and $d(u_1) \geq d(v)$. The minimum cardinality of a dom strong dominating set is called the dom strong domination number and is denoted by $\gamma_{ds}(G)$. A dom strong dominating set S is said to be a non split dom strong dominating set if the induced subgraph $\langle V - S \rangle$ is connected. The minimum cardinality of a non split dom strong dominating set is called the non split dom strong domination number of a graph and is denoted by $\gamma_{nsds}(G)$. The connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected or trivial graph. In this paper, we find an upper bound for the sum of nonsplit dom strong domination number and connectivity of a graph and characterise the corresponding extremal graphs.

Keywords: Nonsplit dom strong domination number and connectivity.

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1 Introduction

In this paper we consider simple and undirected graphs. The sets V and E are the vertex set and the edge set of the graph G respectively. The connectivity $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected or trivial graph. The degree of a vertex u in G is the number of edges incident with u and is denoted by $d(u)$. The minimum and maximum degree of a graph G is denoted by $\delta(G)$ and $\Delta(G)$, respectively. For terminology we refer to Chartrand and Lesniak [1].

A vertex dominates itself and its neighbors. A set $S \subset V$ is a dominating set of G if every vertex of G is dominated by some vertex in S. The minimum cardinality of a dominating set is called the domination number of G and is denoted by $\gamma(G)$. A subset S of V is called a dom strong dominating set if for every $v \in V - S$, there exists $u_1, u_2 \in S$ such that u_1v, $u_2v \in E(G)$ and $(u_1) \geq (v)$. The minimum cardinality of a dom strong dominating set is called the dom strong domination number and is denoted by $\gamma_{ds}(G)$. The nonsplit dom strong domination number was introduced by G.Mahadevan et.al. [5]. A dom strong dominating set S is said to be a non split dom strong dominating set(NSDSD-set) if the induced subgraph $\langle V - S \rangle$ is connected. The minimum cardinality of a non split dom strong dominating set is called the non split dom strong domination number of a graph and is denoted by $\gamma_{nsds}(G)$.

Several authors have studied the problem of obtaining an upper bound for the sum of a dominating parameter and a graph theoretic parameter and characterized the corresponding extremal graphs. In [6], the authors found an upper bound for the sum of the domination number and connectivity of graphs and charecterized the corresponding extremal graphs. Motivated by the above, we find an upper bound for sum

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of the nonsplit dom strong domination number and connectivity of graphs and characterize the corresponding extremal graphs. Also we characterize the graphs with this sum is greater than or equal to $2n-4$.

2 Preliminaries

Theorem 2.1. [1] *For any graph* G, $\kappa(G) \leq \delta(G)$.

Theorem 2.2. [5] $2 \leq \gamma_{nsds}(G) \leq n$.

Theorem 2.3. [5] *For any connected graph* G, $\gamma_{nsds}(G) = n$ *if and only if* G *is a star.*

Notation 1. *$C_n(P_k)$ is the graph by attaching the end vertices of P_k path graph to any one vertex of the cycle graph. $K_n(P_k)$ is the graph by attaching the end vertices of P_k path graph to any one vertex of the complete graph. $C_n(P_k, P_m, 0, \ldots)$ is the graph by attaching an end vertex of P_k path graph to any one of the vertex C_n and attaching the end vertices of P_m path graph to another vertex of C_n. $K_n(P_k, P_m, 0, \ldots)$ is the graph by attaching the end vertex of P_k path graph to any one of the vertex K_n and attaching the end vertices of P_m path graph to another vertex of K_n. $K_n(u(P_n, P_m), 0, \ldots)$ is the graph obtained from K_n by attaching the end vertices of P_n and P_m path graph to $u \in V(G)$ which is one of the vertex in K_n graph. $K_n(P_n, P_m, \ldots)$ is the graph obtained from K_n by attaching the every end vertices of P_n, $P_m, \ldots$ of path graph to every vertex in K_n graph. $K_n(mP_k)$ is the graph obtained from K_n by attaching the m times P_k path graph to any one vertex of K_n.*

3 Main Results

Theorem 3.1. *For any connected graph* G, $\gamma_{nsds}(G) + \kappa(G) \leq 2n-1$ *and the equality holds if and only if* G *is a complete graph of order* 2.

Proof. $\gamma_{nsds}(G) + \kappa(G) \leq n + \delta \leq n + n - 1 = 2n-1$.
Let $\gamma_{nsds}(G) + \kappa(G) = 2n-1$. Then $\gamma_{nsds}(G) = n$ and $\kappa(G) = n-1$ which gives G is a star as well as a complete graph. Hence $G \cong K_2$. The converse is obvious. □

Theorem 3.2. *For any connected graph* G, $\gamma_{nsds}(G) + \kappa(G) = 2n-2$ *if and only if* $G \cong K_3$ *or* $K_{1,2}$

Proof. Let $\gamma_{nsds}(G) + \kappa(G) = 2n-2$. Then there are two cases to consider.
(i)$\gamma_{nsds}(G) = n$ and $\kappa(G) = n-2$ (ii)$\gamma_{nsds}(G) = n-1$ and $\kappa(G) = n-1$
Case 1.$\gamma_{nsds}(G) = n$ and $\kappa(G) = n-2$
Since $\gamma_{nsds}(G) = n, G$ is a star and hence $\kappa(G) = 1$ which gives $n = 3$. Thus $G \cong K_{1,2}$.

Case 2.$\gamma_{nsds}(G) = n-1$ and $\kappa(G) = n-1$
Since $k(G) = n-1, G$ is a complete graph, this gives $\gamma_{nsds}(G) = 2$. Then $n = 3$ and hence $G \cong K_3$. The converse is obvious.

□

Theorem 3.3. *For any connected graph* G, $\gamma_{nsds}(G) + \kappa(G) = 2n-3$ *if and only if* $G \cong K_4$ *or* $K_4 - e$ *or* $K_{1,3}$ *or* C_4

Proof. Let $\gamma_{nsds}(G) + \kappa(G) = 2n-3$. Then there are three cases to consider.
(i)$\gamma_{nsds}(G) = n$ and $\kappa(G) = n-3$ (ii)$\gamma_{nsds}(G) = n-1$ and $\kappa(G) = n-2$ (iii)$\gamma_{nsds}(G) = n-2$ and $\kappa(G) = n-1$

Case 1.$\gamma_{nsds}(G) = n$ and $\kappa(G) = n-3$
Since $\gamma_{nsds}(G) = n$ we have G is a star and hence $\kappa(G) = 1$ which gives $n = 4$. Thus $G \cong K_{1,3}$.

Case 2.$\gamma_{nsds}(G) = n-1$ and $\kappa(G) = n-2$
Since $\kappa(G) = n-2$ we have $n-2 \leq \delta(G)$. If $\delta = n-1$ then $G \cong K_n$, which is a contradiction. Hence $\delta(G) = n-2$. Then $G \cong K_n - Q$ where Q is a matching in K_n. Then $\gamma_{nsds}(G) \leq 3$. If $\gamma_{nsds}(G) = 3$ then $n = 4$ and hence G is isomorphic to either C_4 *or* $K_4 - e$. If $\gamma_{nsds} = 2$ then $n = 3$ and hence $G \cong K_{1,2}$, which is a contradiction.

Case 3.$\gamma_{nsds}(G) = n-2$ and $\kappa(G) = n-1$

Since $\kappa(G) = n-1$, G is a complete graph. Since $\gamma_{nsds}(K_n) = 2$ we have $n = 4$. Hence $G \cong K_4$. The converse is obvious.

□

Theorem 3.4. *For any connected graph G,$\gamma_{nsds}(G) + \kappa(G) = 2n-4$ if and only if $G \cong K_5$ or $K_{1,4}$ or P_4 or $K_3(1, 0, 0)$ or $C_5 + e$ or $K_5 - Q$ where Q is the maximum matching in K_5 or the graph obtained from $K_{2,3}$ by joining the vertices of degree three by an edge.*

Proof. Let $\gamma_{nsds}(G) + \kappa(G) = 2n-4$. Then there are four cases to consider.
(i)$\gamma_{nsds}(G) = n$ and $\kappa(G) = n-4$ (ii)$\gamma_{nsds}(G) = n-1$ and $\kappa(G) = n-3$ (iii)$\gamma_{nsds}(G) = n-2$ and $\kappa(G) = n-2$ (iv)$\gamma_{nsds}(G) = n-3$ and $\kappa(G) = n-1$

Case 1.$\gamma_{nsds}(G) = n$ and $\kappa(G) = n-4$

Since $\gamma_{nsds}(G) = n$ we have G is a star and hence $\kappa(G) = 1$ which gives $n = 5$. Thus $G \cong K_{1,4}$.

Case 2.$\gamma_{nsds}(G) = n-1$ and $\kappa(G) = n-3$

Since $\kappa(G) = n-3$ we have $n-3 \leq \delta$. If $\delta = n-1$ then $G \cong K_n$ which is a contradiction. If $\delta = n-2$ then $G \cong K_n - Q$ where Q is a matching in K_n. Then $\gamma_{nsds}(G) \leq 3$. If $\gamma_{nsds}(G) = 3$ then $n = 4$. Hence $G \cong K_4 - e$ or C_4. For these two graphs $\kappa(G) \neq n-3$ which is a contradiction. Hence $\delta(G) = n-3$. Let $S = \{u_1, u_2, \ldots, u_{n-3}\}$ be the minimum vertex cut of G and let $V - S = \{y_1, y_2, y_3\}$.

Subcase 2.1. $< V - S >= \bar{K_3}$

Let us assume $< S >$ be connected. If $| S |= 1$ then G is isomorphic to $K_{1,3}$ which is a contradiction. If $| S |= 2$ then G is isomorphic to the graph obtained from $K_{2,3}$ by joining the vertices of degree 3 by an edge. Let $| S |\geq 3$ and let $d(u_2) \geq d(u_i)$, i = 1 or 3. Then $\{u_1, u_2, y_1, y_2\}$ is a NSDSD-set of G. Hence $\gamma_{nsds} \leq 4$. Then $n \leq 5$ which is a contradiction.

Subcase 2.2. $< V - S >= K_1 \cup K_2$

Let $y_1y_2 \in E(G)$. Then y_3 is adjacent to all the vertices of S and y_1, y_2 are not adjacent to at most one vertex of S. Let us assume $d(y_1) = n-2$. Suppose $\langle S \rangle$ is disconnected. Then $| S |\leq 3$. If $| S |= 2$ then G is isomorphic to $C_5 + e$. If $| S |= 3$ then we get the graphs with $\gamma_{nsds} \leq 4$ which is a contradiction. Suppose $\langle S \rangle$ is connected. If $d(u_i) \leq n-2$ for all i and $| S |\geq 2$ then $\{y_1, y_2, y_3\}$ is a NSDSD-set of G which is a contradiction. If $| S |= 1$ then G is isomorphic to P_4. Let $u_1 \in S$ such that $d(u_1) = n-1$. If $| S |\geq 3$ then $V - \{y_1, y_2\}$ is a NSDSD-set of G which is a contradiction. If $| S |= 1$ then G is isomorphic to $C_3(1, 0, 0)$. If $| S |= 2$ then we obtain the graphs with $\gamma_{nsds}(G) \neq n-1$.

Suppose $d(y_1) = d(y_2) = n-3$. If $| S |\geq 3$ then $V - \{y_1, y_2\}$ is a NSDSD-set of G which is a contradiction. If $| S |= 2$ then we obtain the graphs with $\gamma_{nsds}(G) + \kappa(G) \neq 2n-4$

Case 3.$\gamma_{nsds}(G) = n-2$ and $\kappa(G) = n-2$

If $\kappa(G) = n-2$, then $n-2 \leq \delta(G)$. If $\delta(G) = n-1$ then $G \cong K_n$, which is a contradiction and we have $\delta(G) = n-2$. Then $G \cong K_n - Q$ where Q is the matching in K_n. Then $\gamma_{nsds}(G) \leq 3$. If $\gamma_{nsds}(G) = 3$ then $n = 5$. Hence G is $K_5 - Q$.If $| Q |= 1$ then $G \cong K_5 - Q$ where Q is a matching in K_5 with $| Q |= 2$. If $\gamma_{nsds}(G) = 2$ then $n = 4$. Thus $G \cong K_4 - e$ *or* C_4

Case 4.$\gamma_{nsds}(G) = n-3$ and $\kappa(G) = n-1$

If $\kappa(G) = n-1$, then $G \cong K_n$ on n vertices. But $\gamma_{nsds}(G) = 2$ then $n = 5$ and hence $G \cong K_5$. The converse is obvious.

□

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