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Double quadrilateral snakes on k-odd sequential harmonious labeling of graphs

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Abstract

The objective of this paper is to investigate some k-odd sequential harmonious labeling of graphs. In particular, we show that k-odd sequential harmonious labeling of double quadrilateral snakes ($2Q_x$-snakes) for each $x \geq 1$. We also prove that, $2mQ_x$-snakes are k-odd sequential harmonious labeling of graphs for each $m, x \geq 1$. Finally, we present some examples and verified to illustrate proposed theories.

Keywords: Labeling, Harmonious, k-odd sequential harmonious, Double quadrilateral snake.

2010 MSC: 34G20.

1 Introduction

All the graphs in this paper are finite,simple and undirected.The symbols $V(G)$ and $E(G)$ denote the vertex set and the edge set of a graph G.

The cardinality of the vertex set is called the order of G. The cardinality of the edge set is called the size of G. A graph with p vertices and q edges is called a (p, q) graph.

Definition : 1.1

If the vertices are assigned values subject to certain condition(s) then it is known as graph labeling.

Definition : 1.2

A graph G is said to be harmonious if there exist an injection $f : V(G) = \{0, 1, 2, \cdots, 2q-1\}$ such that the induced function $f^+ : E(G)$ =$\{0, 1, 2, \cdots, 2q-1\}$ defined by $f^+(uv) = (f(u) + f(v))(\mod q-1)$ is a bijection and f is said to be harmonious labeling of G.

Definition : 1.3

An odd sequential harmonious labeling if there exist an injection f from the vertex set V to $\{0, 1, 2, \cdots, 2q-1\}$ such that the induced mapping f^+ from the edge set E to $\{1, 3, 5, \cdots, 2q-1\}$ defined by

$$f^+(uv) = \begin{cases} f(u) + f(v), & \text{if } f(u) + f(v) \text{ is even} \\ f(u) + f(v) + 1, & \text{if } f(u) + f(v) \text{ is odd are distinct.} \end{cases}$$

A graph G is said to be an odd sequential harmonious graph if it admits an odd sequential harmonious labeling.

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Definition : 1.4

For any integer $k \geq 1$, A labeling is an k-odd sequential harmonious labeling if there exist an injection f from the vertex set V to $\{k-1, k, k+1, \cdots, k+2q-2\}$ such that the induced mapping f^+ from the edge set E to $\{2k-1, 2k+1, 2k+3, \cdots, 2k+2q-3\}$ defined by

$$f^+(uv) = \begin{cases} f(u)+f(v), & \text{if } f(u)+f(v) \text{ is even} \\ f(u)+f(v)+1, & \text{if } f(u)+f(v) \text{ is odd are distinct.} \end{cases}$$

A graph G is said to be an k-odd sequential harmonious graph if it admits an k-odd sequential harmonious labeling.

In this paper, we investigate Double Quadrilateral Snakes on k-odd sequential harmonious labeling of graphs. Throughout this paper, k denote any positive integer ≥ 1. For brevity, we use k-OSHL for k-odd sequential harmonious labeling.

2 Main Results

Definition : 2.1

The Quadrilateral snake Q_x is obtained from the $v_1, v_2, v_3 \ldots, v_n$ by joining v_i and v_{i+1} to new vertices u_{2i-1} and u_{2i}. That is, every edge of a path is replaced by a cycle C_4.

Definition : 2.2

Let Q_x be the Quadrilateral snake is obtained from the path $v_1, v_2, v_3, \cdots, v_n$. Then the double quadrilateral snake $D(Q_x)$ is obtained from Q_x by adding new vertices $w_1, w_2, w_3, \cdots, w_{2n-2}$ and new edges $v_i w_{2i-2}$ for $2 \leq i \leq n$ and $w_{2i-1}\ w_{2i}$, $v_i\ w_{2i-1}$ for $1 \leq i \leq n-1$.

Theorem : 2.3

Double quadrilateral snake is a k-odd sequential harmonious graph for each $x \geq 1$.

Proof. Let $2Q_x$ - snake be a double quadrilateral snake.

Let the vertices of $2Q_x$ be $\{v_i : 1 \leq i \leq n\} \cup \{u_i : 1 \leq i \leq 2n-2\} \cup \{w_i : 1 \leq i \leq 2n-2\}$.

The edges of $2Q_x$ be $\{v_i v_{i+1} : 1 \leq i \leq n-1\} \cup \{u_{2i-1} u_{2i} : 1 \leq i \leq n-1\} \cup \{w_{2i-1} w_{2i} : 1 \leq i \leq n-1\} \cup \{v_i u_{2i-1} : 1 \leq i \leq n-1\} \cup \{v_i u_{2i-2} : 2 \leq i \leq n\} \cup \{v_i w_{2i-1} : 1 \leq i \leq n-1\} \cup \{v_i w_{2i-2} : 2 \leq i \leq n\}$, which are denoted in Fig.2.3(a).

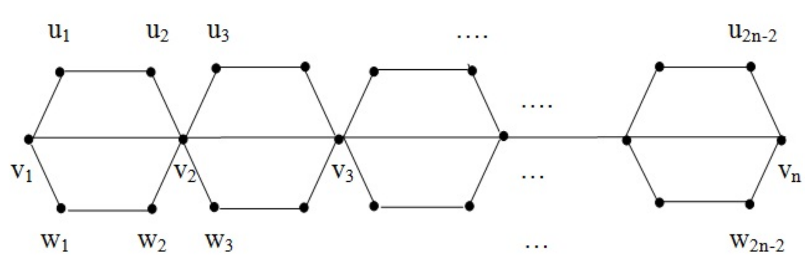

Fig.2.3(a) : D(Q$_x$) with ordinary labeling

We first, label the vertices of $D(Q_x)$ as follows,

Define $f : V(DQ_x)\mathring{\{}k-1, k, k+1, \cdots, k+2q-2\}$ by

$f(v_i) = 3i - 1 + k \qquad 1 \leq i \leq n$

$f(u_i) = 3i - 2 + k \qquad 1 \leq i \leq 2n-2$

$f(w_i) = 3i - k + 1 \qquad 1 \leq i \leq 2n-2$

Then the induced edge labels are

$$\left.\begin{aligned} f^+(v_iv_{i+1}) &= 2i+2k+1 \\ f^+(u_{2i-1}u_{2i}) &= 4i-2k+1 \end{aligned}\right\} 1 \le i \le n-1 \left\{\begin{aligned} f^+(w_{2i-1}w_{2i}) &= 6i-2k+1 \\ f^+(v_iu_{2i-1}) &= 6i+2k-1 \end{aligned}\right.$$

$f^+(v_iu_{2i-2}) = 6i+2k+1 \qquad 2 \le i \le n$
$f^+(v_iw_{2i-1}) = 6i+2k+3 \qquad 1 \le i \le n-1$
$f^+(v_iw_{2i-2}) = 6i+2k-3 \qquad 2 \le i \le n$

Clearly, the edge labels are odd and distinct, $f^+(E) = \{2k-1, 2k+1, 2k+3, \cdots, 2k+2q-3\}$. Hence, the graph $D(Q_x)$ is a k-odd sequential harmonious graph. □

EXAMPLE : 2-OSHL of $D(Q_3)$ is shown in Fig.2.3(b).

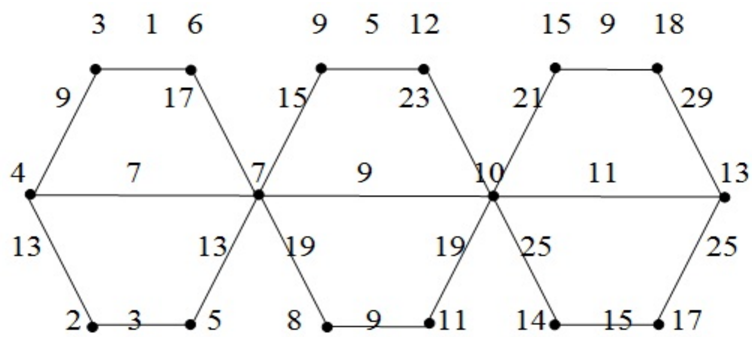

Fig.2.3(b) : 2-OSHL of 2(Q$_3$)

Theorem : 2.4

Double m-quadrilateral snake is a k-odd sequential harmonious graph for each $m, x \ge 1$.

Proof. Let $2mQ_x$ -snake be a double m-quadrilateral snake.

Let the vertices of $2mQ_x$ be $\{v_i : 1 \le i \le n\} \cup \{u_i : 1 \le i \le 2n-2\} \cup \{u_i^1 : 1 \le i \le 2n-2\} \cup \{u_i^2 : 1 \le i \le 2n-2\} \cup \{w_i : 1 \le i \le 2n-2\} \cup \{w_i^1 : 1 \le i \le 2n-2\} \cup \{w_i^2 : 1 \le i \le 2n-2\}$.

The edges of $2mQ_x$ be $\{v_iv_{i+1} : 1 \le i \le n-1\} \cup \{u_{2i-1}u_{2i} : 1 \le i \le n-1\} \cup \{u_{2i-1}^1u_{2i}^1 : 1 \le i \le n-1\} \cup \{u_{2i-1}^2u_{2i}^2 : 1 \le i \le n-1\} \cup \{w_{2i-1}w_{2i} : 1 \le i \le n-1\} \cup \{w_{2i-1}^1w_{2i}^1 : 1 \le i \le n-1\} \cup \{w_{2i-1}^2w_{2i}^2 : 1 \le i \le n-1\} \cup \{v_iu_{2i-1} : 1 \le i \le n-1\} \cup \{v_i^1u_{2i-1}^1 : 1 \le i \le n-1\} \cup \{v_i^2u_{2i-1}^2 : 1 \le i \le n-1\} \cup \{v_iu_{2i-2} : 2 \le i \le n\} \cup \{v_i^1u_{2i-2}^1 : 2 \le i \le n\} \cup \{v_i^2u_{2i-2}^2 : 2 \le i \le n\} \cup \{v_iw_{2i-1} : 1 \le i \le n-1\}\{v_i^1w_{2i-1}^1 : 1 \le i \le n-1\}\{v_i^2w_{2i-1}^2 : 1 \le i \le n-1\} \cup \{v_iw_{2i-2} : 2 \le i \le n\} \cup \{v_i^1w_{2i-2}^1 : 2 \le i \le n\} \cup \{v_i^2w_{2i-2}^2 : 2 \le i \le n\}$, which are denoted in Fig.2.4(a).

We first, label the vertices of $2m(Q_x)$ as follows,

Define $f : V(2mQ_x)\mathring{\{}k-1, k, k+1, \cdots, k+2q-2\}$ by

$f(v_i) = 4i-6+k \qquad 1 \le i \le n$

$$\left.\begin{aligned} f(u_i) &= 4i-4+k \\ f(u_i^1) &= 4i+4+k \\ f(u_i^2) &= 4i-2-k \end{aligned}\right\} 1 \le i \le 2n-2 \left\{\begin{aligned} f(w_i) &= 4i+k-1 \\ f(w_i^1) &= 4i+k+1 \\ f(w_i^2) &= 4i-k+1 \end{aligned}\right.$$

Then the induced edge labels are

$f^+(v_iv_{i+1}) = 8i+2k+3 \qquad 1 \le i \le n-1$

$$\left.\begin{aligned} f^+(u_{2i-1}u_{2i}) &= 8i+4k-1 \\ f^+(u_{2i-1}^1u_{2i}^1) &= 8i-4k+1 \\ f^+(u_{2i-1}^2u_{2i}^2) &= 8i-4k+3 \end{aligned}\right\} 1 \le i \le n-1 \left\{\begin{aligned} f^+(w_{2i-1}w_{2i}) &= 6i-2k-1 \\ f^+(w_{2i-1}^1w_{2i}^1) &= 6i-2k+3 \\ f^+(w_{2i-1}^2w_{2i}^2) &= 8i-2k+3 \end{aligned}\right.$$

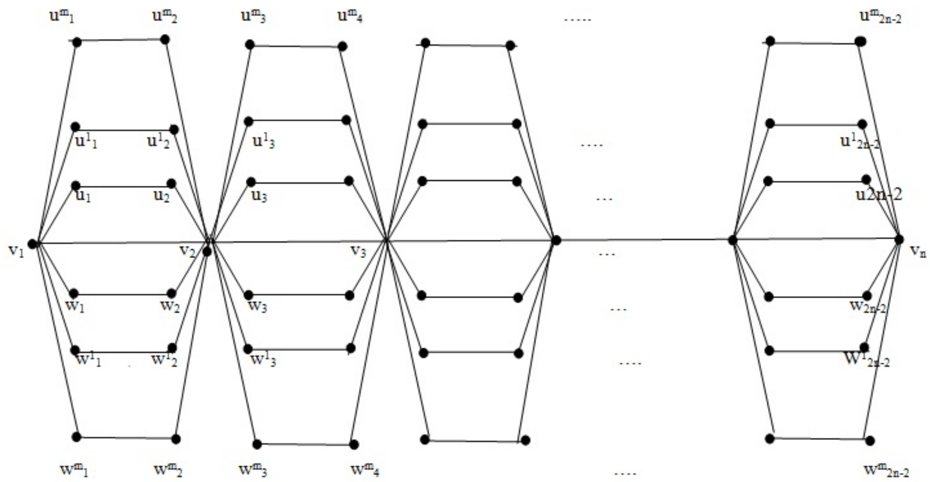

Fig.2.4(a) : 2m(Q_x) with ordinary labeling

$$\left.\begin{aligned} f^+(v_iu_{2i-1}) &= 8i+4k-3\\ f^+(v_iu^1_{2i-1}) &= 2i+2k+3\\ f^+(v_iu^2_{2i-1}) &= 2i+2k-3 \end{aligned}\right\}\ 1\le i\le n-1\ \left\{\begin{aligned} f^+(v_iw_{2i-1}) &= 8i+2k-3\\ f^+(v_iw^1_{2i-1}) &= 8i-2k-3\\ f^+(v_iw^2_{2i-1}) &= 10i-2k+1 \end{aligned}\right.$$

$$\left.\begin{aligned} f^+(v_iu_{2i-2}) &= 2i+2k-1\\ f^+(v_iu^1_{2i-2}) &= 4i+2k+1\\ f^+(v_iu^2_{2i-2}) &= 4i+2k-1 \end{aligned}\right\}\ 2\le i\le n\ \left\{\begin{aligned} f^+(v_iw_{2i-2}) &= 10i-2k-1\\ f^+(v_iw^1_{2i-2}) &= 10i+2k+1\\ f^+(v_iw^2_{2i-2}) &= 10i+2k-1 \end{aligned}\right.$$

Clearly, the edge labels are odd and distinct, $f^+(E) = \{2k-1, 2k+1, 2k+3, \cdots, 2k+2q-3\}$. Hence, the graph $2m(Q_x)$ is a k-odd sequential harmonious graph. □

EXAMPLE :

2-OSHL of $6(Q_3)$ is shown in Fig.2.4(b)

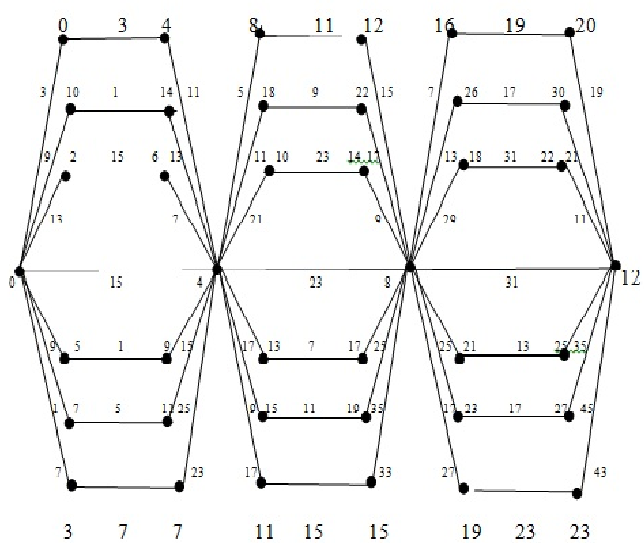

Fig.2.4(b) : 2-OSHL of 6(Q_3)

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