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THE DOM-CHROMATIC NUMBER OF A GRAPH

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Abstract

For a given χ-coloring of a graph $G = (V, E)$. A dominating set $S \subseteq V(G)$ is said to be *dom-coloring set* if it contains at least one vertex from each color class of G. The dom-chromatic number $\gamma_{dc}(G)$ is the minimum cardinality taken over all dom-coloring sets of G. In this paper, we initiate a study on $\gamma_{dc}(G)$ and its exact values for some classes of graphs have been established. Also its relationship with other graph theoretic parameters are investigated.

Keywords: Graph; Chromatic number; Domination number; Dom-chromatic number.

2010 MSC: 05C15, 05C70.

1 Introduction

All the graphs $G = (V, E)$ considered here are simple, finite and undirected, where $|V| = p$ denotes number of vertices and $|E| = q$ denotes number of edges of G. In general we use $\langle X \rangle$ to denote subgraph induced by the set of vertices X and $N(v)$ and $N[v]$ denote open and closed neighborhood of a vertex v, respectively. Let $deg(v)$ be the degree of vertex v and usual $\delta(G)$ the minimum degree and $\Delta(G)$ the maximum degree of a graph G. A subgraph H of a graph G is called a component of G, if H is maximally connected sub graph of G. Any undefined term in this paper may be found in Harary [5].

A coloring of a graph G is an assignment of colors to its vertices. So, that no two adjacent vertices have the same color. The set of all vertices with any one color is independent and is called a *color class*. An k-coloring of a graph G uses k-colors. The *chromatic number* $\chi(G)$ is defined as the minimum k for which G has an k-coloring. For complete review on theory of coloring we refer [8] and [10].

A set D of vertices in a graph G is a *dominating set* if every vertex in $V - D$ is adjacent to some vertex in D. *The domination number* $\gamma(G)$ is the minimum cardinality of a dominating set of G. A minimum dominating set of a graph G is called a γ*-set* of G. For more information on domination and its related parameters, we refer [1], [6], [7] and [14].

Analogously, we initiate the study on domination and coloring theory in terms of dom-chromatic number as follows: For a given χ-coloring of G, a dominating set $S \subseteq V(G)$ is said to be *dom-coloring set* if it contains at least one vertex from each color class of G. The *dom-chromatic number* $\gamma_{dc}(G)$ is the minimum cardinality taken over all dom-coloring sets of a graph G.

2 Bounds and characterization

First, we begin with couple of observations.

Observation 1. *In a graph G with χ-coloring, not all dominating sets are dom-coloring sets.*

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For example, consider a complete graph on five vertices say v_1, v_2, v_3, v_4, v_5. The dominating set is v_1, which is not dom-coloring set. The set $S = \{v_1, v_2, v_3, v_4, v_5\}$ is the dom-coloring set.

Observation 2. *Let G be a nontrivial graph with $n \geq 2$ components. Then*

$$\gamma_{dc}(G) \leq \gamma_{dc}(G_1) + \gamma_{dc}(G_2) + \cdots + \gamma_{dc}(G_n).$$

Theorem 2.1. *For any graph G,*

$$max\{\gamma(G), \chi(G)\} \leq \gamma_{dc}(G) \leq \gamma(G) + \chi(G) - 1.$$

The bounds are sharp.

Proof. Since every dom-coloring set is a dominating set of a graph G and hence $\gamma_{dc}(G) \geq \gamma(G)$. Then a dom-coloring set contains at least one vertex from each color class, we have $\gamma_{dc}(G) \geq \chi(G)$. Thus the lower bound follows.

Since every minimum dominating set contains at least one vertex with any color classes say c_1. Clearly, $S = D \cup T$ is a dom-coloring set with $|S| = \gamma(G)\chi(G) - 1$, where T consists of $\chi(G) - 1$ vertices with distinct colors, distinct from c_1 used in D. Hence the upper bound.

The lower bound attains for P_p, $p \geq 2$ vertices and upper bound attains for K_p or $\overline{K}_p$. □

To prove our next result we make use of the following definition.

Definition 2.1. *In a graph G, the minimum dominating is said to be optimized dominating set if it contains maximum number of vertices with distinct colors, where maximum is taken over all minimum dominating set. The optimized dominating set denoted by D_μ, where μ is the number of colors used in the optimized dominating set.*

For illustrative example of optimized dominating set D_μ. We consider a cycle C_4 with vertices in the form of $\{v_1v_2v_3v_4v_1\}$. The set of all minimum dominating sets are $D_1 = \{v_1, v_2\}$, $D_2 = \{v_2, v_3\}$, $D_3 = \{v_3, v_4\}$, $D_4 = \{v_4, v_1\}$, $D_5 = \{v_1, v_3\}$ and $D_6 = \{v_2, v_4\}$. Among all above said the minimum dominating set we can take any of D_1, D_2, D_3 and D_4 as D_μ. Since these contains vertices with two colors. D_5 and D_6 can not be taken as D_μ. Since these contains vertices with only one color.
Note: In any nontrivial graph G, $\mu \geq 1$.

Theorem 2.2. *For any graph G,*

$$\gamma_{dc}(G) \leq \gamma(G) + \chi(G) - \mu.$$

Proof. Let G be any nontrivial graph with optimized dominating set D_μ. We claim $S = D_\mu \cup T$ is a dom-coloring set, where T is the set of vertices with all distinct colors which are not used in D_μ. Since $D_\mu \subseteq S$, clearly S is a dominating set and also S contains at least one vertex from each color class. Hence, S is a dom-coloring set. The number of vertices in S is given by $|S| = |D_\mu| + \chi(G) - \mu$. Hence the result follows. □

Theorem 2.3. *For a graph G, $\gamma_{dc}(G) = 1$ if and only if $G = K_1$.*

Theorem 2.4. *For any graph G, $\gamma_{dc}(G) = p$ if and only if $G \cong K_p$ or $\overline{K}_p$.*

Proof. Suppose $\gamma_{dc}(G) = p$. On the contrary, if $G \neq K_p$ or $\overline{K}_p$, then the following cases arise.
Case 1. G is a connected graph.
In G, there exist at least two non adjacent vertices say u, v and each with degree at least one which receive same color. Hence, the set $V(G) - \{u\}$ is the dominating set which contain at least one vertex from each color class. Hence, $\gamma_{dc}(G) < p$, a contradiction.
Case 2. G is a disconnected graph.
Suppose G contains n components, say $G_1, G_2, \ldots, G_n$ and let G_j be the component which uses maximum number of colors in the χ-coloring of G. The set $V(G_j) - \{u\} \cup S$ is the dominating set, where u is any vertex in G_j and S is the union of all components other than G_j. There fore, $\gamma_{dc}(G) < p$, a contradiction. Hence $G = K_p$ or $\overline{K}_p$.
Conversely, suppose $G = K_p$ then $\chi(G) = p$. Hence $\gamma_{dc}(G) = p$. Now suppose $G = \overline{K}_p$ then $\gamma(G) = p$. Thus $\gamma_{dc}(G) = p$. □

Theorem 2.5. *Let G be a connected graph of order at least three with $\delta(G) \geq 2$. Then $\gamma_{dc}(G) = p - 1$ if and only if G is a noncomplete graph containing K_{p-1} as its induced subgraph.*

Proof. Let G be a connected graph of order at least three, $\delta(G) \geq 2$ and $\gamma_{dc}(G) = p - 1$. On the contrary suppose G contains no K_{p-1} as its induced subgraph then there exist at least four vertices v_1, v_2, v_3 and v_4 such that the edges $e_1 = (v_1 v_2)$ and $e_2 = (v_3 v_4)$ does not belongs to G. Hence by assigning the same color, say, c_1 to the vertices v_1, v_2 and by assigning the same color say c_2 to the vertices v_3, v_4, we get a $k \leq p - 2$ coloring of G. The set $V(G) - \{v_1, v_3\}$ is a dominating set containing at least one vertex from each color class. Hence $\gamma_{dc}(G) \leq p - 2$, which is a contradiction.
Conversely, suppose G is a non complete graph containing K_{p-1} as its induced subgraph. The graph G require exactly $p - 1$ colors. Thus the set $V(K_{p-1})$ is a minimum dom-coloring set. Hence the proof. □

Theorem 2.6. *Let G be a connected graph with $p \geq 4$ vertices. If $\delta(G) \geq 2$ satisfying the following conditions:*
(i) G contains K_{p-2} as its induced subgraph.
(ii) G contains four vertices v_1, v_2, v_3 and v_4 such that the edges $e_1 = (v_1 v_2)$ and $e_2 = (v_3 v_4)$ does not belongs to G, then $\gamma_{dc}(G) = p - 2$.

Proof. Let G be a connected graph of order at least four vertices with $\delta(G) \geq 2$. Then by condition (i) the vertices of K_{p-2} require $p - 2$ colors and dominate other two vertices say v_1 and v_3 and by condition (ii) the colors given to v_1 and v_3 are already used in $V(K_{p-2})$. Thus, $V(K_{p-2})$ is the minimum dom-coloring set with cardinality $p - 2$ vertices. Hence the result follows. □

Definition 2.2. *A dominator coloring of a graph G is a proper coloring of G in which every vertex dominates every vertex of at least one color class. The minimum number of colors required for a dominator coloring of G is called the dominator chromatic number of G and is denoted by $\chi_d(G)$.*

To prove our next result we make use of the following result.

Theorem 2.7. *[4] Let G be a connected graph. Then*

$$max\{\chi(G), \gamma(G)\} \leq \chi_d(G) \leq \chi(G) + \gamma(G).$$

The bounds are sharp.

Theorem 2.8. *Let G be a connected graph with $\chi(G) = \chi_d(G)$. Then*

$$\gamma_{dc}(G) = \chi_d(G).$$

Proof. Suppose $\chi(G) = \chi_d(G)$, then by Theorem 2.7, we have for each color class of G, let x_i be a vertex in the class i, where $1 \leq i \leq \chi_d(G)$. We show that the set $S = \{x_i : 1 \leq i \leq \chi_d(G)\}$ is a dominating set. Let $v \in V(G)$. Then v dominates a color class i, for some i $(1 \leq i \leq \chi(G))$. Clearly, D is also a dom-coloring set, which can not be minimized further as it contains exactly one vertex from each color class. Hence the result follows. □

3 Bipartite graph

Theorem 3.9. *For any bipartite graph, with $p \geq 2$ vertices,*

$$2 \leq \gamma_{dc}(G) \leq \lceil \frac{p}{3} \rceil.$$

Further, lower bound exists if in each partite set there exists a vertex with degree equal to cardinality of the other set and upper bound exist if the graph is isomorphic with path P_p, $p \geq 2$ vertices.

Observation 3. *If G is isomorphic to P_2 or P_3, then $\gamma_{dc}(G) \neq \gamma(G)$.*

Theorem 3.10. *For any path P_p with $p \geq 4$ vertices,*

$$\gamma_{dc}(P_p) = \gamma(P_p).$$

Proof. Let a path P_p, $p \geq 4$ be labeled as $1, 2, 3, \ldots, p$. First we prove $\gamma_{dc}(P_p) = \gamma(P_p)$ for $p \geq 6$. We know that $\gamma(P_p) = \lceil \frac{p}{3} \rceil$ for $p \geq 1$, and hence it is true for $p \geq 6$ is also. Now we show the existence of dom-coloring set with cardinality equal to $\lceil \frac{p}{3} \rceil$. Here two cases arise.
Case 1. p is a multiple of 3.
The set $D = \{3m - 1/1 \leq m \leq \frac{p}{3}\}$ is the minimum dominating set which contains at least one vertex from each color class. Hence, $\gamma_{dc}(P_p) = \gamma(P_p)$.
Case 2. p is not a multiple of 3.
Take a largest subpath $P^{'}$ of order multiple of 3, starting from the first vertex of P_p. Form minimum dominating set $D^{'}$ of this path $P^{'}$ as defined in the above case. The set $D = D^{'} \cup \{v\}$ is the minimum dominating set of P_p with cardinality $\lceil \frac{p}{3} \rceil$, where v is the any vertex of P_p not in $P^{'}$. The set D contains at least one vertex from each color class. Hence, $\gamma_{dc}(P_p) = \gamma(P_p)$.
Now we prove the result is true for $p = 4, 5$.
For $p = 4$, $D = \{v_2, v_3\}$ is the minimum dominating set of P_4 which is also dom-coloring set. Then $\gamma_{dc}(P_4) = \gamma(P_4)$.
For $p = 5$, $D = \{v_2, v_5\}$ is the minimum dominating set of P_5 which is also dom-coloring set. Then, $\gamma_{dc}(P_5) = \gamma(P_5)$. □

Theorem 3.11. *For any cycle C_{2p} with $p \geq 2$ vertices,*

$$\gamma_{dc}(C_{2p}) = \gamma(C_p).$$

Theorem 3.12. *For any complete bipartite graph $K_{m,n}$ with $2 \leq m \leq n$ vertices,*

$$\gamma_{dc}(K_{m,n}) = \gamma(K_{m,n}).$$

Proof. By taking one vertex from each partite set. We get a minimum dominating set of $K_{m,n}$ with $2 \leq m \leq n$ vertices, which is also dom-coloring set of $K_{m,n}$. Thus the result follows. □

4 Splitting graph

Definition 4.3. *The splitting graph $S^{'}(G)$ of a graph G is obtained by adding a new vertex $v^{'}$ corresponding to each vertex v of G such that $N(v) = N(v^{'})$, where $N(v)$ and $N(v^{'})$ are the neighborhood sets of v and $v^{'}$ respectively in $S^{'}(G)$.*

Theorem 4.13. *For any non trivial graph G,*

$$\gamma_{dc}(S^{'}(G)) \leq 2\gamma_{dc}(G).$$

Proof. Let G be any graph with $\chi(G)$-coloring and $D = \{v_1, v_2, \ldots, v_{\gamma_{dc}(G)}\}$ be the minimum dom-coloring set of G. The splitting graph $S^{'}(G)$ can also be colored with $\chi(G)$-colors by assigning each $v^{'}$, as that of its the same color corresponding copy in G. In $S^{'}(G)$, D dominates all the vertices except possibly the copies $v_1^{'}, v_2^{'}, \ldots, v_{\gamma_{dc}(G)}^{'}$ of the vertices in D. Hence, $D \cup \{v_1^{'}, v_2^{'}, \ldots, v_{\gamma_{dc}(G)}^{'}\}$ dominates all the vertices of $S^{'}(G)$. Also $D \cup \{v_1^{'}, v_2^{'}, \ldots, v_{\gamma_{dc}(G)}^{'}\}$ contains at least one vertex from each color class as D contains so. Hence, $\gamma_{dc}(S^{'}(G)) \leq |D \cup \{v_1^{'}, v_2^{'}, \ldots, v_{\gamma_{dc}(G)}^{'}\}| = 2\gamma_{dc}(G)$. □

Observation 4. *If G is isomorphic to C_3 or C_4 or C_5 then, $\gamma_{dc}(S^{'}(G)) = \gamma_{dc}(G)$.*

Theorem 4.14. *For any cycle C_p, $p \geq 6$ vertices,*

$$\gamma_{dc}(S^{'}(C_p)) = \begin{cases} 2\gamma_{dc}(C_p), & \text{if } p = 3n + 3, n \geq 1 \\ 2\gamma_{dc}(C_p) - 1, & \text{if } p = 3n + 5, n \geq 1 \\ 2\gamma_{dc}(C_p) - 2, & \text{if } p = 3n + 4, n \geq 1 \end{cases}$$

Proof. Let C_p be labeled as $v_1, v_2, \ldots, v_p$. Here three cases arise.
Case 1. $p = 3n + 3$, $n \geq 1$.
Subcase 1.1. p is even.

Here the cycle C_p is bi-colorable and hence $S^{'}(C_p)$ is also bi-colorable. The set $D = \{v_{3m-2}/1 \leq m \leq n+1\}$ is the minimum dominating set of C_p. Clearly the set D contains at least one vertex from each color class, D dominates all the vertices of $S^{'}(C_p)$ except $D^{'} = \{v^{'}_{3m-2}/1 \leq m \leq n+1\}$. Since there exist no common neighbor ($v \in V(C_p)$ of any two of the vertices in $D^{'}$, to dominate the vertices of $D^{'}$ we must include all the vertices of $D^{'}$ into the dominating set. That is $D_s = D \cup D^{'}$, hence D_s is the minimum dominating set of $S^{'}(C_p)$ containing at least one vertex from each color class. Hence $\gamma_{dc}(S^{'}(C_p)) = 2\gamma_{dc}(C_p)$.

Subcase 1.2. p is odd.

Here the cycle C_p is 3-colorable and hence $S^{'}(C_p)$ is also 3-colorable. Let the first vertex v_1 be colored with color 3, then the set $D_s = D \cup D^{'}$ is the minimum dominating set of $S^{'}(C_p)$ containing at least one vertex from each color class, where D and $D^{'}$ are the sets as defined in the above subcase. Hence $\gamma_{dc}(S^{'}(C_p)) = 2\gamma_{dc}(C_p)$.

Case 2. $p = 3n+5, n \geq 1$.

Subcase 2.1. p is even.

Here the cycle C_p is bi-colorable and hence $S^{'}(C_p)$ is also bi-colorable. The set $D = \{v_{3m-2}/1 \leq m \leq n+2\}$ is the minimum dominating set of C_p. Clearly, the set D contains at least one vertex from each color class, D dominates all the vertices of $S^{'}(C_p)$ except $D^{'} = \{v^{'}_{3m-2}/1 \leq m \leq n+2\}$. There exists a common neighbor v_p of the vertices $v^{'}_{p-1}$ and $v^{'}_1$, hence we take v_p into dominating set and there exist no common neighbor of other vertices of $D^{'}$. We must include the vertices of $D^{'}$ except $v^{'}_1$ and $v^{'}_{p-1}$ into the dominating set, That is $D_s = D \cup \{v_p\} \cup D^{'} \backslash \{v^{'}_1, v^{'}_{p-1}\}$ is the minimum dominating set of $S^{'}(C_p)$ containing at least one vertex from each color class. Hence, $\gamma_{dc}(S^{'}(C_p)) = \gamma_{dc}(C_p) + 1 + \gamma_{dc}(C_p) - 2 = 2\gamma_{dc}(C_p) - 1$.

Subcase 2.2. p is odd.

Here the cycle C_p is 3-colorable and hence $S^{'}(C_p)$ is also 3-colorable. Let the first vertex v_1 be colored with color 3, then the set $D_s = D \cup \{v_p\} \cup D^{'} \backslash \{v^{'}_1, v^{'}_{p-1}\}$ is the minimum dominating set of $S^{'}(C_p)$ containing at least one vertex from each color class, where D and $D^{'}$ are the sets as defined in the above subcase. Hence, $\gamma_{dc}(S^{'}(C_p)) = 2\gamma_{dc}(C_p) - 1$.

Case 3. $p = 3n+4, n \geq 1$.

Subcase 3.1. p is even.

Here the cycle C_p is bi-colorable and hence $S^{'}(C_p)$ is also bi-colorable. The set $D = \{v_{3m-2}/1 \leq m \leq n+2\}$ is the minimum dominating set of C_p. Clearly the set D contains at least one vertex from each color class. The set D dominates all the vertices of $S^{'}(C_p)$ except $D^{'} \backslash \{v^{'}_1, v^{'}_p\} = \{v^{'}_{3m-2}/1 \leq m \leq n+2\} \backslash \{v^{'}_1, v^{'}_p\} = \{v^{'}_{3m-2}/2 \leq m \leq n+1\}$. Since there exist no common neighbor of any of the vertices in $D^{'} \backslash \{v^{'}_1, v^{'}_p\}$, we must include the vertices of $D^{'} \backslash \{v^{'}_1, v^{'}_p\} = \{v^{'}_{3m-2}/2 \leq m \leq n+1\}$ in to the dominating set. i.e., $D_s = D \cup D^{'} \backslash \{v^{'}_1, v^{'}_p\}$ is the minimum dominating set of $S^{'}(C_p)$ containing at least one vertex from each color class. Hence, $\gamma_{dc}(S^{'}(C_p)) = \gamma_{dc}(C_p) + \gamma_{dc}(C_p) - 2 = 2\gamma_{dc}(C_p) - 2$.

Subcase 3.2. p is odd.

Here the cycle C_p is 3-colorable and hence $S^{'}(C_p)$ is also 3-colorable. Let the first vertex v_1 be colored with color 3, then the set $D_s = D \cup D^{'} \backslash \{v^{'}_1, v^{'}_p\}$ is the minimum dominating set of $S^{'}(C_p)$ containing at least one vertex from each color class, where D and $D^{'}$ are the sets as defined in the above subcase 3.1. Hence, $\gamma_{dc}(S^{'}(C_p)) = 2\gamma_{dc}(C_p) - 2$. □

5 Mycielski's graph

Definition 5.4. *From a simple graph G, Mycielski's construction produces a simple graph $\mu(G)$ containing G. Begining with G having vertex set $V = \{v_1, v_2, \ldots, v_p\}$, add vertices $U = \{u_1, u_2, \ldots, u_p\}$ and one more vertex w. Add edges to make u_i adjacent to all of $N_G(v_i)$, and finally let $N(w) = U$.*

To prove our next result we make use of the following results.

Theorem 5.15. *[9] If G is any graph, then*

$$\chi(\mu(G)) = \chi(G) + 1.$$

Theorem 5.16. *[3] If G is any graph, then*

$$\gamma(\mu(G)) = \chi(G) + 1.$$

Theorem 5.17. *For any nontrivial graph G,*

$$\gamma_{dc}(\mu(G)) = \gamma_{dc}(G) + 1.$$

Proof. If T is a dom-coloring set of G then in $\mu(G)$, $T \cup \{w\}$ is the dom-coloring of set. Since w dominates the vertices of U and the vertices in U receives the same color as that of their respective preimages in V. Thus,

$$\gamma_{dc}(\mu(G)) \leq \gamma_{dc}(G) + 1.$$

Let D be a γ_{dc}-set of $\mu(G)$. Clearly, D contains w. Then $D' = D - \{w\}$ dominates V, since $N_{\mu(G)}(v_i) = N_G(v_i) \cup B$, where B is the set of all mirror images of neighbors of v_i. The set D' contains at least one vertex from each color class except the color used for w. Let $D'_{(G)}$ consists of those vertices v_i where either $v_i \in D'$ or $u_i \in D'$. Thus $D'_{(G)}$ is dom-coloring set. Hence

$$\gamma_{dc}(G) \leq |D'_{(G)}| \leq |D'| = \gamma_{dc}(\mu(G)) - 1$$

$$\gamma_{dc}(G) + 1 = \gamma_{dc}(\mu(G)).$$

By virtue of the above facts, we have

$$\gamma_{dc}(\mu(G)) = \gamma_{dc}(G) + 1.$$

□

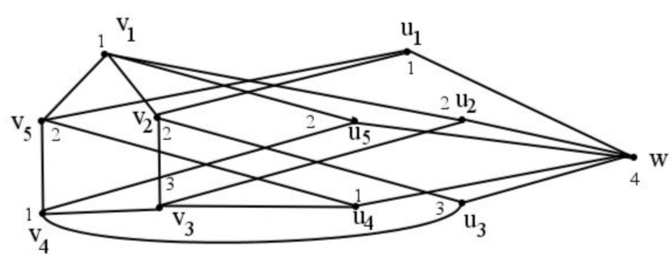

Figure 1: Mycielski's graph of $\mu(C_5)$

For example, consider $\mu(C_5)$ as shown in Figure 1. The minimum dom-coloring set in C_5 is $\{v_1, v_3, v_5\}$ and the minimum dom-coloring set in $\mu(C_5)$ is $\{v_1, v_3, v_5, w\}$. Hence $\gamma_{dc}(C_5) = 3$ and $\gamma_{dc}(\mu(C_5)) = 4$.

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References

[1] B. Chaluvaraju, C. Nandeesh Kumar and C. Appajigowda, The neighbor coloring set in graphs, *Int. J. of Applied Mathematics and Computation*, 4(3), (2012), 307-311.

[2] B. Chaluvaraju, C. Nandeesh Kumar and C. Appajigowda, The Dom-Chromatic index of a graph, *International J. of Math. Sci. & Engg. Appls.*, 8(6) (2014), 43-49.

[3] D. C. Fisher, P. A. Mckenna and E. D. Boyer, Hamiltoncity, diameter, domination, packing and biclique partition of Mycielski's graphs, *Discrete Appl, Math.* 84 (1998), 93-105.

[4] R. M. Gera, On dominator coloring in graphs, *Graph Theory Notes New York*. LII (2007), 25-30.

[5] Harary. F., Graph theory, *Addison-Wesley, Reading Mass* (1969).

[6] T. W. Haynes, S. T. Hedetniemi and P. J. Slater; Fundamentals of domination in graphs, *Marcel Dekker. New York* (1998).

[7] T. W. Haynes, S. T. Hedetniemi and P. J. Slater; Domination in graphs; Advanced topics, *Marcel Dekker. New York* (1998).

[8] T. R. Jensen and B. Toft, Graph Coloring Problem, *John Wiley & Sons, Inc, New York* (1995).

[9] J. Mycielski, Sur le colorings de graphes, *Colloq. Math*. 3 (1955), 161-162.

[10] E. Sampathkumar and G. D. Kamath A generalization of chromatic index, *Discrete Mathematics* 124 (1994), 173-177.

[11] E. Sampathkumar and H. B. Walikar, On Splitting Graph of a Graph, *J. Karnatak Univ. Sci.*, 25(13),(1980), 13-16.

[12] S. K. Vaidya and Rakhimol V. Isaac, b-chromatic number of some degree splitting graphs, *Malaya Journal of Matematik*, 2(3),(2014), 249-253.

[13] S. K. Vaidya and M. S. Shukla, b-Chromatic number of some cycle related graphs, *International Journal of Mathematics and Soft Computing*, 4(2)(2014), 113-127.

[14] H. B. Walikar, B. D Acharya, E. Sampathkumar, Recent developments in the theory of domination in graphs. In:MRI Lecture Notes in Mathematics. Allahabad: *The Mehta Research Institute of Mathematics and Mahematical Physics* 1979.

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