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Minimal and Maximal Soft Open Sets

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Abstract

In this paper, we introduce new types of minimal and maximal sets via soft topological spaces namely minimal and maximal soft open sets and their complements. These sets are depended on the soft open sets. Many interested result are presented to reveal some properties of these new sets.

Keywords: Soft set, soft topology, minimal soft open, maximal soft open.

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1 Introduction

The phenomena of uncertainty can be emerged in many fields such as economy, social and medical sciences, engineering and so on. To deal with such uncertainties many mathematical tools have been introduced such as probability, fuzzy sets, rough sets and etc. However, these tools have their own limitations. In fact the limitations here always associated with the inadequacy of the parametrization tools. Molodtsov [1] initiated another efficient tool, soft set theory, which is more flexible to deal with uncertainty and to treat some limitation obstacles that other tools suffered to handle them. The theory of the soft set has been being investigating intensively and various applications of this theory have been done in many different fields.

Shabir and Naz [2] introduced the concept of the soft topological space. Heavily investigations were followed to this new kind of topological space and many generalizations depending on the generalizations of soft open and closed sets were introduced as well.

On the other hand, the notation of maximal open sets and minimal open sets were introduced by F. Nakaoka and N. Oda in [3] and [4] . Many generalizations of these concepts have been introduced depending on the various generalizations of the concept of open set. In this paper we introduced the concepts of maximal and minimal soft open sets.

2 Preliminaries

Definition 2.1. *[1] Let E be a set of parameters and A be a subset of E, a soft set F_A on the universe set U is denoted by the set of ordered pairs:*

$$F_A = \{(x, f_A(x)) : x \in E, f_A(x) \in P(U)\}$$

where $f_A : E \longrightarrow P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$.
f_A is called an approximate function of the soft set F_A. The value of f_A may be arbitrary, some of them may be empty, or may have nonempty intersection.

Remark 2.1. *The set of all soft sets over U will be denoted by $S(U)$.*

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Example 2.1. *Suppose that there are eight cars in the universe $U = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\}$ and let $E = \{x_1, x_2, x_3, x_4, x_5\}$ is the set of decision parameters such that $x_1 =$ new,$x_2 =$ expensive, $x_3 = high - tech$, $x_4 =$ model, $x_5 =$ interor desgin . Consider the map $f_A \equiv cars(atributes)$, so $f_A(x_3)$ means "cars(high-tech)". Thus the functional value of $f_A(x_3)$ is the set $\{c \in U : c \text{ is a high-tech }\}$. Now let $A = \{x_2, x_3, x_5\}$ and $f_A(x_2) = \{c_2, c_6\}$, $f_A(x_3) = \{c_1, c_3, c_4\}$ and $f_A(x_5) = \{c_1, c_7, c_8\}$. Then the soft set $F_A = \{(x_2, \{c_2, c_6\}), (x_3, \{c_1, c_3, c_4\}), (x_5, \{c_1, c_7, c_8\})\}$.*

Definition 2.2. *[5] Let $F_A \in S(U)$, if $f_A(x) = \emptyset$ for all $x \in E$, then F_A is called an empty set, and denoted by F_Φ.*

Example 2.2. *Let $U = \{u_1, u_2, u_3, u_4\}$ and $E = \{x_1, x_2, x_3\}$, then $F_\Phi = \{(x_1, \emptyset), (x_2, \emptyset), (x_3, \emptyset)\}$*

Definition 2.3. *[5] Let $F_A \in S(U)$, if $f_A(x) = U$ for all $x \in A$, then F_A is called an A-univers soft set and is denoted by $F_{\tilde{A}}$.*
If $A = E$, then $F_{\tilde{E}}$ is called a universe soft set.

Example 2.3. *Let $U = \{u_1, u_2, u_3, \}$, $E = \{x_1, x_2, x_3\}$ and $A = \{x_1, x_2\}$, then $F_{\tilde{A}} = \{(x_1, U), (x_2, U)\}$ and $F_{\tilde{E}} = \{(x_1, U), (x_2, U), (x_3, U)\}$.*

Definition 2.4. *[5] Let $F_A, F_B \in S(U)$. Then F_A is a soft subset of F_B if $f_A(x) \subseteq f_B(x)$ for all $x \in E$ and is denotd by $F_A \tilde{\subseteq} F_B$.*
If $F_A \neq F_B$, then F_A is a proper soft subset of F_B and is denoted by $F_A \tilde{\subset} F_B$.

Example 2.4. *Let $U = \{u_1, u_2, u_3, u_4, u_5\}$, $E = \{x_1, x_2, x_3, x_4, x_5\}$, $A = \{x_1, x_4\}$, $B = \{x_4\}$, $F_A = \{(x_1, \{u_1, u_5\}), (x_4, \{u_2, u_3, u_4\})\}$ and $F_B = \{(x_4, \{u_2, u_3\})\}$. It is clear that $F_B \tilde{\subset} F_A$.*

Definition 2.5. *[5] Let $F_A, F_B \in S(U)$. Then F_A and F_B are soft equal if $f_A(x) = f_B(x)$ for all $x \in E$ and is denoted by $F_A = F_B$.*

Definition 2.6. *[5] Let $F_A, F_B \in S(U)$. Then the soft union of F_A and F_B (denoted by $F_A \tilde{\bigcup} F_B$) is defined by the following: $F_A \tilde{\bigcup} F_B = f_A(x) \bigcup f_B(x)$ for all $x \in E$.*

Example 2.5. *Let $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\}$, $B = \{x_2, x_3\}$, $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$ and $F_B = \{(x_2, \{u_1, u_3\}), (x_3, \{u_3\})\}$. Then $F_A \tilde{\bigcup} F_B = \{(x_1, \{u_1, u_2\}), (x_2, U), (x_3, \{u_3\})\}$*

Definition 2.7. *[5] Let $F_A, F_B \in S(U)$. Then the soft intersection of F_A and F_B (denoted by $F_A \tilde{\bigcap} F_B$) is defined by the following: $F_A \tilde{\bigcap} F_B = f_A(x) \bigcap f_B(x)$ for all $x \in E$.*

Example 2.6. *Let $U = \{u_1, u_2, u_3, u_4\}$, $E = \{x_1, x_2, x_3, x_4\}$, $A = \{x_1, x_4\}$, $B = \{x_1\}$, $F_A = \{(x_1, \{u_1\}), (x_4, \{u_2, u_3, u_4\})\}$ and $F_B = \{(x_1, \{u_1, u_3\})\}$. So $F_A \tilde{\bigcap} F_B = \{(x_1, \{u_1\}\}$.*

Definition 2.8. *[5] Let $F_A \in S(U)$. Then the soft complement of F_A (denoted by $F_A^{\tilde{c}}$) is defined by the approximate function: $F_A^{\tilde{c}} = f_A^c(x)$, where $f_A^c(x) = U - f_A(x)$ for all $x \in A$.*

Example 2.7. *Let $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_3\}$ and $F_A = \{(x_1, \{u_2\}), (x_3, \{u_1, u_3\})\}$, then $F_A^{\tilde{c}} = \{(x_1, \{u_1, u_3\}), (x_3, \{u_2\})\}$.*

Definition 2.9. *[6] Let $F_A \in S(U)$. $\alpha = (x, \{u\})$ is a nonempty soft element of F_A , denoted by $\alpha \tilde{\in} F_A$ if $x \in E$ and $u \in f_A(x)$.*

Remark 2.2. *The pair $(x, \emptyset)$, where $x \in E$, is called the empty soft element of F_A.*

Example 2.8. *Let $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$ and $A = \{x_2, x_3\}$ and let $F_A = \{(x_2, \{u_2, u_3\}), (x_3, \{u_1, u_2\})\}$, then the following are nonempty elements in F_A:*
$\alpha_1 = (x_2, \{u_2\}) \tilde{\in} F_A$; since $u_2 \in f_A(x_2) = \{u_2, u_3\}$
$\alpha_2 = (x_2, \{u_3\}) \tilde{\in} F_A$; since $u_3 \in f_A(x_2) = \{u_2, u_3\}$
$\alpha_3 = (x_3, \{u_1\}) \tilde{\in} F_A$; since $u_1 \in f_A(x_3) = \{u_1, u_2\}$
$\alpha_4 = (x_3, \{u_2\}) \tilde{\in} F_A$; since $u_2 \in f_A(x_3) = \{u_1, u_2\}$

Definition 2.10. *[2] Let $F_E \in S(U)$. A soft topology on F_E, denoted by $\tilde{\tau}$, is a collection of soft subsets of F_E satisfing the following properties*

1. $F_\Phi, F_E \in \tilde{\tau}$.
2. *If* $\{F_{E_i} \tilde{\subseteq} F_E : i \in I \subseteq \mathbb{N}\} \subset \tilde{\tau}$, *then* $\tilde{\bigcup}_{i \in I} F_{E_i} \in \tilde{\tau}$.
3. *If* $\{F_{E_i} \tilde{\subseteq} F_E : 1 \leq i \leq n, n \in \mathbb{N}.\} \subset \tilde{\tau}$, *then* $\tilde{\bigcap}_{i=1}^{n} F_{E_i} \in \tilde{\tau}$.

Then $\tilde{\tau}$ *is called a soft topology and the pair* $(F_E, \tilde{\tau})$ *is called a soft topological space.*

Example 2.9. *Let* $U = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$, $A = \{x_1, x_2\}$, *then* $(F_A, \tilde{\tau}) = \{F_\phi, F_{A_1}, F_{A_2}, F_{A_3}, F_A\}$ *is a soft topological space,*
where $F_{A_1} = \{(x_1, \{u_2\})\}$,$F_{A_2} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$,$F_{A_3} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$.

Definition 2.11. *Let* $(F_A, \tilde{\tau})$ *be a soft topological space and* $\alpha \tilde{\in} F_A$. *If there is a soft open set* F_B *such that* $\alpha \tilde{\in} F_B$, *then* F_B *is called a soft open neighbourhood (or soft neighbourhood) of* α.

Definition 2.12. *[2] Let* $(F_A, \tilde{\tau})$ *be a soft topological space and* $F_B \tilde{\subseteq} F_A$. *Then* F_B *is said to be a soft closed if* $F_B^{\tilde{c}}$ *is a soft open.*

Definition 2.13. *[2] Let* $(F_A, \tilde{\tau})$ *be a soft topological space and* $F_B \tilde{\subseteq} F_A$. *Then the soft closure of* F_B *is the intersection of all soft closed set that contain* F_B *and it is denoted by* $\tilde{\overline{F}}_B$.

3 Minimal and maximal soft open sets

Definition 3.14. *A proper nonempty soft open subset* F_K *of a soft topological space* $(F_A, \tilde{\tau})$ *is said to be minimal soft open set if any soft open set which is contained in* F_K *is* F_Φ *or* F_K.

Definition 3.15. *A proper nonempty soft open subset* F_K *of a soft topological space* $(F_A, \tilde{\tau})$ *is said to be maximal soft open set if any soft open set which contains* F_K *is* F_A *or* F_K.

Example 3.10. *Let* $U = \{u_1, u_2, u_3, u_4\}$, $E = \{x_1, x_2, x_3, x_4\}$, $A = \{x_1, x_2, x_3\}$, *and let* $(F_A, \tilde{\tau}) = \{F_\phi, F_{A_1}, F_{A_2}, F_{A_3}, F_A\}$ *be a soft topological space, where* $F_{A_1} = \{(x_1, \{u_1, u_3\}), (x_2, \{u_2, u_4\})\}$, $F_{A_2} = \{(x_2, \{u_2\})\}$, $F_{A_3} = \{(x_1, \{u_1, u_2, u_3\}), (x_2, U), (x_3, \{u_2\})\}$. *Then* F_{A_2} *is a minimal soft open set and* F_{A_3} *is a maximal soft open set.*

Proposition 3.1. *Let* F_K *and* F_H *be soft open subsets of a soft topological space* $(F_A, \tilde{\tau})$, *if* F_K *is minimal soft open then* $F_K \tilde{\bigcap} F_H = F_\phi$ *or* $F_K \tilde{\subseteq} F_H$.

Proof. Suppose that $F_K \tilde{\bigcap} F_H \neq F_\phi$, so $F_K \tilde{\bigcap} F_H \tilde{\subseteq} F_H$. But F_H is minimal soft open, hence $F_K \tilde{\bigcap} F_H = F_K$. Therefore $F_K \tilde{\subseteq} F_H$. □

Proposition 3.2. *Let* F_K *and* F_H *be minimal soft open subsets of a soft topological space* $(F_A, \tilde{\tau})$,*then* $F_K \tilde{\bigcap} F_H = F_\phi$ *or* $F_K = F_H$.

Proof. Suppose that $F_K \tilde{\bigcap} F_H \neq F_\phi$, so $F_K \tilde{\bigcap} F_H \tilde{\subseteq} F_H$. But F_H is minimal soft open, hence $F_K \tilde{\bigcap} F_H = F_H$. Therefore $F_K \tilde{\subseteq} F_H$.
By using the same argument,we get $F_H \tilde{\subseteq} F_K$. Therefore $F_K = F_H$. □

Proposition 3.3. *Let* F_H *be a minimal soft open set. If* $\alpha \tilde{\in} F_H$, *then* $F_H \tilde{\subset} F_K$ *for any soft open neighbourhood* F_K *of* α.

Proof. Let F_K be a soft open neighbourhood of α and suppose $F_H \tilde{\not\subset} F_K$, then $F_H \tilde{\bigcap} F_K \neq F_\phi$ and it is proper soft open subset of F_H. So we get a contradiction of being F_H is minimal. □

Proposition 3.4. *Let* F_K *be a nonempty finite soft open subset of a soft topological space* $(F_A, \tilde{\tau})$. *Then there exists at least one finite minimal soft open set* F_H *such that* $F_H \tilde{\subset} F_K$.

Proof. Let F_K be a nonempty finite soft open set. If F_K is minimal then set $F_K = F_H$. Otherwise, there exists a soft open set F_{K_1} such that $F_\emptyset \neq F_{K_1}\tilde{\subset}F_K$. So if F_{K_1} is minimal then set $F_H = F_{K_1}$. Otherwise, there exists F_{K_2} such that $F_\emptyset \neq F_{K_2}\tilde{\subset}F_{K_1}\tilde{\subset}F_K$. Now, if F_{K_2} is minimal then set $F_H = F_{K_2}$. Otherwise there exists a finite open soft set F_{K_3} such that $F_\emptyset \neq F_{K_3}\tilde{\subset}F_{K_2}\tilde{\subset}F_{K_1}\tilde{\subset}F_K$. Indeed, since F_K is finite, so if we continue this process we will reach to a final soft open set, say F_{K_n} for some $n \in \mathbb{N}$, which is of course minimal such that $F_\emptyset \neq F_{K_n}\tilde{\subset}F_{K_{n-1}}\tilde{\subset}...\tilde{\subset}F_{K_2}\tilde{\subset}F_{K_1}\tilde{\subset}F_K$. Set $F_H = F_{K_n}$ as required. □

Proposition 3.5. *Let F_K and F_H be soft open subsets of a soft topological space $(F_A, \tilde{\tau})$. If F_K is maximal soft open, then $F_K\tilde{\bigcup}F_H = F_A$ or $F_H\tilde{\subseteq}F_K$.*

Proof. Suppose that $F_K\tilde{\bigcup}F_H \neq F_A$, so $F_K\tilde{\subseteq}F_K\tilde{\bigcup}F_H$. But F_K is maximal soft open, hence $F_K = F_K\tilde{\bigcup}F_H$. Therefore $F_H\tilde{\subseteq}F_K$. □

Proposition 3.6. *Let F_K and F_H be maximal soft open sets of a soft topological space $(F_A, \tilde{\tau})$,then $F_K\tilde{\bigcup}F_H = F_A$ or $F_K = F_H$.*

Proof. Suppose that $F_K\tilde{\bigcup}F_H \neq F_A$, so $F_H\tilde{\subseteq}F_K\tilde{\bigcup}F_H$ and $F_K\tilde{\subseteq}F_K\tilde{\bigcup}F_H$. But F_H is maximal soft open, hence $F_K\tilde{\bigcup}F_H = F_H$. Therefore $F_K\tilde{\subseteq}F_H$.
Using the same argument we get $F_H\tilde{\subseteq}F_K$. Therefore $F_K = F_H$. □

Proposition 3.7. *Let F_M be a proper nonempty cofinite soft open set of a soft topological space $(F_A, \tilde{\tau})$. Then there exists at least one cofinite maximal soft open set F_N such that $F_M\tilde{\subset}F_N$.*

Proof. Let F_M be a proper nonempty cofinite soft open set. If F_M is maximal then set $F_M = F_N$. Otherwise, there exists a proper soft open set F_{N_1} such that $F_M\tilde{\subset}F_{N_1}$. So if F_{N_1} is maximal then set $F_N = F_{N_1}$. Otherwise, there exists a proper soft open set F_{N_2} such that $F_M\tilde{\subset}F_{N_1}\tilde{\subset}F_{N_2}$. Now, if F_{N_2} is maximal then set $F_M = F_{N_2}$. In fact, since F_M is cofinite, so if we continue this process we will reach to a cofinal soft open set, say F_{N_n} for some $n \in \mathbb{N}$, which is of course maximal such that $F_M\tilde{\subset}F_{N_1}\tilde{\subset}F_{N_2}\tilde{\subset}...\tilde{\subset}F_{N_n} \neq F_A$. Set $F_N = F_{N_n}$ as required. □

Proposition 3.8. *Let F_K be a maximal soft open subset of a soft topological space $(F_A, \tilde{\tau})$ and $\alpha\tilde{\notin}F_K$. Then $F_K^{\tilde{c}}\tilde{\subseteq}F_H$ for any soft open set F_H containing α.*

Proof. Since $\alpha\tilde{\notin}F_K$, then for any F_H containing α, so we have $F_H\tilde{\not\subset}F_K$. Hence by using proposition 3.5, we get $F_K\tilde{\bigcup}F_H = F_A$ and this means $F_K^{\tilde{c}}\tilde{\subseteq}F_H$. □

Proposition 3.9. *Let F_K be a maximal soft open subset of a soft topological space $(F_A, \tilde{\tau})$. Then either the following holds:*

1. *For each $\alpha\tilde{\in}F_K^{\tilde{c}}$ and soft open set F_H containing α, we have $F_H = F_A$.*
2. *There exists a soft open set F_H such that $F_K^{\tilde{c}}\tilde{\subseteq}F_H$ and $F_H\tilde{\subset}F_A$.*

Proof. Suppose (1) does not hold, so there exists $\alpha\tilde{\in}F_K^{\tilde{c}}$ and a soft open set F_H containing α such that $F_H\tilde{\subset}F_A$. So by proposition 3.8 we have that $F_K^{\tilde{c}}\tilde{\subseteq}F_H$. □

Proposition 3.10. *Let F_K be a maximal soft open subset of a soft topological space $(F_A, \tilde{\tau})$. Then either the following holds:*

1. *For each $\alpha\tilde{\in}F_K^{\tilde{c}}$ and each soft open neighbourhood set F_H containing α, we have $F_K^{\tilde{c}}\tilde{\subset}F_H$.*
2. *There exists a proper soft open set F_H such that $F_K^{\tilde{c}} = F_H$*

Proof. Suppose that(2) does not hold, so by proposition 3.8, we get $F_K^{\tilde{c}}\tilde{\subset}F_H$ for each $\alpha\tilde{\in}F_K^{\tilde{c}}$ and each soft open neighbourhood F_H of α. □

Proposition 3.11. *Let F_K be a maximal soft open subset of a soft topological space $(F_A, \tilde{\tau})$. Then $\tilde{\overline{F}}_K = F_A$ or $\tilde{\overline{F}}_K = F_K$*

Proof. Let F_K be a maximal soft open set. By using proposition 3.10, so we have only two cases:

1. From the first condition of proposition 3.10;let $\alpha\tilde{\in}F_K^{\tilde{c}}$ and each soft open neighbourhood F_H of α, then $F_K^{\tilde{c}}\tilde{\subset}F_H$. So $F_K\tilde{\bigcap}F_H \neq F_\phi$. i.e. $\alpha\tilde{\in}\tilde{\overline{F}}_K$. Thus $F_K^{\tilde{c}}\tilde{\subset}\tilde{\overline{F}}_K$. But $F_A = F_K\bigcup F_K^{\tilde{c}}\tilde{\subset}F_K\bigcup\tilde{\overline{F}}_K = \tilde{\overline{F}}_K\tilde{\subset}F_A$. Consequently we get $\tilde{\overline{F}}_K = F_A$.

2. From the second condition of proposition 3.10; there exits a soft open set F_H such that $F_K^{\tilde{c}} = F_H \neq F_A$, so $F_K^{\tilde{c}}$ is soft open set and thus F_K is soft closed, i.e. $\tilde{\overline{F}}_K = F_K$.

□

Definition 3.16. *A proper nonempty soft closed subset F_C of a soft topological space $(F_A, \tilde{\tau})$ is said to be minimal soft closed set if any soft closed set which is contained in F_C is F_Φ or F_C.*

Definition 3.17. *A proper nonempty soft closed subset F_C of a soft topological space $(F_A, \tilde{\tau})$ is said to be maximal soft closed set if any soft closed set which contains F_C is F_A or F_C.*

Example 3.11. *Consider example 2.9, then $F_B = \{(x_1, \{u_1, u_3\})\}$ is a minimal soft closed set and $F_C = \{(x_1, \{u_1, u_3\}), (x_2, \{u_1, u_3\})\}$ is a maximal soft closed set.*

Proposition 3.12. *Let F_C and F_D be soft closed sets of a soft topological space $(F_A, \tilde{\tau})$.*

1. *If F_C is minimal, then $F_C\tilde{\bigcap}F_D = F_\Phi$ or $F_C\tilde{\subseteq}F_D$.*

2. *If F_C and F_D are minimal, then $F_C\tilde{\bigcap}F_D = F_\Phi$ or $F_C = F_D$.*

3. *If F_C is maximal, then $F_C\tilde{\bigcup}F_D = F_A$ or $F_D\tilde{\subseteq}F_C$.*

4. *If F_C and F_D are maximal, then $F_C\tilde{\bigcup}F_D = F_A$ or $F_C = F_D$.*

Proof. (1) Suppose that $F_C\tilde{\bigcap}F_D \neq F_\phi$, so $F_C\tilde{\bigcap}F_D\tilde{\subseteq}F_C$. But F_C is minimal soft closed, hence $F_C\tilde{\bigcap}F_D = F_C$. Therefore $F_C\tilde{\subseteq}F_D$.
(2) Suppose that $F_C\tilde{\bigcap}F_D \neq F_\phi$, so $F_C\tilde{\bigcap}F_D\tilde{\subset}F_C$. But F_D is minimal soft closed, hence $F_C\tilde{\bigcap}F_D = F_D$. Therefore $F_D\tilde{\subseteq}F_C$. But from (1) we have $F_C\tilde{\subseteq}F_D$. Therefore $F_C = F_D$.
(3) Suppose that $F_D\tilde{\bigcup}F_C \neq F_A$, so $F_C\tilde{\subseteq}F_C\tilde{\bigcup}F_D$. But F_C is maximal soft closed, hence $F_C = F_C\tilde{\bigcup}F_D$. Therefore $F_D\tilde{\subseteq}F_C$.
(4) Suppose that $F_C\tilde{\bigcup}F_D \neq F_A$, so $F_D\tilde{\subseteq}F_C\tilde{\bigcup}F_D$ and $F_C\tilde{\subseteq}F_C\tilde{\bigcup}F_D$. But F_D is maximal soft closed, hence $F_C\tilde{\bigcup}F_D = F_D$. Therefore $F_C\tilde{\subseteq}F_D$.
Using the same argument, we get $F_D\tilde{\subseteq}F_C$. Therefore $F_C = F_D$. □

Proposition 3.13. *Let $(F_A, \tilde{\tau})$ be a soft topological space. If F_K is a proper maximal soft open subset of F_A, then $F_K^{\tilde{c}}$ is a minimal soft closed set.*

Proof. Suppose $F_K^{\tilde{c}}$ is not minimal soft closed set, so there exists a soft closed set F_C such that $F_\Phi \neq F_C\tilde{\subset}F_K^{\tilde{c}}$. Hence $F_K\tilde{\subset}F_C^{\tilde{c}}\tilde{\subset}F_A$. This means that F_K is not maximal which is contradicting of being F_K is maximal. □

Proposition 3.14. *Let $(F_A, \tilde{\tau})$ be a soft topological space. If F_K be a proper minimal soft open subset of F_A then $F_K^{\tilde{c}}$ is a maximal soft closed set.*

Proof. Suppose $F_K^{\tilde{c}}$ is not maximal soft closed set, so there exists a soft closed set F_C such that $F_K^{\tilde{c}}\tilde{\subset}F_C\tilde{\subset}F_A$. Hence $F_\Phi \neq F_C^{\tilde{c}}\tilde{\subset}F_K$. This means that F_K is not minimal which is contradicting of being F_K is minimal. □

Proposition 3.15. *Let F_C and $\{F_{D_\alpha} : \lambda \in \Lambda\}$be minimal soft closed subsets of a soft topological space $(F_A, \tilde{\tau})$. If $F_C \neq F_{D_\lambda}$ for each λ, then $(\tilde{\bigcup}_{\lambda\in\Lambda} F_{D_\lambda})\tilde{\bigcap}F_C = F_\Phi$.*

Proof. Suppose that $(\tilde{\bigcup}_{\lambda\in\Lambda} F_{D_\lambda})\tilde{\bigcap}F_C \neq F_\Phi$, so there exist some $\lambda_o \in \Lambda$ such that $F_{D_{\lambda_o}}\tilde{\bigcap}F_C \neq F_\Phi$. But F_C and $F_{D_{\lambda_o}}$ are minimal so by 3.12 we get $F_C = F_{D_{\lambda_o}}$ which is a contradiction. □

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