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Interval-Valued Fuzzy Ideals in Ternary Semirings

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Abstract

In this paper we introduce the notion of interval-valued fuzzy ternary subsemirings and interval-valued fuzzy ideals in ternary semirings and investigate some of the properties. Also the homomorphism image and inverse image are investigated.

Keywords: Interval-valued fuzzy ternary subsemirings, interval-valued fuzzy ideal.

2010 MSC: 08A72, 16Y60, 20M12, 17A40.

1 Introduction

The notion of ternary algebraic system was introduced by Lehmer [12] in 1932. He investigated certain ternary algebraic systems called triplexes. In 1971, Lister [13] characterized additive semigroups of rings which are closed under the triple ring product and he called this algebraic system a ternary ring. Dutta and Kar [1] introduced a notion of ternary semirings which is a generalization of ternary rings and semirings, and they studied some properties of ternary semirings [1–7, 10]. The theory of fuzzy sets was first studied by Zadeh [15] in 1965. Many papers on fuzzy sets appeared showing the importance of the concept and its applications to logic, set theory, group theory, ring theory, real analysis, topology, measure theory, etc. Interval-valued fuzzy sets were introduced independently by Zadeh [16], Grattan-Guiness [8], Jahn [9], sambuc [14] in the same year 1975 as a generalization of fuzzy set. In the field of application, the success of the use of fuzzy set theory depends on the choice of the membership function that we take. However there are application in which experts do not have precise knowledge of the function that should be taken. In the cases, it is appropriate to represent the membership degree of each element to the fuzzy set by means of an interval. From these considerations arises the extension of fuzzy sets called theory of Interval-valued fuzzy set (IVFS) that is, fuzzy sets such that the membership degree of each element of the fuzzy set is given by a closed subinterval of the interval $[0,1]$. Thus an interval-valued fuzzy set is defined by an interval-valued membership function. It is important to note that not only vagueness (lack of sharp class boundaries), but also a feature of uncertainty (lack of information) can be addressed intuitively by interval valued fuzzy set. Kavikumar et al.[11] studied fuzzy ideals in ternary semirings. In this paper we introduce the notion of interval-valued fuzzy ternary subsemirings and interval-valued fuzzy ideals in ternary semirings and investigate some of the properties. Also the homomorphism image and inverse image are investigated.

2 Preliminaries

In this section, we first give some basic definitions of the theory of ternary semirings which will be used in this paper.

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Definition 2.1. *A nonempty set S together with a binary operation called, addition $+$ and a ternary multiplication, denoted by juxtaposition, is said to be a ternary semiring if $(S,+)$ is a commutative semigroup satisfying the following conditions:*
(i) $(abc)de = a(bcd)e = ab(cde)$,
(ii) $(a+b)cd = acd + bcd$,
(iii) $a(b+c)d = abd + acd$
and (iv) $ab(c+d) = abc + abd$ for all $a, b, c, d, e \in S$.

Throughout this paper S denotes a ternary semiring with zero.

Definition 2.2. *Let S be a ternary semiring. If there exists an element $0 \in S$ such that $0 + x = x = x + 0$ and $0xy = x0y = xy0 = 0$ for all $x, y \in S$, then 0 is called the zero element or simply the zero of the ternary semiring S. In this case we say that S is a ternary semiring with zero.*

Definition 2.3. *An additive subsemigroup T of S is called a ternary subsemiring of S if $t_1t_2t_3 \in T$ for all $t_1, t_2, t_3 \in T$.*

Definition 2.4. *An additive subsemigroup I of S is called a left [resp. right, lateral] ideal of S if $s_1s_2i \in I$ [resp. $is_1s_2 \in I, s_1is_2 \in I$] for all $s_1, s_2 \in S$ and $i \in I$. If I is a left, right and lateral ideal of S, then I is called an ideal of S.*

It is obvious that every ideal of a ternary semiring with zero contains the zero element.

Definition 2.5. *Let S_1 and S_2 be ternary semirings. A mapping $f : S_1 \to S_2$ is said to be a homomorphism if $f(x+y) = f(x) + f(y)$ and $f(xyz) = f(x)f(y)f(z)$ for all $x, y, z \in S_1$.*

Let $f : S_1 \to S_2$ be an onto homomorphism of ternary semirings. Note that if I is an ideal of S_1, then $f(I)$ is an ideal of S_2. If S_1 and S_2 be ternary semirings with zero 0, then $f(0) = 0$.

Definition 2.6. *An interval number on $[0,1]$, denoted by $\widetilde{a}$, is defined as the closed sub interval of $[0,1]$, where $\widetilde{a} = [a^-, a^+]$ satisfying $0 \le a^- \le a^+ \le 1$.*

The set of all interval numbers is denoted by $D[0,1]$. The interval $[a,a]$ is identified with the number $a \in [0,1]$.

Definition 2.7. *Let $\widetilde{a} = [a^-, a^+]$ and $\widetilde{b} = [b^-, b^+]$ be two interval numbers in $D[0,1]$. Then*
i) $\widetilde{a} \le \widetilde{b}$ if and only if $a^- \le b^-$ and $a^+ \le b^+$
ii) $\widetilde{a} = \widetilde{b}$ if and only if $a^- = b^-$ and $a^+ = b^+$
iii) $inf\widetilde{a_i} = [\bigwedge_{i\in I} {a_i}^-, \bigwedge_{i\in I} {a_i}^+]$, $sup\widetilde{a_i} = [\bigvee_{i\in I} {a_i}^-, \bigvee_{i\in I} {a_i}^+]$ for interval numbers $\widetilde{a_i} = [{a_i}^-, {a_i}^+] \in D[0,1], i \in I$.

Definition 2.8. *Let $\{\widetilde{a_i}\}, i = 1, 2, \cdots, n$ for some $n \in Z^+$ be a finite number of interval numbers, where $\widetilde{a_i} = [{a_i}^-, {a_i}^+]$. Then we define $Max^i\{\widetilde{a_i}\} = [\, max\{{a_i}^-\},\ max\{{a_i}^+\}\,]$ and $Min^i\{\widetilde{a_i}\} = [\, min\{{a_i}^-\},\ min\{{a_i}^+\}\,]$.*

In this paper we assume that any two interval numbers in $D[0,1]$ are comparable. i.e. for any two interval numbers $\widetilde{a}$ and $\widetilde{b}$ in $D[0,1]$, we have either $\widetilde{a} \le \widetilde{b}$ or $\widetilde{a} > \widetilde{b}$. It is clear that $(D[0,1], \le, \vee, \wedge)$ is a complete lattice with $\widetilde{0} = [0,0]$ as the least element and $\widetilde{1} = [1,1]$ as the greatest element.
Let X be a non-empty set. A map $\widetilde{\mu} : X \to D[0,1]$ is called an interval-valued fuzzy subset of X.
Note: We can write $\widetilde{\mu}(x) = [\mu^-(x), \mu^+(x)]$ for all $x \in X$, for any interval-valued fuzzy subset $\widetilde{\mu}$ of a non empty set X, where μ^- and μ^+ are some fuzzy subsets of X.

Definition 2.9. *Let $\widetilde{\mu}_1$ and $\widetilde{\mu}_2$ be two interval-valued fuzzy subsets of a non-empty set X. Then $\widetilde{\mu}_1$ is said to be a subset of $\widetilde{\mu}_2$, denoted by $\widetilde{\mu}_1 \subseteq \widetilde{\mu}_2$ if $\widetilde{\mu}_1(x) \le \widetilde{\mu}_2(x)$, i.e., $\mu_1^-(x) \le \mu_2^-(x)$ and $\mu_1^+(x) \le \mu_2^+(x)$, for all $x \in X$ where $\widetilde{\mu}_1(x) = [\mu_1^-(x), \mu_1^+(x)]$ and $\widetilde{\mu}_2(x) = [\mu_2^-(x), \mu_2^+(x)]$.*

Definition 2.10. *Let $\widetilde{\mu}$ be an interval-valued fuzzy subset of a non-empty set X and $[\alpha, \beta] \in D[0,1]$. Then the level subset of $\widetilde{\mu}$, denoted by $\overline{U}(\widetilde{\mu}, [\alpha, \beta])$, is defined as $\overline{U}(\widetilde{\mu}, [\alpha, \beta]) = \{x \in X : \widetilde{\mu}(x) \ge [\alpha, \beta]\}$.*

If we consider two interval numbers $[\alpha_1, \beta_1]$ and $[\alpha_2, \beta_2]$ such that $[\alpha_1, \beta_1] > [\alpha_2, \beta_2]$, then we have $[\alpha_1, \beta_1] \ge [\alpha_2, \beta_2]$ and $[\alpha_1, \beta_1] \ne [\alpha_2, \beta_2]$. In this case, we find that $\overline{U}(\widetilde{\mu}, [\alpha_1, \beta_1]) \subseteq \overline{U}(\widetilde{\mu}, [\alpha_2, \beta_2])$, since for any $x \in \overline{U}(\widetilde{\mu}, [\alpha_1, \beta_1]) \Rightarrow \widetilde{\mu}(x) \ge [\alpha_1, \beta_1] \ge [\alpha_2, \beta_2] \Rightarrow x \in \overline{U}(\widetilde{\mu}, [\alpha_2, \beta_2])$.

Definition 2.11. *Let I be any subset of a ternary semiring S. The interval-valued characteristic function of I denoted by $\widetilde{\chi}_I$ is defined as*

$$\widetilde{\chi}_I = \begin{cases} \widetilde{1} & if \;\; x \in I \\ \widetilde{0} & otherwise \end{cases}$$

Definition 2.12. *Let $\widetilde{\mu}_1$,$\widetilde{\mu}_2$ and $\widetilde{\mu}_3$ be any three interval-valued fuzzy subsets of a ternary semiring S. Then $\widetilde{\mu}_1 \cap \widetilde{\mu}_2$, $\widetilde{\mu}_1 \cup \widetilde{\mu}_2$, $\widetilde{\mu}_1 + \widetilde{\mu}_2$, $\widetilde{\mu}_1 \circ \widetilde{\mu}_2 \circ \widetilde{\mu}_3$ are interval-valued fuzzy subsets of S defined by for all $x \in S$*

$(\widetilde{\mu}_1 \cap \widetilde{\mu}_2)(x) = Min^i\{\widetilde{\mu}_1(x),\ \widetilde{\mu}_2(x)\}$

$(\widetilde{\mu}_1 \cup \widetilde{\mu}_2)(x) = Max^i\{\widetilde{\mu}_1(x),\ \widetilde{\mu}_2(x)\}$

$$(\widetilde{\mu}_1 + \widetilde{\mu}_2)(x) = \begin{cases} sup\{Min^i\{\widetilde{\mu}_1(y), \widetilde{\mu}_2(z)\}\} & if \;\; x = y + z \\ \widetilde{0} & otherwise \end{cases}$$

$$(\widetilde{\mu}_1 \circ \widetilde{\mu}_2 \circ \widetilde{\mu}_3)(x) = \begin{cases} sup\{Min^i\{\widetilde{\mu}_1(u), \widetilde{\mu}_2(v), \widetilde{\mu}_3(w)\}\} & if \;\; x = uvw, \\ \widetilde{0} & otherwise \end{cases}$$

3 Interval-Valued Fuzzy ideals

Definition 3.13. *Let $\widetilde{\mu}$ be an interval-valued fuzzy subset of S. Then $\widetilde{\mu}$ is called an interval-valued fuzzy ternary subsemiring of S if*
1. $\widetilde{\mu}(x + y) \geq Min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}$
2. $\widetilde{\mu}(xyz) \geq Min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), \widetilde{\mu}(z)\}$ for all $x, y, z \in S$.

Theorem 3.1. *Let $\widetilde{\mu}$ be an interval-valued fuzzy subset of a ternary semiring S. Then $\widetilde{\mu}$ is an interval-valued fuzzy ternary subsemiring of S if and only if $\overline{U}(\widetilde{\mu}, [\alpha, \beta])$ is a ternary subsemiring of S for all $[\alpha, \beta] \in Im\widetilde{\mu}$.*

Proof: Let $\widetilde{\mu}$ be an interval-valued fuzzy ternary subsemiring of S and $[\alpha, \beta]$ be an arbitrary element in $Im\widetilde{\mu}$. Let $x, y, z \in \overline{U}(\widetilde{\mu}, [\alpha, \beta])$ then $\widetilde{\mu}(x) \geq [\alpha, \beta]$, $\widetilde{\mu}(y) \geq [\alpha, \beta]$, $\widetilde{\mu}(z) \geq [\alpha, \beta]$. Now $\widetilde{\mu}(x + y) \geq Min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\} \geq [\alpha, \beta]$. Thus $\widetilde{\mu}(x + y) \geq [\alpha, \beta]$. Hence $x + y \in \overline{U}(\widetilde{\mu}, [\alpha, \beta])$. Similarly $xyz \in \overline{U}(\widetilde{\mu}, [\alpha, \beta])$. Hence $\overline{U}(\widetilde{\mu}, [\alpha, \beta])$ is a ternary subsemiring of S.
Conversely, let $\overline{U}(\widetilde{\mu}, [\alpha, \beta])$ be a ternary subsemiring of S for all $[\alpha, \beta] \in Im\widetilde{\mu}$. If there exist $x, y, z \in S$ such that $\widetilde{\mu}(x + y) < [\alpha, \beta] = Min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}$ then $[\alpha, \beta] \in Im\widetilde{\mu}$ and $x, y \in \overline{U}(\widetilde{\mu}, [\alpha, \beta])$ with $x + y \notin \overline{U}(\widetilde{\mu}, [\alpha, \beta])$ this contradicts to that $\overline{U}(\widetilde{\mu}, [\alpha, \beta])$ is a ternary subsemiring. Hence $\widetilde{\mu}(x + y) \geq Min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}$. Similarly we have $\widetilde{\mu}(xyz) \geq Min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), \widetilde{\mu}(z)\}$. Therefore $\widetilde{\mu}$ is an interval-valued fuzzy ternary subsemiring of S.

Example 3.1. *Consider the set of integer modulo 6, non-positive integer $Z_6^- = \{0, -1, -2, -3, -4, -5\}$ with the addition modulo 6 and ternary multiplication modulo 6. Then $(Z_6^-, \oplus_6, \odot_6)$ is a ternary semiring. Let an interval-valued fuzzy subset $\widetilde{\mu} : Z_6^- \rightarrow D[0, 1]$ be defined by $\widetilde{\mu}(0) = [0.7, 0.9]$, $\widetilde{\mu}(-1) = [0.1, 0.2]$, $\widetilde{\mu}(-2) = [0.7, 0.9]$, $\widetilde{\mu}(-3) = [0.1, 0.2]$, $\widetilde{\mu}(-4) = [0.7, 0.9]$ and $\widetilde{\mu}(-5) = [0.1, 0.2]$. Then $\widetilde{\mu}$ is an interval-valued fuzzy ternary subsemiring of S.*

Definition 3.14. *Let $\widetilde{\mu}$ be an interval-valued fuzzy subset of a ternary semiring S. $\widetilde{\mu}$ is called an interval-valued fuzzy right (resp. left, lateral) ideal of S if*
1. $\widetilde{\mu}(x + y) \geq Min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}$
2. $\widetilde{\mu}(xyz) \geq \widetilde{\mu}(x)$ (resp. $\widetilde{\mu}(xyz) \geq \widetilde{\mu}(z)$, $\widetilde{\mu}(xyz) \geq \widetilde{\mu}(y)$) for all $x, y, z \in S$.

Definition 3.15. *Let $\widetilde{\mu}$ be an interval-valued fuzzy subset of a ternary semiring S. $\widetilde{\mu}$ is called an interval-valued fuzzy ideal of S if*
1. $\widetilde{\mu}(x + y) \geq Min^i\{\widetilde{\mu}(x), \widetilde{\mu}(y)\}$
2. $\widetilde{\mu}(xyz) \geq Max^i\{\widetilde{\mu}(x), \widetilde{\mu}(y), \widetilde{\mu}(z)\}$ for all $x, y, z \in S$.

Theorem 3.2. *Let $\widetilde{\mu}$ be an interval-valued fuzzy subset of a ternary semiring S. Then $\widetilde{\mu}$ is an interval-valued fuzzy right (resp. left, lateral) ideal of S if and only if $\overline{U}(\widetilde{\mu}, [\alpha, \beta])$ is a right (resp. left, lateral) ideal of S for all $[\alpha, \beta] \in Im\widetilde{\mu}$.*

Proof: Let $\widetilde{\mu}$ be an interval-valued fuzzy right ideal of S and $[\alpha,\beta] \in Im\widetilde{\mu}$. By Theorem 3.1, $\overline{U}(\widetilde{\mu},[\alpha,\beta])$ is a ternary subsemiring. Let $x \in \overline{U}(\widetilde{\mu},[\alpha,\beta])$ and $y,z \in S$. Then $\widetilde{\mu}(xyz) \geq \widetilde{\mu}(x) \geq [\alpha,\beta]$. Thus $\widetilde{\mu}(xyz) \geq [\alpha,\beta]$, then $xyz \in \overline{U}(\widetilde{\mu},[\alpha,\beta])$. Hence $\overline{U}(\widetilde{\mu},[\alpha,\beta])$ is a right ideal of S. Conversely, let $\overline{U}(\widetilde{\mu},[\alpha,\beta])$ be a right ideal for all $[\alpha,\beta] \in Im\widetilde{\mu}$. By Theorem 3.1, $\widetilde{\mu}$ is an interval-valued fuzzy ternary subsemiring. If there exist $x,y,z \in S$ such that $\widetilde{\mu}(xyz) < [\alpha,\beta] = \widetilde{\mu}(x)$, then $x \in \overline{U}(\widetilde{\mu},[\alpha,\beta])$ and $y,z \in S$ with $xyz \notin \overline{U}(\widetilde{\mu},[\alpha,\beta])$. This contradicts that $\overline{U}(\widetilde{\mu},[\alpha,\beta])$ is a right ideal. Hence $\widetilde{\mu}(xyz) \geq \widetilde{\mu}(x)$. Therefore $\widetilde{\mu}$ is an interval-valued fuzzy right ideal of S.

Example 3.2. *Let S be a ternary semiring consists of non-positive integers with usual addition and ternary multiplication. Let*

$$\widetilde{\mu}(x) = \begin{cases} [0.9,1] & if \;\; x = 0 \\ [0.5,0.8] & if \;\; x \text{ is even} \\ [0.2,0.3] & if \;\; x \text{ is odd} \end{cases}$$

Then $\widetilde{\mu}$ is an interval-valued fuzzy right ideal of S.

Theorem 3.3. *$\widetilde{\chi}_I$ is an interval-valued fuzzy ideal of S if and only if I is an ideal of S.*

Proof: Let $\widetilde{\chi}_I$ be an interval-valued fuzzy ideal of a ternary semiring S. Let $x,y \in I$. Then $\widetilde{\chi}_I(x+y) \geq Min^i\{\widetilde{\chi}_I(x),\widetilde{\chi}_I(y)\} = \widetilde{1}$ which implies $\widetilde{\chi}_I(x+y) = \widetilde{1}$. Thus $x+y \in I$. Similarly if $x \in I$; $y,z \in S$ we have $xyz,\ yzx,\ yxz \in I$. Therefore I is an ideal of S.
Conversely, if there exist $x,y \in S$ such that $\widetilde{\chi}_I(x+y) < \widetilde{1} = Min^i\{\widetilde{\chi}_I(x),\widetilde{\chi}_I(y)\}$ then $\widetilde{\chi}_I(x+y) = \widetilde{0},\ \widetilde{\chi}_I(x) = \widetilde{1},\ \widetilde{\chi}_I(y) = \widetilde{1}$. Thus $x,y \in I$ and $x+y \notin I$ which is a contradiction. Thus $\widetilde{\chi}_I(x+y) \geq Min^i\{\widetilde{\chi}_I(x),\widetilde{\chi}_I(y)\}$. Similarly we prove that $\widetilde{\chi}_I(xyz) \geq Max^i\{\widetilde{\chi}_I(x),\widetilde{\chi}_I(y),\widetilde{\chi}_I(z)\}$. Therefore $\widetilde{\chi}_I$ is an interval-valued fuzzy ideal of S.

Theorem 3.4. *Let $f : S_1 \to S_2$ be an epimorphism of ternary semirings. Let $\widetilde{\mu}$ be an interval-valued fuzzy subset of S_2. Then $\widetilde{\mu}$ is an interval-valued fuzzy ideal of S_2 if and only if $f^{-1}(\widetilde{\mu})$ is an interval-valued fuzzy ideal of S_1 where $[f^{-1}(\widetilde{\mu})](x) = \widetilde{\mu}(f(x))$ for all $x \in S_1$.*

Proof: Let $\widetilde{\mu}$ be an interval-valued fuzzy ideal of S_2. Let $x_1,x_2,x_3 \in S_1$. Now
$f^{-1}(\widetilde{\mu})(x_1+x_2) = \widetilde{\mu}(f(x_1+x_2)) = \widetilde{\mu}(f(x_1)+f(x_2))$
$\geq Min^i\{\widetilde{\mu}(f(x_1)),\widetilde{\mu}(f(x_2))\} = Min^i\{f^{-1}(\widetilde{\mu})(x_1),f^{-1}(\widetilde{\mu})(x_2)\}$.
Now $f^{-1}(\widetilde{\mu})(x_1x_2x_3) = \widetilde{\mu}(f(x_1x_2x_3)) = \widetilde{\mu}(f(x_1)f(x_2)f(x_3))$
$\geq Max^i\{\widetilde{\mu}(f(x_1)),\widetilde{\mu}(f(x_2)),\widetilde{\mu}(f(x_3))\}$
$= Max^i\{f^{-1}(\widetilde{\mu})(x_1),f^{-1}(\widetilde{\mu})(x_2),f^{-1}(\widetilde{\mu})(x_3)\}$. Thus $f^{-1}(\widetilde{\mu})$ is an interval-valued fuzzy ideal of S_1. Conversely let $f^{-1}(\widetilde{\mu})$ is an interval-valued fuzzy ideal of S_1. Let $y_1,y_2,y_3 \in S_2$ such that $f(x_1) = y_1,\ f(x_2) = y_2$ and $f(x_3) = y_3$ where $x_1,x_2,x_3 \in S_1$.
$\widetilde{\mu}(y_1+y_2) = \widetilde{\mu}(f(x_1)+f(x_2)) = \widetilde{\mu}(f(x_1+x_2)) = f^{-1}(\widetilde{\mu})(x_1+x_2)$
$\geq Min^i\{f^{-1}(\widetilde{\mu})(x_1),f^{-1}(\widetilde{\mu})(x_2)\} = Min^i\{\widetilde{\mu}(f(x_1)),\widetilde{\mu}(f(x_2))\} = Min^i\{\widetilde{\mu}(y_1),\widetilde{\mu}(y_2)\}$. Similarly we have $\widetilde{\mu}(y_1y_2y_3) \geq Max^i\{\widetilde{\mu}(y_1),\widetilde{\mu}(y_2),\widetilde{\mu}(y_3)\}$. Thus $\widetilde{\mu}$ is an interval-valued fuzzy ideal of S_2.

Theorem 3.5. *Let $f : S_1 \to S_2$ be an epimorphism of ternary semirings and let $\widetilde{\mu}$ be an interval-valued fuzzy subset of S_1. If $\widetilde{\mu}$ is an interval-valued fuzzy ideal of S_1 then $f(\widetilde{\mu})$ is an interval-valued fuzzy ideal of S_2 where $f(\widetilde{\mu})(y) = sup\{\widetilde{\mu}(x)/f(x) = y\}$.*

Proof: Let us assume that $\widetilde{\mu}$ is an interval-valued fuzzy ideal of S_1. Let $y_1,y_2,y_3 \in S_2$ then there exist $x_1,x_2,x_3 \in S_1$ such that $f(x_1) = y_1,\ f(x_2) = y_2$ and $f(x_3) = y_3$. Now $f(\widetilde{\mu})(y_1+y_2) = \sup\limits_{x \in S_1}\{\widetilde{\mu}(x)/f(x) = y_1+y_2\}$
$\geq \sup\limits_{x_1,x_2 \in S_1}\{\widetilde{\mu}(x_1+x_2)/f(x_1+x_2) = y_1+y_2\}$
$= \sup\limits_{x_1,x_2 \in S_1}\{\widetilde{\mu}(x_1+x_2)/f(x_1)+f(x_2) = y_1+y_2\}$
$\geq \sup\{Min^i\{\widetilde{\mu}(x_1),\widetilde{\mu}(x_2)\}/f(x_1) = y_1,\ f(x_2) = y_2\}$
$= Min^i\{\sup\{\widetilde{\mu}(x_1)/f(x_1) = y_1\},\sup\{\widetilde{\mu}(x_2)/f(x_2) = y_2\}\}$
$= Min^i\{f(\widetilde{\mu})(y_1),f(\widetilde{\mu})(y_2)\}$.
Now $f(\widetilde{\mu})(y_1y_2y_3) \geq \sup\{\widetilde{\mu}(x_1x_2x_3)/f(x_1x_2x_3) = y_1y_2y_3\}$

$= \sup\{\widetilde{\mu}(x_1x_2x_3)/f(x_1)f(x_2)f(x_3) = y_1y_2y_3\}$
$\geq \sup\{Max^i\{\widetilde{\mu}(x_1), \widetilde{\mu}(x_2), \widetilde{\mu}(x_3)\}/f(x_1) = y_1,\ f(x_2) = y_2,\ f(x_3) = y_3\}$
$= Max^i\{\sup\{\widetilde{\mu}(x_1)/f(x_1) = y_1\}, \sup\{\widetilde{\mu}(x_2)/f(x_2) = y_2\},$
$\quad \sup\{\widetilde{\mu}(x_3)/f(x_3) = y_3\}\}$
$= Max^i\{f(\widetilde{\mu})(y_1), f(\widetilde{\mu})(y_2), f(\widetilde{\mu})(y_3)\}.$
Thus $f(\widetilde{\mu})$ is an interval-valued fuzzy ideal of S_2. The following example shows that converse of the above theorem need not be true.

Example 3.3. *Let Z_0^- and Z_6^- be the ternary semirings of negative integers and negative integer modulo 6 respectively. The mapping f defined by $f : Z_0^- \to Z_6^-$, $f(x) = k$ (mod 6) where $k \equiv x$(mod 6), $-5 \leq k \leq 0$ is a homomorphism and f is onto. Let*

$$\widetilde{\mu}(x) = \begin{cases} [0.7, 0.8] & if\ \ x = -18 \\ [0.4, 0.5] & if\ \ x \in \langle -9 \rangle\ \ and\ \ x \neq -18 \\ [0.6, 0.7] & if\ \ x \in \langle -3 \rangle\ \ and\ \ x \notin \langle -9 \rangle \\ [0.1, 0.2] & otherwise \end{cases}$$

Then

$$f(\widetilde{\mu})(x) = \begin{cases} [0.7, 0.8] & if\ \ x = 0 \\ [0.6, 0.7] & if\ \ x = -3 \\ [0.1, 0.2] & otherwise \end{cases}$$

Clearly $f(\widetilde{\mu})$ is an interval-valued fuzzy ideal of Z_6^- but $\widetilde{\mu}$ is not an interval-valued fuzzy ideal of Z_0^-, since $\widetilde{\mu}(-18 + (-18)) < Min^i\{\widetilde{\mu}(-18), \widetilde{\mu}(-18)\}$.

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