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Lyapunov Approach for Stability of Integro-Differential Equations with Non Instantaneous Impulse Effect

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Abstract

In this paper an integro-differential system of equations, with fixed moments of non instantaneous impulse effects is considered. Sufficient conditions for stability and asymptotic stability of this system have been worked out. The investigations are carried out by means of piecewise continuous functions, analogous to Lyapunov functions and by means of the theory of differential inequalities for such functions. A new comparison lemma, connecting the solution of the given impulsive integro-differential system to the solution of a scalar impulsive differential system is also established.

Keywords: Impulsive integro-differential systems, non instantaneous impulses, Lyapunov stability, asymptotic stability, Lyapunov function.

2010 MSC: 34C20; 34D20; 34A37; 34K4; 92D25.

1 Introduction

The literature on abstract impulsive differential equations considers basically the problems on existence and qualitative properties of solutions of equations of the type

$$x' = f(t, x), \quad t \neq t_i, \tag{1.1}$$

$$\Delta x = I_i(x), \quad t = t_i, \tag{1.2}$$

where, $i \in N, t \in R^+, I_i(x) = x(t_i^+) - x(t_i^-), x \in R^n, f : R^+ \times R^n \to R^n$ and $0 = t_0 < t_1 < t_2 < t_3 < \cdots < \infty$. Here I_i:$R^n \to R^n$ is a sequence of instantaneous impulse operators and have been used to describe abrupt changes such as shocks, harvesting , natural disasters etc.

It seems that the above instantaneous impulsive differential equations models can not characterize the dynamics of evolution process completely in pharmacotherapy. For example as in [1], consider the hemodynamical equilibrium of a person. In the case of decompensation (e.g. high or low levels of glucose), one can prescribe some intravenous drugs(insulin) and the introduction of the drugs in the bloodstream and the consequent absorption for the body are gradual and continuous processes. In fact the above situation has fallen in new impulsive action which starts abruptly and stays active on a finite time interval. Thus we have to use a new model to describe such an evolution process.

To the best of our knowledge, Hernandez and O'Regan [1] in 2013, initially offered to study a new class of abstract impulsive differential equations with non instantaneous impulses in a PC_α-normed Banach space.Then Pierre and Rolnik[2] continued the work in a PC_α- normed Banach space and developed the results in [1].UlamHyers stability and Lyapunov stability of this type of non instantaneous differential

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systems were established recently in [6,8] and [4] respectively.Existence of solutions for integer/fractional differential and integro-differential equations with non instantaneous impulses was presented in[3, 5,7].

Motivated by the above stated work on non instantaneous impulsive differential systems, in this paper, we consider the following, new model of impulsive integro -differential equations to describe an evolution process, in which an impulse action starts at an arbitrary fixed point and keeps active on a finite time interval and establish sufficient conditions its stability and asymptotic stability

$$x'(t) = f(t, x(t), Lx), \quad t \in (s_i, t_{i+1}], \quad i = 0, 1, 2, 3, \cdots, m, \tag{1.3}$$

$$x(t) = g_i(t, x(t), Mx), \quad t \in (t_i, s_i], \quad i = 0, 1, 2, 3, \cdots, m, \tag{1.4}$$

where $0 = t_0 = s_0 < t_1 \leq s_1 \leq t_2 \leq \cdots s_{m-1} \leq t_m \leq s_m \leq t_{m+1} = T$ are pre fixed numbers, $f : J \times R^n \times R^n \to R^n$, where $J = [0, T]$ is continuous and g_i:$[t_i, s_i] \times R^n \times R^n \to R^n$ is continuous for all $i = 1, 2, \cdots, m$, which are called non instantaneous impulses.

$Lx = \int_{t_0}^{t} K(s, x(s))ds$, K: $J \times R^n \to R^n$ and $Mx = \int_{t_0}^{t} I(s, x(s))ds$, I: $J \times R^n \to R^n$ are continuous functions. As far as existence of solution of system (1.3)-(1.4), is concerned, we refer [3,7]. Assume that we can measure the state of the process at any time to get a function $x(\cdot)$ as a solution of (1.3)-(1.4). To ensure the existence of trivial solution of the system (1.3)-(1.4), let us assume that $f(t, 0, 0) = 0$, $K(t, 0) = 0$, $g_i(t, 0, 0) = 0$, $I(t, 0)$.

The novelty of our paper is to establish stability and asymptotic stability of solutions of integro-differential system of equations with non instantaneous impulses. A new comparison lemma for this non instantaneous impulsive systems is proved and by using this, the study of the solution of impulsive integro-differential system is replaced by the study of the solutions of a scalar Impulsive integro-differential system as done in [12,15] .

In section 2, some preliminaries notes and definitions are given. In section 3, a new comparison lemma, connecting the solution of the given impulsive integro-differential system to the solution of a scalar impulsive integro-differential system is worked out. This lemma plays an important role in establishing the main results of the paper. Sufficient conditions for stability and asymptotic stability of impulsive integro-differential system of equations with non instantaneous fixed time impulse effect, are established by using the lemma.

2 PRELIMINARIES

Let $C(J, R^n)$ be the Banach space of all continuous function from J into R^n with the norm $\|x\| = Max\{\|x_1\|_C, \|x_2\|_C, \|x_3\|_C, \cdots, \|x_n\|_C\}$ for $x \in C(J, R^n)$, where $\|x_k\|_C = \sup |x_k(t)|$. Also we use the Banach space $PC(J, R^n) = \{x : J \to R^n : x \in C((t_k, t_{k+1}], R^n) : \text{k} = 0, 1, 2, \cdots, m\}$ and for $k = 1, 2, \cdots, m$ there exists $x(t_k^-)$ and $x(t_k^+)$ such that $x(t_k^-) = x(t_k^+)$ with the norm $\|x\|_{PC} = \max\{\|x_1\|_{PC}, \|x_2\|_{PC}, \|x_3\|_{PC}, \cdots, \|x_n\|_{PC}\}$. Denote $PC^1(J, R^n) = \{x \in PC(J, R^n) : x' \in PC(J, R^n)\}$. Set $\|x\|_{PC^1} = \max\{\|x\|_{PC}, ||x'||_{PC}$. Clearly $PC^1(J, R^n)$ endowed with the norm $|| \cdot ||_{PC^1}$ is also a Banach space. If $x, y \in R^n$, $x = (x_1, x_2, \cdots, x_n)$, $y = (y_1, y_2, \cdots, y_n)$ by $x \leq y$ we mean that $x_i \leq y_i$ $\forall i = 1, 2, \cdots, n$.

Referring [3,5,7] , a function $x \in PC^1(J, R^n)$ is called classical solution of the impulsive Cauchy problem

$$\begin{aligned} x'(t) &= f(t, x(t), Lx), \quad t \in (s_i, t_{i+1}], \quad i = 0, 1, 2, \cdots, m, \\ x(t) &= g_i(t, x(t), Mx), \quad t \in (t_i, s_i], \quad i = 0, 1, 2, \cdots, m, \\ x(0) &= x_0. \end{aligned}$$

If satisifes $x(0) = x_0$, $x(t) = g_i(t, x(t), Mx)$, $t \in (t_i, s_i]$, $i = 1, 2, \cdots, m$. And

$$\begin{aligned} x(t) &= x_0 + \int_{t_0}^{t} f\left(s, x(s), \int_{t_0}^{t} K(\tau, x(\tau))d\tau\right)ds, \quad t \in (0, t_1], \\ x(t) &= g_i\left(s_i, x(s_i), \int_{t_0}^{t} I(\tau, x(\tau))d\tau\right) + \int_{t_0}^{t} f\left(s, x(s), \int_{t_0}^{t} K(\tau, x(\tau))d\tau\right), \quad t \in (s_i, t_{i+1}], \\ &i = 1, 2, \cdots, m. \end{aligned}$$

Let us introduce the intervals : $G_i = (s_i, t_{i+1}] : i = 1, 2, \cdots, m$ with $G = \cup_{i=0}^{m} G_i$ and $H_i = (t_i, s_i] : i = 0, 1, 2, \cdots, m$ with $H = \cup_{i=0}^{m} H_i$.

Definition 2.1. *[10, 14] A function V: $J \times R^n \to R^+$ is said to belong to class V_0 if* *(i)* V *is continuous in* $G_i \cup H_i$, $i = 0, 1, 2, \cdots, m$.

(ii) V *is locally Lipschitz continuous in its second argument on each of* G_i, $i = 0, 1, 2, \cdots, m$.

(iii) $V(t+0, g_i(t,x)) \leq V(t,x)$ *for each* $x \in H_i$, $i = 0, 1, 2, \cdots, m$.

(iv) *For* $i = 1, 2, \cdots, m$ *in* , $V(t_i - 0, x) = V(t_i, x)$ *and* $V(t_i + 0, x) = \lim_{t \to t_i + 0} V(t_i, x)$.

Further for $t \in G_i$ *and* $x \in PC(J, R^n)$,*we define the following derivative* ,

$$D^+_{(2)} V(t,x) = \lim_{h \to 0} \sup \frac{1}{h} \left[V\left(t+h, x + hf\left(t, x(t), \int_{t_0}^{t} K(s, x(s)) ds \right) \right) - V(t,x) \right]$$

Note that if $x(t)$ *is a solution of the system (1.3)-(1.4), then* $D^+_{(2)} V(t,x) = V'_{(2)}(t,x)$. *We shall now use the following classes of functions:*

$\mathcal{K} = \{ a \in C[J, R^+] : a(\cdot)$ *is monotonically increasing and* $a(0) = 0 \}$.

$C\mathcal{K} = \{ a \in C[J \times R^+, R^+] : a(t, \cdot) \in \mathcal{K}$ *for each* $t \in J \}$.

Together with system (1.3)-(1.4), we consider the following scalar impulsive differential system of equations:

$$u'(t) = g(t, u(t), Pu), \quad t \in (s_i, t_{i+1}], \quad i = 0, 1, 2, \cdots, m, \tag{2.1}$$

$$u(t) = f_i(t, u(t), Qu), \quad t \in (t_i, s_i], \quad i = 0, 1, 2, \cdots, m, \tag{2.2}$$

$$x(t_0^+) = u_0 \geq 0, \tag{2.3}$$

where g: $J \times R^+ \times R^+ \to R^+$, f_i: $[t_i, s_i] \times R^+ \times R^+ \to R^+$, $Pu = \int_{t_0}^{t} K_1(\tau, u(\tau)) d\tau$, K_1: $J \times R^+ \to R^+$ *and* $Qu = \int_{t_0}^{t} I_1(\tau, u(\tau)) d\tau$, I_1: $J \times R^+ \to R^+$. *Let* $S(\rho) = \{ (t,x) \in J \times R^n : ||x|| < \rho, \rho > 0 \}$. *We shall say that the conditions (A) are satisfied if the following hold:*

(A1) $f \in PC(S(\rho) \times R^n, R^n)$.

(A2) $K \in C(S(\rho), R^n)$.

(A3) $g(t, 0, 0) = 0$ *for* $t \in J$.

(A4) $\psi_k \in C[R^+, R^+]$ *is non decreasing function with* $\psi_k(0) = 0$ *and* $\psi_k(r_{k-1}(t_k; t_{k-1}, u^+_{k-1})) = u^+_k$, *where* $r_k(t, t_k, u^+_k)$ *is the maximal solution of the system (2.1)-(2.3), if it occurs in* $(t_k, t_{k+1}] = H_k \cup G_k$, $k = 0, 1, 2, \cdots, m$.

(A5) *Let* $||g_i(t, x(t), Mx)|| < \rho$ *for each* $t \in H_i$, $i = 0, 1, 2, \cdots, m$.

Definition 2.2. *[14] The system (1.3)-(1.4) is said to be stable, if for each* $\epsilon > 0$, $\exists$ *a* $\delta = \delta(t_0, \epsilon) > 0$ *such that for any solution* $x(t) = x(t, t_0, x_0)$ *of (1.3)-(1.4), the inequality* $||x_0|| \leq \delta \Rightarrow ||x(t)|| < \epsilon, \forall t \geq t_0$.

Definition 2.3. *[14] The system (1.3)-(1.4) is said to be attractive, if for each* $\epsilon > 0$, $\exists$ *two numbers* $\delta = \delta(t_0) > 0$ *and* $\Gamma = \Gamma(t_0, \epsilon) > 0$ *such that for any solution* $x(t) = x(t, t_0, x_0)$ *of (1.3)-(1.4), the inequality* $||x_0|| \leq \delta \Rightarrow ||x(t)|| < \epsilon$, $\forall t \geq t_0 + \Gamma$.

Definition 2.4. *[14] The system (1.3)-(1.4) is said to be asymptotically stable if it is stable and attractive.*

Definition 2.5. *[14] A function* $V \in V_0$*is said to be :*

(i) *positive definite if there exists a* $\delta > 0$ *and a function* $a \in \mathcal{K}$ *such that* $||x|| < \delta \Rightarrow a(||x||) \leq V(t,x)$.

(ii) *weakly decrescent if there exists a* $\delta > 0$ *and a function* $b \in C\mathcal{K}$ *such that* $||x|| < \delta \Rightarrow V(t,x) \leq b(t, ||x||)$.

3 MAIN RESULTS

Before establishing the main results of the paper, we will prove the following lemma:

Lemma 3.1. *Let the following conditions be fulfilled:*

1. *Conditions (A1)-(A4) hold.*

2. *Let $V \in S(\rho) \to R^+$ and $V \in V_0$. Assume that*
   (i) $D^+_{(2)} V(t, x(t)) \leq g(t, V(t, x(t))) : t \in G_i$.
   (ii) $V(t_k + 0, x(t_k + 0)) \leq \psi_k(V(t_k, x(t_k))) : k = 1, 2, \cdots, m$.
   (iii) $V(s, x(s)) \leq V(t, x(t))$ *:for $t, s \in H_i$ such that $0 \leq t \leq s$.*
3. *The solution $x(t) = x(t; t_0, x_0)$ of system (1.3)-(1.4) is such that $(t, x(t + 0, t_0, x_0)) \in S(\rho)$ for $t \in J$.*
4. *Let $r(t, t_0, u_0)$, the maximal solution of (2.1)-(2.3) satisfying $u_0 \geq V(t_0 + 0, x_0)$exists on J.*

Then

$$V(t, x(t; t_0, x_0)) \leq r(t; t_0, u_0) : t \in J. \tag{3.1}$$

Proof. The maximal solution $r(t, t_0, u_0)$ of the system (2.1)-(2.3) can be defined as follows:

$$r(t, t_0, u_0) = \begin{cases} r_0(t, t_0, u_0^+), & t_0 < t \leq t_1 \\ r_1(t, t_1, u_1^+), & t_1 < t \leq t_2 \\ \cdots \\ \cdots \\ r_m(t, t_m, u_m^+), & t_m < t \leq t_{m+1}, \end{cases}$$

where $r_i(t, t_1, u_i^+)$ is the maximal solution of the system (2.1)-(2.3), in $(t_i, t_{i+1}] = H_i \cup G_i$, for which $\psi_i(r_{i-1}(t_i; t_{i-1}, u_{i-1}^+)) = u_i^+$, $i = 1, 2, \cdots, m$ and $u_0^+ = u_0$. We claim (3.1) by considering the following three cases;

Case 1: For $t \in (s_i, t_{i+1}) = G_i - \{t_{i+1}\}$ let us say $m(t) = V(t, x(t; t_0, x_0))$ so that for small $h > 0$ we have

$$\begin{aligned} m(t+h) - m(t) &= V(t+h, x(t+h)) - V(t, x(t)) \\ &= V(t+h, x(t+h)) - V\left(t+h, x + hf\left(t, x(t), \int_{t_0}^{t} K(s, x(s))ds\right)\right) \\ &\quad + V\left(t+h, x + hf\left(t, x(t), \int_{t_0}^{t} K(s, x(s))ds\right)\right) - V(t, x(t)). \end{aligned}$$

As in Definition 2.1, $V(t, x(t))$ is locally Lipschitzian in x for $t \in G$, using assumption 2(i) of the statement of lemma, we arrive at $D^+_{(2)} m(t) \leq g(t, m(t)) : t \in G_i - \{t_{i+1}\}$. Then by theorem 3.1.1 in [13] we observe that

$$m(t) \leq r(t), i.e. V(t, x(t; t_0, x_0)) \leq r(t; t_0, u_0) : t \in G_i - \{t_{i+1}\}. \tag{3.2}$$

Case 2: For $t \in H_i = (t_i, s_i]$,Using condition 2(iii) of the lemma and the fact that $u_0 \geq V(t_0 + 0, x_0)$, it is clear that

$$V(t, x(t; t_0, x_0)) \leq V(t_0 + 0, x_0) \leq u_0 \leq r(t; t_0, u_0). \tag{3.3}$$

Case 3: For $t \in \{t_1, t_2, \cdots, t_{m+1}\}$, i.e. the moments of impulses .w.l.o.g. let us assume that $t = t_k$. Then by using assumption 2(ii) of the lemma, we see

$$V(t_i + 0, x(t_i + 0; t_0, x_0)) \leq \psi_i(V(t_i, x(t_i; t_0, x_0))) \leq \psi_i(r_i(t_i, t_{i-1}, u_{i-1}^+)) = u_i^+.$$

Again using this condition $V(t, x(t; t_i + 0, x_0)) \leq u_i^+$ in place of $u_0 \geq V(t_0 + 0, x_0)$ and as done in case 1,we get

$$V(t, x(t; t_0, x_0)) \leq r(t; t_0, u_0) \text{for each} = 1, 2, \cdots, m. \tag{3.4}$$

Thus from (3.2), (3.3) and (3.4), we conclude that $V(t, x(t; t_0, x_0)) \leq r(t; t_0, u_0) : t \in J$ and hence (3.1) is established. □

Theorem 3.1. *Let the following conditions hold:*

1. *Conditions (A) are satisfied.*
2. *Let $V \in S(\rho) \to R^+$ and $V \in V_0$ such that it is positive definite and weakly decrescent.*
3. *Assume that for $t > t_0 \geq 0$*
   - *(i)* $D^+_{(2)} V(t, x(t)) \leq g(t, V(t, x(t))) : t \in G_i$,
   - *(ii)* $V(t_k + 0, x(t_k + 0)) \leq \psi_k(V(t_k, x(t_k)))$: $k = 1, 2, \cdots, m$
   - *(iii)* $V(s, x(s)) \leq V(t, x(t))$: *for* $t, s \in H_i$ *such that* $0 \leq t \leq s$

   Then,

   - *(a) if the zero solution of the system (2.1)-(2.3) is stable then the system (1.3)-(1.4) will also be stable.*
   - *(b) if the zero solution of the system (2.1)-(2.3) is asymptotically stable then the system (1.3)-(1.4) will also be asymptotically stable.*

Proof. **proof of (a):**
Since $V \in V_0$ is positively definite in $S(\rho)$, there exists a function $a \in \mathcal{K}$ and a such that $\delta_1 > 0, 0 < \delta_1 \leq \rho ||x|| < \delta_1$ implies

$$a(||x||) \leq V(t, x). \tag{3.5}$$

Since $V \in V_0$ is weakly decrescent, there exists a function $a \in C\mathcal{K}$ and a $\delta_2 > 0$ such that $||x|| < \delta_2$ implies

$$V(t + 0, x) \leq b(t + 0, ||x||). \tag{3.6}$$

Now as assertion in the statement of theorem, zero solution of the system (2.1)-(2.3) is stable, by definition of stability that there exists a $\delta_3(t_0, \epsilon) > 0$ such that $||u_0|| < \delta_3$ implies $\big|\big|u(t; t_0, u_0)\big|\big| < a(\epsilon)$ for $t \geq t_0$ and in particular $\big|\big|r(t; t_0, u_0)\big|\big| < a(\epsilon)$ for $t \geq t_0$, where $r(t; t_0, u_0)$ is the maximal solution of (2.1)-(2.3) for which

$$r(t_0 + 0; t_0, u_0) = u_0. \tag{3.7}$$

Choose $\delta_4 = \delta_4(t_0, \epsilon)$ satisfying $b(t_0 + 0, \delta_4) < \delta_3$ and let $\delta = \delta(t_0, \epsilon) = \min\{\delta_3, \delta_4\}$. Then for $||x_0|| < \delta$, we have $V(t_0 + 0, x_0) \leq b(t_0 + 0, ||x||) \leq (t_0 + 0, \delta) < \delta_3$. Therefore we see that

$$V(t_0 + 0, x_0) < \delta_3, \text{ for } ||x|| < \delta. \tag{3.8}$$

Thus $\big|\big|r(t; t_0, V(t_0 + 0, x_0))\big|\big| < a(\epsilon)$ for $t \geq t_0$. Let $x(t) = x(t; t_0, x_0)$ be a solution of system (1.3)-(1.4). Then to prove that zero solution of system (1.3)-(1.4) will be stable,we claim that for above mentioned $\delta = \delta(t_0, \epsilon) > 0$, $||x|| < \delta$ implies that for every $\epsilon > 0$ we have

$$||x(t)|| < \epsilon, \text{ for } t \geq t_0. \tag{3.9}$$

If possible let this be false. Then there exists some $t^* > t_0$ such that $t_k < t^* \leq t_{k+1}$ for some k satisfying $\| x(t^*) \| \geq \epsilon$ and

$$\| x(t) \| < \epsilon, \text{ for } t_0 < t \leq t_k. \tag{3.10}$$

Again for $\| x(t_k^+) \| = \| g_i(t, x(t), Mx) \| < \rho$ and $\| x(t_k) \| < \epsilon$ from(3.9) . Hence we can find a t^0 such that $t_k < t^0 \leq t^*$ and $\epsilon \leq \| x(t^0) \| < \rho$ with

$$\| x(t) \| < \rho, \text{ for } t_0 \leq t \leq t^0. \tag{3.11}$$

Set $m(t) = V(t, x(t; t_0, x_0))$ for $t_0 \leq t \leq t^0$. We note here that $m(t_0^+) = V(t_0^+, x(t; t_0, x_0)) = u_0$ and all the conditions of lemma 3.1, are fulfilled in the interval $[t_0, t^0]$. Therefore, applying the lemma for the system (2.1)-(2.3) of integro-differntial equations in the interval $[t_0, t^0]$, instead of interval $J = [0, T]$, we have the following inequality:

$$\begin{aligned} m(t) &= V(t, x(t; t_0, x_0)) \leq r(t; t_0, u_0) \\ &= r\big(t; t_0, V(t_0^+, x_0)\big) : t \in \big[t_0, t^0\big], \end{aligned} \tag{3.12}$$

where $r(t;t_0,u_0)$ is the maximal solution of (2.1)-(2.3) in $[t_0,t^0]$. We then have, by using(3.10), (3.5), (3.12), (3.11) and (3.7)

$$a(\epsilon) \leq (\|x(t^0)\|) \leq V(t^0,x(t^0)) \leq r(t;t_0,V(t_0^+,x_0)) < a(\epsilon)$$

which is a contradiction. Therefore (3.9) holds true and hence for every given $\epsilon > 0$, there exists a $\delta = \delta(t_0,\epsilon)$, such that $||x_0|| < \delta$ implies $||x(t)|| < \epsilon$ for $t \geq t_0$. Thus the zero solution of the system (1.3)-(1.4) is stable.
proof of (b): It is given that the zero solution of the system (2.1)-(2.3) is asymptotically stable, which means it is stable and attractive. As proved in part (a), stability of system (2.1)-(2.3) implies that the system (1.3)-(1.4) is stable. Therefore, by definition,there exists a $\delta_{01} = \delta_{01}(t_0,\epsilon) > 0$, such that $||x_0|| < \delta_{01}$ implies $||x(t)|| < \epsilon$ for $t \geq t_0$. In particular if we take $\epsilon = \rho$ we have, $||x(t)|| < \rho$ for $||x_0|| < \delta_{01}$, $t \geq t_0$.

Again as zero solution of system (2.1)-(2.3) is attractive, by definition 2.3, for each $\epsilon > 0$, $\exists$ two numbers $\delta_{02} = \delta_{02}(t_0) > 0$ and $\Gamma = \Gamma(t_0,\epsilon)$ such that

$$||u_0|| \leq \delta_{02} \Rightarrow \|r(t,t_0,u_0)\| < a(\epsilon), t \geq t_0 + \Gamma. \tag{3.13}$$

Choose $\delta_{03} = \delta_{03}(t_0) > 0$ such that

$$\delta_{03} < \delta_{02}, \text{ with } b(t_0,\delta_{03}) < \delta_{02}. \tag{3.14}$$

Then from (3.6) and (3.14), we get,

$$V(t_0+0,x_0) \leq b(t_0+0,||x_0||) \leq b(t_0+0,\delta_{03}) < \delta_{02}.$$

Therefore, $\|r(t;t_0,V(t_0+0,x_0))\| < a(\epsilon)$ for $t \geq t_0 + \Gamma$. Set $\delta_0 = \delta_0(t_0) = \min\{\delta_{01},\delta_{02},\delta_{03}\}$. Now let $||x_0|| < \delta_0$ and apply lemma, it follows that if $x(t) = x(t;t_0,x_0)$ is a solution of the system(1.3)-(1.4), then

$$V(t,x(t;t_0+x_0)) \leq r(t;t_0,V(t_0+0,x_0)) : t \geq t_0 + \Gamma. \tag{3.15}$$

Now, to prove that system (1.3)-(1.4) is attractive , we claim that for above mentioned $\delta_0 = \delta_0(t_0) > 0$, $||x_0|| < \delta$ implies that for every $\epsilon > 0$, we have $\|x(t)\| < \epsilon$ for $t \geq t_0 + \Gamma$. If possible let it be false. Then as done in the proof of part(a), by using (3.13), (3.14) and (3.15) we will arrive at contradiction and hence the system (1.3)-(1.4) is attractive. Thus the system (1.3)-(1.4) is asymptotically stable. □

4 CONCLUSION

A variety of results concerning Lyapunov stability , eventual stability and practical stability for the impulsive differential systems of type (1.1)-(1.2) with instantaneous impulses(fixed time and variable time impulses), are established in literature by using Lyapunov functions along with comparison theorems ([9,10,11,14] and the references there in). in this paper, we established stability and asymptotic stability for a new impulsive integro-differential system in which, impulses are non instantaneous and that to the best of our knowledge, are proved for the first time. A new lemma by which the study of the solution of impulsive integro-differential system is replaced by the study of the solutions of a scalar Impulsive integro-differential system is also proved. The desired results are obtained by using Lyapunov functions and comparison differential inequalities.

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