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A note on inventory model for perishable items with trapezoidal type market demand and time-varying holding cost under partial backlogging

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Abstract

In this paper, we studied an inventory model for perishable items with time dependent trapezoidal type demand. Shortages are allowed and partially backlogged with a constant rate. Holding cost is assumed to be linearly dependent with time. The rate of deterioration of the items dependent on both time and life of the products. The numerical solution of the model is obtained. Sensitivity analysis is performed to show the effect of changes in the parameter on the optimum solution.

Keywords: Trapezoidal type demand function, Partial backlogging, Time dependent deterioration rate, Time-varying holding cost.

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1 Introduction

An inventory system with time dependent deteriorating items is one of considerable attention in the recent years. In daily situations such as failure of electric bulb, batteries as they age, expiry of drugs, evaporation of volatile liquids, are common problem to all of us, so, we should not neglect the effect of deterioration on the replenishment policies. In fact, the stock level of inventory is continuously depleting because of the combined effects of its demand and deterioration. In the last few years, many attention has been given to the inventory control system involve with deteriorating items. Ghare and Schrader [1] developed an inventory model by taking into account the effect of deterioration of items in storage. In their model, they introduced a constant deterioration rate, while the demand rate was also taken to be constant. Afterward, Covert and Philip [2] and Tadikamalla [3] extended Ghare and Schrader's work by introducing variable rates of deterioration. Then, immediately Shah [29] provided a further generalization of all these models by considering shortages and using a general distribution for the deterioration rate.

All the above inventory models are based on static environment where the demand is assumed to be constant and steady over a finite replenishment cycle. However, in the real business market scenario, demand should not be constant which is increasing with time during the market growth phase. Then, after reaching its peak, the demand becomes stable for a finite time period. Thereafter, the demand starts decreasing with time. For example, we can easily think that some kind of winter season winter products. In the beginning of the winter season, about October or November, the sale increases up to the month of December and the sale reaches its climatic and maintain this climate sales situation until the end of the winter season. This type of market demand may be approximated by a ramp type demand function. The inventory control with ramp type demand rate first time proposed by Hill [5], they introduced the inventory models for increasing demand followed by a constant demand. Thereafter, Hariga [6] developed an

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inventory lot sizing model for deteriorating items with general continuous time varying demand over a finite planning horizon under three replenishment policies and considered deterioration rate is a constant fraction of the on hand inventory, shortages are allowed and completely back-ordered. Subsequently, several authors discussed inventory models with ramp type demand rates from various aspects. Here, (please see the table-1) we listed some authors those who have used ramp type demand function to study inventory systems in different environments.

Apart from the above discussion, we may think, when some seasonal goods are coming to the market, the demand rate of such type of items may increase with time up to the certain time and then reaches a peak, the demand becomes stable for a finite time period, and finally the market demand rate gradually decreases to a constant or zero. We hope such type of demand is more realistic to construct some EOQ model. This type of demand is named as trapezoidal type demand. Cheng and Wang [27] first introduced trapezoidal type demand. They extended Hill's [5] ramp type demand rate to trapezoidal type demand. Subsequently, several authors discussed inventory models with trapezoidal type demand rates from different angles. Here we listed (please see the table-II) some authors those who have considered a trapezoidal type demand function to formulate some EOQ models in different domain.

All articles given in table-2, shows that, all the researchers studied economic order quantity model by considering the trapezoidal type demand, deterioration (constant/linear/Weibull), shortages (allowed/not allowed), backlogging (partial/complete), and constant holding costs. However, always constant holding cost may not help to develop a better approximate EOQ model in real life scenario, perhaps. So, holding cost may not be constant over time always, as there is a change in time value of money and change in the price index.

Hence, the motivation behind this article is, to prepare a more general inventory model, which includes; (a) Trapezoidal type demand, which is piecewise linear continuous function with time (b) Shortages are allowed with partially backlogged, and backlogging rate is constant (c) Deterioration depends on both time and life on an item, which is reflected more realistic than constant. (d) Linear increasing holding cost with time.

2 Notations and assumptions

The model is based on following assumptions and notations:

1. The demand rate $D(t)$ is assumed to be a trapezoidal type function, which is piecewise linear continuous with time , defined as follows;
$$D(t) = \begin{cases} a_1 + b_1 t & \text{if } 0 \le t \le \mu_1 \\ D_0 & \text{if } \mu_1 \le t \le \mu_2 \\ a_2 - b_2 t & \text{if } \mu_2 \le t \le T \le \frac{a_2}{b_2} \end{cases}$$
where μ_1 is the point in time axis, when demand reaches peak position and maintain constant, and μ_2 is the point in time axis, when demand start decreases.

2. The replenishment rate is infinite, thus replenishment is instantaneous, i. e. lead time is zero.

3. T is the length of each ordering cycle.

4. $I(t)$ is level of inventory at time t, $0 \le t \le T$.

5. $S = I(0)$ is the maximum inventory level for the ordering cycle.

6. $\theta(t) = \frac{1}{1+R-t}$ is the deteriorating rate of inventory items, where R is the maximum life time of item.

7. t_1 is the time when the inventory level reaches zero due to both demand and deterioration.

8. Shortage is allowed and partially backlogged.

9. β is the backlogging rate; $0 \le \beta \le 1$, if β is 1 or 0, then shortage is completely backlogged or lost.

10. $H(t) = h + \alpha t$ is the holding cost, where $\alpha > 0$, $h > 0$.

11. c_1 is the constant shortage cost per unit per unit time.

12. c is the constant purchasing cost per unit.

13. L is the constant lost sale cost per unit.

14. A is the fixed ordering cost per order.

15. $C_1(t_1)$ is the total average cost per unit (when $0 \leq t_1 \leq \mu_1$).

16. $C_2(t_1)$ is the total average cost per unit (when $\mu_1 \leq t_1 \leq \mu_2$).

17. $C_3(t_1)$ is the total average cost per unit (when $\mu_2 \leq t_1 < T$).

18. t_1^* is the optimal time, when the inventory level reaches zero.

3 Formulation of mathematical model and its solutions

Here, we consider the time dependent deteriorating inventory model with trapezoidal type demand rate. Inventory level attains maximum at $t = 0$, when replenishment occurs. From $t = 0$ to $t = t_1$, the level of inventory reduces due to both demand and deterioration. At t_1 the inventory level reaches zero, then shortage starts occurring during the time interval (t_1, T), and all the demand during the shortage period (t_1, T) is partially backlogged with constant backlogging rate (β), $(0 \leq \beta \leq 1)$. The total number of backlogged items is replaced by the next replenishment. The rate of change of the inventory during the positive stock period $(0, t_1)$ and shortage period (t_1, T) is described by the following differential equations:

$$\frac{dI(t)}{dt} = -\theta(t)I(t) - D(t),\ \ 0 < t < t_1 \tag{3.1}$$

and

$$\frac{dI(t)}{dt} = -\beta D(t),\ \ t_1 < t < T. \tag{3.2}$$

with boundary condition $I(t_1) = 0$.
As per the nature of the demand function, our work can be completed through three cases, because, the shortage of inventory may occur during $(0, \mu_1]$, or $[\mu_1, \mu_2]$, or $[\mu_2, T)$. Hence, to make a complete study of the inventory model, we should take care about all three cases. These three cases are given as follows.

3.1 Case-I $(0 < t \leq \mu_1)$

Due to demand and deterioration, the inventory level gradually decreases during the time interval $(0, t_1]$ and finally falls to zero at time $t = t_1$, i. e. shortage starts during $(0, \mu_1]$. Hence equations (3.1) and (3.2) reduce to

$$\frac{dI(t)}{dt} = -\frac{I(t)}{1+R-t} - (a_1 + b_1 t),\ \ 0 < t < t_1, \tag{3.3}$$

$$\frac{dI(t)}{dt} = -(a_1 + b_1 t)\beta,\ \ t_1 < t < \mu_1, \tag{3.4}$$

$$\frac{dI(t)}{dt} = -D_0\beta,\ \ \mu_1 < t < \mu_2, \tag{3.5}$$

and

$$\frac{dI(t)}{dt} = -(a_2 - b_2 t)\beta,\ \ \mu_2 < t < T. \tag{3.6}$$

Solving the above differential equations (3.3-3.6) with the condition $I(t_1) = 0$ and continuity property of $I(t)$, we get

$$I(t) = (1+R-t)\left[a_1 \ln\left(\frac{1+R-t}{1+R-t_1}\right) + b_1(1+R)\ln\left(\frac{1+R-t}{1+R-t_1}\right) + b_1(t-t_1)\right],\ 0 \leq t \leq t_1, \tag{3.7}$$

$$I(t) = \beta a_1(t_1 - t) + \beta\frac{b_1}{2}(t_1^2 - t^2),\ t_1 \leq t \leq \mu_1, \tag{3.8}$$

$$I(t) = -D_0\beta t + a_1\beta t_1 + \beta\frac{b_1}{2}(t_1^2 + \mu_1^2),\ \mu_1 \leq t \leq \mu_2, \tag{3.9}$$

and

$$I(t) = \beta a_1 t_1 - \beta a_2 t + \beta \frac{b_2}{2}(t^2 + \mu_2^2) + \beta \frac{b_1}{2}(t_1^2 + \mu_1^2), \ \mu_2 \le t \le T. \tag{3.10}$$

The beginning inventory level can be obtained as

$$S = I(0) = (1+R)\left[\ln\left(\frac{1+R}{1+R-t_1}\right)(a_1 + b_1(1+R)) - b_1 t_1\right]. \tag{3.11}$$

Inventory is available in the system during the time period $(0, t_1)$. So, the cost for holding inventory in stock is computed for time period $(0, t_1)$ only.
Holding cost is as follows:

$$\begin{aligned}
HC &= \int_0^{t_1} H(t)I(t)dt \\
&= \int_0^{t_1} (h+\alpha t)(1+R-t)\left[\ln\left(\frac{1+R-t}{1+R-t_1}\right)(a_1 + b_1(1+R)) + b_1(t-t_1)\right]dt \\
&= (a_1 + b_1(1+R))(t_1 - (1+R))\left[t_1 + (1+R)\ln\left(\frac{1+R-t_1}{1+R}\right) + \frac{(R\alpha + \alpha - h)}{2}\left[(1+R)t_1 + \frac{t_1^2}{2}\right.\right. \\
&+ \left.\left.(1+R)^2 \ln\left(\frac{1+R-t_1}{1+R}\right)\right] - \alpha\left[(1+R)^2 t_1 + (1+R)\frac{t_1^2}{2} + \frac{t_1^3}{3} + (1+R)^3 \ln\left(\frac{1+R-t_1}{1+R}\right)\right]\right] \\
&+ b_1\left(\frac{\alpha}{12}t_1^4 - \frac{(R\alpha + \alpha - h)}{6}t_1^3 - \frac{h(1+R)}{2}t_1^2\right).
\end{aligned} \tag{3.12}$$

Shortage due to stock out is accumulated in the system during the time period (t_1, T). The optimum level of shortage is occur at $t = T$, hence, the total shortage cost during the above mentioned time period is as follows:

$$\begin{aligned}
SC &= c_1 \int_{t_1}^{T} -I(t)dt \\
&= c_1\left[-\int_{t_1}^{\mu_1} I(t)dt - \int_{\mu_1}^{\mu_2} I(t)dt - \int_{\mu_2}^{T} I(t)dt\right] \\
&= -c_1\left[\int_{t_1}^{\mu_1}\left(\beta a_1(t_1 - t) + \beta\frac{b_1}{2}(t_1^2 - t^2)\right)dt + \int_{\mu_1}^{\mu_2}\left(-D_0\beta t + \beta a_1 t_1 + \beta\frac{b_1}{2}(t_1^1 + \mu_1^2)\right)dt\right. \\
&+ \left.\int_{\mu_2}^{T}\left(\beta a_1 t_1 - \beta a_2 t + \beta\frac{b_2}{2}(t^2 + \mu_2^2) + \beta\frac{b_1}{2}(t_1^2 + \mu_1^2)\right)dt\right] \\
&= c_1\left[\beta\frac{a_1}{2}(t_1 - \mu_1)(t_1 + \mu_1 - 2T) + \beta\frac{b_1}{6}(2t_1^3 - 2\mu_1^3 + 3T\mu_1^2 - 3Tt_1^2) + \beta\frac{a_2}{2}(\mu_2^2 - T^2)\right. \\
&+ \left.\beta\frac{b_2}{6}(3T\mu_2^2 - T^3 - 2\mu_2^3) + \beta\frac{D_0}{2}(\mu_1 - \mu_2)(\mu_1 + \mu_2 - 2T)\right].
\end{aligned} \tag{3.13}$$

Due to stock out during the time period (t_1, T), shortage is accumulated, but not all customers are willing to wait for the next lot size to arrive. Hence, this results in some loss of sale which accounts to loss in profit.
Lost sale cost is calculated as follows:

$$\begin{aligned}
LSC &= L\int_{t_1}^{T}(1-\beta)D(t)dt \\
&= L(1-\beta)\left[\int_{t_1}^{\mu_1} D(t)dt + \int_{\mu_1}^{\mu_2} D(t)dt + \int_{\mu_2}^{T} D(t)dt\right] \\
&= L(1-\beta)\left[a_1(\mu_1 - t_1) + \frac{b_1}{2}(\mu_1^2 - t_1^2) + D_0(\mu_2 - \mu_1) + a_2(T - \mu_2) - \frac{b_2}{2}(T^2 - \mu_2^2)\right].
\end{aligned} \tag{3.14}$$

Purchase cost is as follows:

$$\begin{aligned}
PC &= c\left[I(0) + \int_{t_1}^{T}\beta D(t)dt\right] \\
&= c(1+R)\left[a_1 \ln\left(\frac{1+R}{1+R-t_1}\right) + b_1(1+R)\ln\left(\frac{1+R}{1+R-t_1}\right) - b_1 t_1\right] + c\beta\left[a_1(\mu_1 - t_1)\right. \\
&+ \left.\frac{b_1}{2}(\mu_1^2 - t_1^2) + D_0(\mu_2 - \mu_1) + a_2(T - \mu_2) - \frac{b_2}{2}(T^2 - \mu_2^2)\right].
\end{aligned} \tag{3.15}$$

The total average cost is given by

$$\begin{aligned}
C_1(t_1) &= \frac{1}{T}\Big[A + PC + HC + SC + SLC\Big] \\
&= \frac{1}{T}\Big[A + c(1+R)\Big[a_1 \ln\left(\frac{1+R}{1+R-t_1}\right) + b_1(1+R)\ln\left(\frac{1+R}{1+R-t_1}\right) - b_1 t_1\Big] \\
&+ c\beta\Big[a_1(\mu_1 - t_1) + \frac{b_1}{2}(\mu_1^2 - t_1^2) + D_0(\mu_2 - \mu_1) + a_2(T-\mu_2) - \frac{b_2}{2}(T^2 - \mu_2^2)\Big] \\
&+ L(1-\beta)\Big[a_1(\mu_1 - t_1) + \frac{b_1}{2}(\mu_1^2 - t_1^2) + D_0(\mu_2 - \mu_1) + a_2(T-\mu_2) - \frac{b_2}{2}(T^2 - \mu_2^2)\Big] \\
&+ c_1\beta\frac{a_1}{2}(t_1 - \mu_1)(t_1 - \mu_1 - 2T) + c_1\frac{\beta b_1}{6}(2t_1^3 - 2\mu_2^3 + 3T\mu_1^2 - 3Tt_1^2) \\
&+ c_1\beta\frac{a_2}{2}(\mu_2 - T)^2 + c_1\beta\frac{b_2}{6}(3T\mu_2^2 - T^3 - 2\mu_2^3) + c_1\beta\frac{D_0}{2}(\mu_1 - \mu_2)(\mu_1 + \mu_2 - 2T) \\
&+ (a_1 + b_1(1+R))(t_1 - (1+R))\Big[t_1 + (1+R)\ln\left(\frac{1+R-t_1}{1+R}\right) + \frac{(R\alpha + \alpha - h)}{2}\Big[(1+R)t_1 + \frac{t_1^2}{2} \\
&+ (1+R)^2\ln\left(\frac{1+R-t_1}{1+R}\right)\Big] - \alpha\left((1+R)^2 t_1 + (1+R)\frac{t_1^2}{2} + \frac{t_1^3}{3} + (1+R)^3\ln\left(\frac{1+R-t_1}{1+R}\right)\right)\Big] \\
&+ b_1\left(\frac{\alpha}{12}t_1^4 - \frac{(R\alpha + \alpha - h)}{6}t_1^3 - \frac{h(1+R)}{2}t_1^2\right)\Big]. \qquad (3.16)
\end{aligned}$$

Equation (3.16) is highly non linear in nature with t_1. We can find the optimum values of t_1 for minimum average cost $C_1(t_1)$ from the solutions of the following equations by the help of Mathematica 10,

$$\frac{dC_1(t_1)}{dt_1} = 0. \qquad (3.17)$$

3.2 Case-II (for $t_1 \in [\mu_1, \mu_2]$)

The differential equations governing the inventory model can be expressed as follows:

$$\frac{dI(t)}{dt} = -\frac{I(t)}{1+R-t} - (a_1 + b_1 t),\ \ 0 < t < \mu_1, \qquad (3.18)$$

$$\frac{dI(t)}{dt} = -\frac{I(t)}{1+R-t} - D_0,\ \ \mu_1 < t < t_1, \qquad (3.19)$$

$$\frac{dI(t)}{dt} = -D_0\beta,\ \ t_1 < t < \mu_2 \qquad (3.20)$$

and

$$\frac{dI(t)}{dt} = -\beta(a_2 - b_2 t),\ \ \mu_2 < t < T. \qquad (3.21)$$

Solving the above differential equations (3.18-3.21) with the help of $I(t_1) = 0$ and continuity property of $I(t)$, we obtain

$$\begin{aligned}
I(t) &= (1+R-t)\Big[a_1\ln\left(\frac{1+R-t}{1+R-t_1}\right) + b_1(1+R)\ln\left(\frac{1+R-t}{1+R-\mu_1}\right) \\
&+ b_1\mu_1\ln\left(\frac{1+R-\mu_1}{1+R-t_1}\right) + b_1(t-\mu_1)\Big],\ \ 0 \le t \le \mu_1, \qquad (3.22)
\end{aligned}$$

$$I(t) = (1+R-t)D_0\ln\left(\frac{1+R-t}{1+R-t_1}\right),\ \ \mu_1 \le t \le t_1, \qquad (3.23)$$

$$I(t) = \beta D_0(t_1 - t),\ \ t_1 \le t \le \mu_2 \qquad (3.24)$$

and

$$I(t) = -\beta a_2(t - t_1) - \beta b_2\mu_2 t_1 + \frac{\beta b_2}{2}(t^2 + \mu_2^2),\ \ \mu_2 \le t \le T. \qquad (3.25)$$

The beginning inventory level can be obtained as

$$\begin{aligned} S &= I(0) \\ &= (1+R)\left[a_1 \ln\left(\frac{1+R}{1+R-t_1}\right) + b_1(1+R) + b_1(1+R)\ln\left(\frac{1+R}{1+R-t_1}\right)\right. \\ &+ \left. b_1\mu_1 \ln\left(\frac{1+R-\mu_1}{1+R-t_1}\right) - b_1\mu_1\right]. \end{aligned} \tag{3.26}$$

The total cost per ordering cycle is consists by following five different costs, these are as follows:

1. Ordering cost.

$$OC = A. \tag{3.27}$$

2. Holding cost

$$\begin{aligned} HC &= \int_0^{\mu_1} (h+\alpha t)(1+R-t)\left[a_1 \ln\left(\frac{1+R-t}{1+R-t_1}\right) + b_1(1+R)\ln\left(\frac{1+R-t}{1+R-\mu_1}\right)\right. \\ &+ \left. b_1\mu_1 \ln\left(\frac{1+R-\mu_1}{1+R-t_1}\right) + b_1(t-\mu_1)\right]dt + \int_{\mu_1}^{t_1}(h+\alpha t)b_0 \ln\left(\frac{1+R-t}{1+R-t_1}\right)dt \\ &= \left[a_1 \ln\left(\frac{1+R-\mu_1}{1+R-t_1}\right) + b_1\mu_1 \ln\left(\frac{1+R-\mu_1}{1+R-t_1}\right) - b_1\mu_1\right]\left[h(1+R)\mu_1 + (R\alpha+\alpha-h)\frac{\mu_1^2}{2} - \frac{\alpha\mu_1^3}{3}\right] \\ &+ \left[ah(1+R)(t_1-(1+R)) + b_1 h(1+R)^2(\mu_1-(1+R))\right]\left[\mu_1 + (1+R)\ln\left(\frac{1+R-\mu_1}{1+R}\right)\right] \\ &+ \left[(1+R)\mu_1 + \frac{\mu_1^2}{2} + (1+R)^2 \ln\left(\frac{1+R-\mu_1}{1+R}\right)\right]\left[a_1(t_1-(1+R))\left(\frac{R\alpha+\alpha-h}{2}\right)\right. \\ &+ \left. 3(1+R)(\mu_1-(1+R))\left(\frac{R\alpha+\alpha-h}{2}\right)\right] - \left[(1+R)^2\mu_1 + \left(\frac{1+R}{2}\right)\mu_1^2 + \frac{\mu_1^3}{3}\right] \\ &\times \left[\frac{\alpha}{3}a_1(t_1-(1+R)) + \frac{\alpha}{3}b_1(1+R)(\mu_1-(1+R))\right] + b_1 h(1+R)\frac{\mu_1^2}{2} + (R\alpha+\alpha-h)b_1\frac{\mu_1^3}{3} \\ &- \frac{\alpha b_1\mu_1^4}{4} + D_0(t_1-(1+R))h\left[(t_1-\mu_1) + (1+R)\ln\left(\frac{1+R-t_1}{1+R-\mu_1}\right)\right] \\ &+ (t_1-(1+R))\frac{\alpha D_0}{2}\left[(1+R)(t_1-\mu_1) + \frac{t_1^2-\mu_1^2}{2} + (1+R)^2\ln\left(\frac{1+R-t_1}{1+R-\mu_1}\right)\right]. \end{aligned} \tag{3.28}$$

3. Purchase cost

$$\begin{aligned} PC &= c\left[(1+R)\left(a_1 \ln\left(\frac{1+R}{1+R-t_1}\right) + b_1(1+R)\ln\left(\frac{1+R}{1+R-t_1}\right) + b_1\mu_1\ln\left(\frac{1+R-\mu_1}{1+R-t_1}\right) - b_1\mu_1\right)\right. \\ &+ \left. \beta D_0(\mu_2-t_1) + \beta a_2(T-\mu_2) - \frac{\beta b_2}{2}(T^2-\mu_2^2)\right]. \end{aligned} \tag{3.29}$$

4. Shortage cost

$$\begin{aligned} SC &= \frac{c_1\beta D_0}{2}(t_1-\mu_2)^2 + \frac{c_1\beta a_2}{2}\left[(T-t_1^2) - (\mu_2-t_1^2)\right] \\ &+ c_1\beta b_2\mu_2 t_1(T-\mu_2) + \frac{c_1\beta b_2}{2}\left[\frac{T^3-\mu_2^3}{3} + \mu_2^2(T-\mu_2)\right]. \end{aligned} \tag{3.30}$$

5. Lost sale cost

$$LSC = L(1-\beta)\left[D_0(\mu_2-t_1) + a_2(T-\mu_2) - \frac{b_2}{2}(T^2-\mu_2^2)\right]. \tag{3.31}$$

The total average cost is given by

$$
\begin{aligned}
C_2(t_1) &= \frac{1}{T}\Big[OC + HC + PC + SC + LSC\Big] \\
&= \frac{1}{T}\Big[A + c\Big[(1+R)\left(a_1\ln\left(\frac{1+R}{1+R-t_1}\right) + b_1(1+R)\ln\left(\frac{1+R}{1+R-t_1}\right) + b_1\mu_1\ln\left(\frac{1+R-\mu_1}{1+R-t_1}\right) - b_1\mu_1\right) \\
&+ \beta D_0(\mu_2 - t_1) + \beta a_2(T-\mu_2) - \frac{\beta b_2}{2}(T^2-\mu_2^2)\Big] \\
&+ L(1-\beta)\Big[D_0(\mu_2 - t_1) + a_2(T-\mu_2) - \frac{b_2}{2}(T^2-\mu_2^2)\Big] \\
&+ \frac{c_1\beta D_0}{2}(t_1-\mu_2)^2 + \frac{c_1\beta a_2}{2}\Big[(T-t_1^2) - (\mu_2 - t_1^2)\Big] \\
&+ c_1\beta b_2\mu_2 t_1(T-\mu_2) + \frac{c_1\beta b_2}{2}\left[\frac{T^3-\mu_2^3}{3} + \mu_2^2(T-\mu_2)\right] \\
&+ \left[a_1\ln\left(\frac{1+R-\mu_1}{1+R-t_1}\right) + b_1\mu_1\ln\left(\frac{1+R-\mu_1}{1+R-t_1}\right) - b_1\mu_1\right]\left[h(1+R)\mu_1 + (R\alpha+\alpha-h)\frac{\mu_1^2}{2} - \frac{\alpha\mu_1^3}{3}\right] \\
&+ \Big[ah(1+R)(t_1-(1+R)) + b_1h(1+R)^2(\mu_1-(1+R))\Big]\left[\mu_1 + (1+R)\ln\left(\frac{1+R-\mu_1}{1+R}\right)\right] \\
&+ \left[(1+R)\mu_1 + \frac{\mu_1^2}{2} + (1+R)^2\ln\left(\frac{1+R-\mu_1}{1+R}\right)\right]\left[a_1(t_1-(1+R))\left(\frac{R\alpha+\alpha-h}{2}\right)\right. \\
&+ \left.3(1+R)(\mu_1-(1+R))\left(\frac{R\alpha+\alpha-h}{2}\right)\right] - \left[(1+R)^2\mu_1 + \left(\frac{1+R}{2}\right)\mu_1^2 + \frac{\mu_1^3}{3}\right] \\
&\times \left[\frac{\alpha}{3}a_1(t_1-(1+R)) + \frac{\alpha}{3}b_1(1+R)(\mu_1-(1+R))\right] + b_1h(1+R)\frac{\mu_1^2}{2} + (R\alpha+\alpha-h)b_1\frac{\mu_1^3}{3} \\
&- \frac{\alpha b_1\mu_1^4}{4} + D_0(t_1-(1+R))h\left[(t_1-\mu_1) + (1+R)\ln\left(\frac{1+R-t_1}{1+R-\mu_1}\right)\right] \\
&+ (t_1-(1+R))\frac{\alpha D_0}{2}\left[(1+R)(t_1-\mu_1) + \frac{t_1^2-\mu_1^2}{2} + (1+R)^2\ln\left(\frac{1+R-t_1}{1+R-\mu_1}\right)\right]. \qquad (3.32)
\end{aligned}
$$

Equation (3.32) is highly non linear in nature with t_1. We can find the optimum values of t_1 for minimum average cost $C_2(t_1)$ from the solutions of the following equations by the help of Mathematica 10,

$$\frac{dC_2(t_1)}{dt_1} = 0. \qquad (3.33)$$

3.3 Case-III ($t_1 \in [\mu_2, T]$)

The differential equation governing the inventory model can be expressed as follows:

$$\frac{dI(t)}{dt} = -\frac{I(t)}{1+R-t} - (a_1 + b_1 t),\ \ 0 < t < \mu_1, \qquad (3.34)$$

$$\frac{dI(t)}{dt} = -\frac{I(t)}{1+R-t} - D_0,\ \ \mu_1 \le t < \mu_2, \qquad (3.35)$$

$$\frac{dI(t)}{dt} = -\frac{I(t)}{1+R-t} - (a_2 - b_2 t),\ \ \mu_2 \le t < t_1 \qquad (3.36)$$

and

$$\frac{dI(t)}{dt} = -\beta(a_2 - b_2 t),\ \ t_1 < t < T. \qquad (3.37)$$

Solving the above differential equations (3.34-3.37) with the help of $I(t_1) = 0$ and continuity property of $I(t)$, we obtain

$$
\begin{aligned}
I(t) &= (1+R-t)\Big[a_1\ln(1+R-t) + b_1(1+R)\ln(1+R-t) + b_1 t + (b_2(1+R) - a_2)\ln(1+R-t_1) \\
&- b_2R\ln(1+R-\mu_2) - b_2(t_1-\mu_2) - b_1R\ln(1+R-\mu_1) - b_1\mu_1\Big],\ \ 0 \le t < \mu_1, \qquad (3.38)
\end{aligned}
$$

$$\begin{aligned} I(t) &= (1+R-t)\Big[D_0 \ln(1+R-t) + (b_2(1+R)-a_2)\ln(1+R-t_1) \\ &- b_2 R\ln(1+R-\mu_2) + b_2(t_1-\mu_2)\Big], \;\; \mu_1 \le t \le \mu_2, \end{aligned} \tag{3.39}$$

$$I(t) = (1+R-t)\left[a_2 \ln\left(\frac{1+R-t}{1+R-t_1}\right) - b_2(1+R)\ln\left(\frac{1+R-t}{1+R-t_1}\right) + b_2(t_1-t),\; \mu_2 \le t \le t_1 \tag{3.40}$$

and

$$I(t) = \beta a_2(t_1-t) + \frac{\beta b_2}{2}(t^2-t_1^2),\;\; t_1 < t < T. \tag{3.41}$$

In this case the begging inventory level can be obtained as

$$\begin{aligned} S &= I(0) \\ &= (1+R)\Big[a_1\ln(1+R) + b_1(1+R)\ln(1+R) + (b_2(1+R)-a_2)\ln(1+R-t_1) \\ &- b_2R\ln(1+R-\mu_2) - b_2(t_1-\mu_2) - b_1R\ln(1+R-\mu_1) - b_1\mu_1\Big]. \end{aligned} \tag{3.42}$$

The total cost per order cycle is consist by following different costs, these are as follows:

1. Ordering cost

$$OC = A. \tag{3.43}$$

2. Holding cost

$$
\begin{aligned}
HC &= \ln(1+R-\mu_1)\left[h(1+R)\mu_1 + (R\alpha+\alpha-h)\frac{\mu_1^2}{2} - \frac{2\mu_1^3}{3}\right](a_1+b_1(1+R)) - (a_1+b_1(1+R))h(1+R) \\
&\times \left[\mu_1 + (1+R)\ln\left(\frac{1+R-\mu_1}{1+R}\right)\right] - (a_1+b_1(1+R))\left(\frac{R\alpha+\alpha-h}{2}\right)\left[(1+R)\mu_1\right. \\
&+ \left.\frac{\mu_1^2}{2} + (1+R)^2\ln\left(\frac{1+R-\mu_1}{1+R}\right)\right] + (a_1+b_1(1+R))\frac{\alpha}{3}\left[(1+R)^2\mu_1 + (1+R)\frac{\mu_1^2}{2}\right. \\
&+ \left.\frac{\mu_1^3}{3} + (1+R)^3\ln\left(\frac{1+R-\mu_1}{1+R}\right)\right] + h(1+R)b_1\frac{\mu_1^2}{2} + (R\alpha+\alpha-h)\frac{b_1\mu_1^3}{3} - \frac{\alpha b_1\mu_1^4}{4} \\
&+ \left[(b_1(1+R)-a_2)\ln(1+R-t_1) - b_2R\ln(1+R-\mu_2) - b_2(t_1-\mu_2) - b_1R\ln(1+R-\mu_1)\right. \\
&- \left.b_1\mu_1\right]\left[h(1+R)\mu_1 + (R\alpha+\alpha-h)\frac{\mu_1^2}{2} - \frac{\alpha\mu_1^3}{3}\right] + D_0\ln\left(\frac{1+R-\mu_2}{1+R-\mu_1}\right)\left[h(1+R)(\mu_2-\mu_1)\right. \\
&+ \left.(R\alpha+\alpha-h)\frac{\mu_2^2-\mu_1^2}{2} - \frac{\alpha(\mu_2^3-\mu_1^3)}{3}\right] + D_0h(1+R)\left[\mu_2-\mu_1+(1+R)\ln\left(\frac{1+R-\mu_2}{1+R-\mu_1}\right)\right] \\
&+ D_0\left(\frac{R\alpha+\alpha-h}{2}\right)\left[(1+R)(\mu_2-\mu_1) + \frac{\mu_2^2-\mu_1^2}{2} + (1+R)^2\ln\left(\frac{1+R-\mu_2}{1+R\mu_1}\right)\right] \\
&- \frac{\alpha}{3}D_0\left[(1+R)^2(\mu_2-\mu_1) + \frac{1+R}{2}(\mu_2^2-\mu_1^2) + \frac{\mu_2^3-\mu_1^3}{3}\right] + \left[(b_2(1+R)-a_2)\ln(1+R-t_1)\right. \\
&- \left.b_2R\ln(1+R-\mu_2) + b_2(t_1-\mu_2)\right]\left[h(1+R)(\mu_2-\mu_1) + \frac{R\alpha+\alpha-h}{2}(\mu_2^2-\mu_1^2) - \frac{\alpha}{3}(\mu_2^3-\mu_1^3)\right] \\
&+ (a_2-b_2(1+R))(1+R-t_1)\frac{\alpha}{3}\left[(1+R)^2(t_1-\mu_2) + \frac{1+R}{2}(t_1^2-\mu_2^2) + (1+R)^3\ln\left(\frac{1+R-t_1}{1+R-\mu_2}\right)\right] \\
&- (a_2-b_2(1+R))\ln\left(\frac{1+R-\mu_2}{1+R-t_1}\right)\left[h(1+R)\mu_2 + (R\alpha+\alpha-h)\frac{\mu_2^2}{2} - \frac{\alpha\mu_2^3}{3}\right] \\
&- (a_2-b_2(1+R))(1+R-t_1)h(1+R)\left[t_1-\mu_2+(1+R)\ln\left(\frac{1+R-t_1}{1+R-\mu_2}\right)\right] \\
&- (a_2-b_2(1+R))(1+R-t_1)\frac{R\alpha+\alpha-h}{2}\left[(1+R)(t_1-\mu_2) + \frac{t_1^2-\mu_2^2}{2} + (1+R)^2\ln\left(\frac{1+R-t_1}{1+R-\mu_2}\right)\right] \\
&+ h(1+R)b_2t_1(t_1-\mu_2) + (R\alpha+\alpha-h)b_2t_1\frac{t_1^2-\mu_2^2}{2} - \alpha b_2t_1\frac{t_1^3-\mu_2^3}{3} \\
&+ h(1+R)b_2\frac{t_1^2-\mu_2^2}{2} + (R\alpha+\alpha-h)b_2\frac{t_1^3-\mu_2^3}{3} - \alpha b_2\frac{t_1^4-\mu_2^4}{4}. \qquad (3.44)
\end{aligned}
$$

3. Purchase cost

$$
\begin{aligned}
PC &= c\left[(1+R)\left[a_1\ln(1+R) + b_1(1+R)\ln(1+R) + (b_2(1+R)-a_2)\ln(1+R-t_1) + \beta a_2(T-t_1)\right.\right. \\
&- \left.\left.b_2R\ln(1+R-\mu_2) - b_2(t_1-\mu_2) - b_1R\ln(1+R-\mu_1) - b_1\mu_1\right] - \beta\frac{b_2}{2}(T^2-t_1^2)\right]. \qquad (3.45)
\end{aligned}
$$

4. Shortage cost

$$
SC = c_1\left[\beta a_2t_1(t_1-T) + \beta a_2\frac{T^2-t_1^2}{2} - \frac{\beta b_2}{6}(T^3-t_1^3) + \frac{\beta b_2}{2}t_1^2(T-t_1)\right]. \qquad (3.46)
$$

5. Lost sale cost

$$
LSC = L(1-\beta)\left[a_2(T-t_1) - \frac{b_2}{2}(T^2-t_1^2)\right]. \qquad (3.47)
$$

The total average cost is given by

$$
\begin{aligned}
C_3(t_1) &= \frac{1}{T}\Bigg[A + \ln(1+R-\mu_1)\Big[h(1+R)\mu_1 + (R\alpha+\alpha-h)\frac{\mu_1^2}{2} - \frac{2\mu_1^3}{3}\Big](a_1+b_1(1+R)) - (a_1+b_1(1+R)) \\
&\times\ h(1+R)bigg[\mu_1 + (1+R)\ln\left(\frac{1+R-\mu_1}{1+R}\right)\Big] - (a_1+b_1(1+R))\left(\frac{R\alpha+\alpha-h}{2}\right)\Big[(1+R)\mu_1 \\
&+\ \frac{\mu_1^2}{2} + (1+R)^2\ln\left(\frac{1+R-\mu_1}{1+R}\right)\Big] + (a_1+b_1(1+R))\frac{\alpha}{3}\Big[(1+R)^2\mu_1 + (1+R)\frac{\mu_1^2}{2} \\
&+\ \frac{\mu_1^3}{3} + (1+R)^3\ln\left(\frac{1+R-\mu_1}{1+R}\right)\Big] + h(1+R)b_1\frac{\mu_1^2}{2} + (R\alpha+\alpha-h)\frac{b_1\mu_1^3}{3} - \frac{\alpha b_1\mu_1^4}{4} \\
&+\ \Big[(b_1(1+R)-a_2)\ln(1+R-t_1) - b_2R\ln(1+R-\mu_2) - b_2(t_1-\mu_2) - b_1R\ln(1+R-\mu_1) \\
&-\ b_1\mu_1\Big]\Big[h(1+R)\mu_1 + (R\alpha+\alpha-h)\frac{\mu_1^2}{2} - \frac{\alpha\mu_1^3}{3}\Big] + D_0\ln\left(\frac{1+R-\mu_2}{1+R-\mu_1}\right)\Big[h(1+R)(\mu_2-\mu_1) \\
&+\ (R\alpha+\alpha-h)\frac{\mu_2^2-\mu_1^2}{2} - \frac{\alpha(\mu_2^3-\mu_1^3)}{3}\Big] + D_0h(1+R)\Big[\mu_2-\mu_1+(1+R)\ln\left(\frac{1+R-\mu_2}{1+R-\mu_1}\right)\Big] \\
&+\ D_0\left(\frac{R\alpha+\alpha-h}{2}\right)\Big[(1+R)(\mu_2-\mu_1) + \frac{\mu_2^2-\mu_1^2}{2} + (1+R)^2\ln\left(\frac{1+R-\mu_2}{1+R\mu_1}\right)\Big] \\
&-\ \frac{\alpha}{3}D_0\Big[(1+R)^2(\mu_2-\mu_1) + \frac{1+R}{2}(\mu_2^2-\mu_1^2) + \frac{\mu_2^3-\mu_1^3}{3}\Big] + \Big[(b_2(1+R)-a_2)\ln(1+R-t_1) \\
&-\ b_2R\ln(1+R-\mu_2) + b_2(t_1-\mu_2)\Big]\Big[h(1+R)(\mu_2-\mu_1) + \frac{R\alpha+\alpha-h}{2}(\mu_2^2-\mu_1^2) - \frac{\alpha}{3}(\mu_2^3-\mu_1^3)\Big] \\
&+\ (a_2-b_2(1+R))(1+R-t_1)\frac{\alpha}{3}\Big[(1+R)^2(t_1-\mu_2) + \frac{1+R}{2}(t_1^2-\mu_2^2) + (1+R)^3\ln\left(\frac{1+R-t_1}{1+R-\mu_2}\right)\Big] \\
&-\ (a_2-b_2(1+R))\ln\left(\frac{1+R-\mu_2}{1+R-t_1}\right)\Big[h(1+R)\mu_2 + (R\alpha+\alpha-h)\frac{\mu_2^2}{2} - \frac{\alpha\mu_2^3}{3}\Big] \\
&-\ (a_2-b_2(1+R))(1+R-t_1)h(1+R)\Big[t_1-\mu_2+(1+R)\ln\left(\frac{1+R-t_1}{1+R-\mu_2}\right)\Big] \\
&-\ (a_2-b_2(1+R))(1+R-t_1)\frac{R\alpha+\alpha-h}{2}\Big[(1+R)(t_1-\mu_2) + \frac{t_1^2-\mu_2^2}{2} + (1+R)^2\ln\left(\frac{1+R-t_1}{1+R-\mu_2}\right)\Big] \\
&+\ h(1+R)b_2t_1(t_1-\mu_2) + (R\alpha+\alpha-h)b_2t_1\frac{t_1^2-\mu_2^2}{2} - \alpha b_2t_1\frac{t_1^3-\mu_2^3}{3} \\
&+\ h(1+R)b_2\frac{t_1^2-\mu_2^2}{2} + (R\alpha+\alpha-h)b_2\frac{t_1^3-\mu_2^3}{3} - \alpha b_2\frac{t_1^4-\mu_2^4}{4} \\
&+\ c\Big[(1+R)\Big[a_1\ln(1+R) + b_1(1+R)\ln(1+R) + (b_2(1+R)-a_2)\ln(1+R-t_1) + \beta a_2(T-t_1) \\
&-\ b_2R\ln(1+R-\mu_2) - b_2(t_1-\mu_2) - b_1R\ln(1+R-\mu_1) - b_1\mu_1\Big] - \beta\frac{b_2}{2}(T^2-t_1^2)\Big] \\
&+\ c_1\Big[\beta a_2t_1(t_1-T) + \beta a_2\frac{T^2-t_1^2}{2} - \frac{\beta b_2}{6}(T^3-t_1^3) + \frac{\beta b_2}{2}t_1^2(T-t_1)\Big] \\
&+\ L(1-\beta)\Big[a_2(T-t_1) - \frac{b_2}{2}(T^2-t_1^2)\Big]\Bigg]. \qquad (3.48)
\end{aligned}
$$

Equation (48) is highly non linear in nature with t_1. We can find the optimum values of t_1 for minimum average cost $C_3(t_1)$ from the solutions of the following equations by the help of Mathematica 10,

$$\frac{dC_3(t_1)}{dt_1} = 0. \qquad (3.49)$$

4 Numerical example and sensitivity analysis

In this section, we use Mathematica 10 to get a numerical solutions and sensitivity analysis of model for different parameters.

Example 4.1. *The parameter values is given as follows:*
$A = \$350$ *per order,* $T = 12$ *weeks,* $\mu_1 = 4$ *weeks,* $\mu_2 = 10$ *weeks,* $a_1 = 150$ *unit,* $b_1 = 50$ *unit,* $a_2 = 450$ *unit,* $b_2 = 10$ *unit,* $c = \$75$ *per unit,* $R = 13$ *weeks,* $L = \$7$ *per unit,* $\alpha = 0.2$ *unit,* $h = 5$ *unit,* $c_1 = \$5$ *per unit,* $\beta = 0.4$, $t_1^* = 3.20578$ *weeks, minimum cost* $C_1(t_1^*) = \$90137.07715$.

Example 4.2. *The parameter values is given as follows:*
$A = \$350$ *per order,* $T = 12$ *weeks,* $\mu_1 = 2.5$ *weeks,* $\mu_2 = 9$ *weeks,* $a_1 = 150$ *unit,* $b_1 = 50$ *unit,* $a_2 = 450$ *unit,* $b_2 = 10$ *unit,* $c = \$75$ *per unit,* $R = 13$ *weeks,* $L = \$7$ *per unit,* $\alpha = 0.2$ *unit,* $h = 5$ *unit,* $c_1 = \$5$ *per unit,* $\beta = 0.4$, $t_1^* = 4.901507$ *weeks, minimum cost* $C_2(t_1^*) = \$62503.3333$.

Example 4.3. *The parameter values is given as follows:*
$A = \$350$ *per order,* $T = 12$ *weeks,* $\mu_1 = 2$ *weeks,* $\mu_2 = 4.5$ *weeks,* $a_1 = 150$ *unit,* $b_1 = 40$ *unit,* $a_2 = 450$ *unit,* $b_2 = 10$ *unit,* $c = \$75$ *per unit,* $R = 13$ *weeks,* $L = \$7$ *per unit,* $\alpha = 0.2$ *unit,* $h = 5$ *unit,* $c_1 = \$5$ *per unit,* $\beta = 0.4$, $t_1^* = 8.051037$ *weeks, minimum cost* $C_3(t_1^*) = \$11873.09731$.

Based on the above numerical examples (1, 2 & 3), the performed sensitivity analysis by $\pm 50\%$ and $\pm 25\%$ changing one parameter at a time and keeping the remaining parameters at their original values. The table (3, 4 & 5) summarize the results of sensitivity analysis. Based on the results of sensitivity analysis from table (3, 4 & 5) the following observations can be done.

From table-3, the total average cost decreases with decrease in the parameters α, β, h, c_1, L, c, a_1, a_2, b_1, b_2 and A, however, the total average cost increases with decrease in the value of R. The total average cost is highly sensitive to changes in R, c_1, c, a_1, a_2, and b_1. It is less sensitive to changes in α, β, h, L, b_2 and A. The time duration of shortage is increases with decrease in the parameters α, β, h, a_2, b_1, A and R, however, the time duration of shortage is decreases with decrease in the parameters c_1, L, c, a_1 and b_2.

From table-4, the total average cost gradually decreases with decrease in the parameters α, β, h, c_1, L, c, a_1, a_2, b_1, b_2 and A, however, the total average cost increases with decrease in the value of R. The total average cost is less sensitive to changes in all parameters. The time duration of shortage is decreases with decrease in the parameters α, whereas, the time duration of shortage is increases with decrease in the parameters c_1, L, c, a_1, b_2.β, h, a_2, b_1, A and R.

From table-5, the total average cost gradually decreases with decrease in the parameters α, β, h, c_1, L, c, a_1, a_2, b_1, b_2 and A, however, the total average cost increases with decrease in the value of the parameter R. The total average cost is less sensitive to changes in all parameters, except R. The model is highly sensitive to change the parameter R. The time duration of shortage is increases with decrease the value of all parameters.

5 Conclusions

In this paper, we developed an economic order quantity model when the market demand is followed by trapezoidal type function. Shortages are allowed and partially backlogged. Time varying holding cost, deterioration rate of the items dependent on both time and life of items are considered. Numerical examples are carried out to illustrate the model and the solution procedure. Subsequently, sensitivity analysis is carried out with respect to all key parameters to observe interesting managerial insights. Finally, in this paper, from numerical example and sensitivity analysis, we conclude that the model without shortage is more profitable than the model allow with shortage. This is happening because of the deterioration rate is not only depend on time but also depend on life of the product. If the life of product is more than the length of replenishment cycle, then the model without shortage is profitable. In this paper, we have taken life of the products is more than the length of replenishment cycle. Therefore, our paper is conclude that the model without shortage is more profitable than shortage.

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Table 1: List of authors those who have used ramp type demand

Authors	Objective	Contribution	Remarks
Hill (1995)	Finding EOQ model	First time introduced ramp type demand	Constant holding cost
Mandal and Pal (1998)	Finding EOQ model	Deterministic and probabilistic demand are discussed and validate their model with numerical techniques	Constant holding cost
Wu et al. (1999)	Finding EOQ model	Weibull distribution, numerical techniques, sensitivity analysis	Constant holding cost
Wu and Ouyang (2000)	Finding EOQ model	Ramp type demand function, shortages are allowed	Constant holding cost
Wu (2001)	Finding EOQ model	Weibull distribution deterioration, variable backlogging rate and dependent on waiting time for the next replenishment.	Constant holding cost
Giri et al. (2003)	Finding EOQ model	Deterioration rate of items are follow Weibull distribution, shortages are allowed and fully backlogged	Constant holding cost
Deng (2005)	Finding EOQ model	Weibull distribution, sensitivity analysis and numerical techniques	Constant holding cost
Chen et al. (2006)	Finding EOQ model	Time dependent deterioration, shortages are allowed, sensitivity analysis and numerical examples.	Constant holding cost
Manna and Chadhuri (2006)	Finding EOQ model	Linear time dependent deterioration and numerical techniques, model with no shortage and with shortages are discussed	Constant holding cost
Deng et al. (2007)	Finding EOQ model	Point out some questionable results of Mandal and Pal (1998) and Wu and Ouyang (2000)	Constant holding cost
Panda et al. (2008)	Finding EOQ model	Constant deterioration, shortages are not allowed	Constnat holding cost
Wu et al. (2008)	Finding EOQ model	Model allows shortage, completely backlogged and constant deterioration	Constant holding cost
Skouri et al. (2009)	Finding EOQ model	Weibull distribution, partial backlogging, model starting with no shortages and with shortages	Constant holding cost
Panda et al. (2009)	Finding EOQ model	Uniqueness and existence of the solution is established	Constnat holding cost
Mahata and Goswami (2009)	Finding EOQ model	Fuzzy cost coefficients, fuzzy replenishment fuzzy multi-objective mathematical programming with the triangular fuzzy number	Constant holding cost
Panda and Saha (2010)	Finding EOQ model	Shortages are not allowed, uniform deterioration rate and sensitivity analysis.	Constant holding cost
Roy and Choudhuri (2011)	Finding EOQ model	Shortages and without shortages are allowed, numerical solutions provided, finite time horizon	Constnat holding cost
Agarwal and Banerjee (2011)	Finding EOQ model	Shortages are allowed and partially backlogged, an algorithm is developed	Constant holding cost
Tripathy and Mishra (2011)	Finding EOQ model	Weibull distribution, sensitivity analysis	Constant holding cost
Goyal et al. (2013)	Finding EOQ model	Genetic algorithm is implemented, finite time horizon	Constant holding cost
Sanni and Chukwa (2013)	Finding EOQ model	Three parameter Weibull distribution, shortages are allowed and complete backlog	Constant holding cost

Table 2: List of authors those who have used trapezoidal type demand

Authors	Objective	Contribution	Remarks
Cheng and Wang (2009)	Finding EOQ model	First time trapezoidal type demand introduced, shortages are allowed and completely backlogged	Constant holding cost, constant deterioration rate
Cheng et al. (2011)	Finding EOQ model	Trapezoidal type demand, shortages are allowed, partial backlogging	Constant holding cost
Saha et al. (2011)	Finding EOQ model	Price discount mathematical modelling, numerical techniques	Constant holding cost
Cheng (2012)	Finding EOQ model	Trapezoidal type demand supply chain management	Constant holding cost, constant deterioration rate
Uthayakumar and Rameswari (2012)	Finding EOQ model	Euler-Lagrang method used, shortages are not allowed	Constant holding cost, constant deterioration rate
Chuang et al. (2013)	Finding EOQ model	Trapezoidal type demand function, with and without shortage, partially backlogged	Constant holding cost, constant deterioration rate
Singh and Pattnayak (2013)	Finding EOQ model	Deterioration rate is Weibull distribution, trapezoidal type demand	Constant holding cost
Zhao (2014)	Finding EOQ model	Trapezoidal type demand, Weibull distribution deterioration rate, numerical techniques	Constant holding cost
Lin et al. (2014)	Finding EOQ model	Deterioration and partial backlogging	Constant holding cost
Dem et al. (2014)	Finding EOQ model	Trapezoidal type demand, numerical techniques	Constant holding cost, constant deterioration rate
Debata et al. (2015)	Finding EOQ model	Quadratic trapezoidal type demand, shortages are allowed	Constant holding cost, constant deterioration rate

Table 3: Sensitivity analysis with respect to the different parameters of the EOQ model for Case-I

Parameter	Change(%)	t_1^*	$C_1(t_1^*)$
α	+50	3.27051	93750.20153
	+25	3.25062	93043.01798
	-25	3.25051	92097.52768
	-50	3.20105	92001.00796
β	+50	3.20031	92508.01392
	+25	3.05780	92001.79231
	-25	3.00921	92001.00701
	-50	2.97013	91909.27592
h	+50	3.25079	92707.09376
	+25	3.20701	92701.12093
	-25	3.19230	92435.01783
	-50	3.10273	92013.87054
c_1	+50	2.97301	93079.01253
	+25	3.00791	93005.73458
	-25	3.15240	92739.00157
	-50	3.19076	92543.07532
L	+50	3.10520	92805.70153
	+25	3.20513	92510.15304
	-25	3.22075	92502.07352
	-50	3.22071	92401.03731
c	+50	3.00253	93517.05943
	+25	3.21075	92759.13709
	-25	3.20148	92050.00195
	-50	3.22057	91907.25079
a_1	+50	2.50713	94725.01536
	+25	2.81346	93079.17254
	-25	3.70193	90137.02549
	-50	3.84076	90731.02716
a_2	+50	3.80173	94053.01732
	+25	3.35162	92750.30457
	-25	2.90125	92080.17062
	-50	2.80157	91652.90173
b_1	+50	3.07523	92503.00171
	+25	2.90536	92275.36072
	-25	2.81603	92032.07480
	-50	———	——-
b_2	+50	2.70532	92901.05736
	+25	3.01257	92712.63527
	-25	3.30764	92204.72065
	-50	3.51480	92201.30531
A	+50	3.22053	92545.30512
	+25	3.21001	92544.30125
	-25	3.20732	92498.07326
	-50	3.01560	92440.69075
R	+50	3.60523	91705.86301
	+25	3.46798	92052.01983
	-25	2.90763	94925.01267
	-50	2.53075	97098.92576

Table 4: Sensitivity analysis with respect to the different parameters of the EOQ model for Case-II

Parameter	Change(%)	t_1^*	$C_2(t_1^*)$
α	+50	4.70127	62015.02357
	+25	4.81013	61973.16743
	-25	4.90072	61862.05732
	-50	4.99053	61802.50643
β	+50	4.95073	61989.20579
	+25	4.90879	61907.57832
	-25	4.90705	61896.42793
	-50	4.89235	61850.06925
h	+50	4.91205	61975.05769
	+25	4.90753	61960.17368
	-25	4.90601	61890.25376
	-50	4.90076	61875.03425
c_1	+50	4.98075	62901.75321
	+25	4.94362	61957.01893
	-25	4.90201	61032.70685
	-50	4.87057	61015.81342
L	+50	5.01379	63079.16983
	+25	4.90739	62582.75032
	-25	4.89074	61037.06517
	-50	4.80153	60979.35792
c	+50	5.10572	62932.05132
	+25	4.98374	61875.23691
	-25	4.90725	61013.71528
	-50	4.70792	60978.06493
a_1	+50	4.96053	61890.05271
	+25	4.94712	61867.79825
	-25	4.90358	61805.08543
	-50	4.89532	61801.73105
a_2	+50	4.98053	62904.50732
	+25	4.80148	61852.73201
	-25	4.67592	61801.05937
	-50	5.60985	61790.75301
b_1	+50	4.95780	61980.25710
	+25	4.92045	61875.05923
	-25	4.90752	61701.17682
	-50	4.89271	61692.87052
b_2	+50	4.97530	61979.01532
	+25	4.93125	61900.33251
	-25	4.90048	61897.01572
	-50	4.90101	61865.79858
A	+50	4.95667	61954.85791
	+25	4.93514	61901.03725
	-25	4.80532	61895.17523
	-50	4.80254	61890.53201
R	+50	5.07521	60073.00125
	+25	4.99079	62573.27918
	-25	4.30253	65379.82460
	-50	3.90792	69024.31026

Table 5: Sensitivity analysis with respect to the different parameters of the EOQ model for Case-III

Parameter	Change(%)	t_1^*	$C_3(t_1^*)$
α	+50	8.70531	12251.07963
	+25	8.31982	11083.70342
	-25	7.90871	10735.04251
	-50	7.89253	10071.96521
β	+50	8.40215	13781.07193
	+25	8.17082	11057.80134
	-25	7.95026	10042.71692
	-50	7.90631	9075.06741
h	+50	8.50921	12705.31206
	+25	8.19052	12069.61283
	-25	8.01304	10317.00791
	-50	7.97302	9419.10742
c_1	+50	8.60931	13924.80349
	+25	8.26310	11739.15723
	-25	8.01972	10705.90618
	-50	7.96052	9107.69042
L	+50	8.20981	15921.09275
	+25	8.19804	12074.83210
	-25	8.00541	10182.98024
	-50	7.89071	9471.07315
c	+50	8.30781	18076.42197
	+25	8.09561	14071.00814
	-25	7.94861	11806.70581
	-50	7.90671	10174.60832
a_1	+50	8.10893	11075.91372
	+25	8.07052	11026.11732
	-25	8.01246	10642.90742
	-50	8.01004	10173.63410
a_2	+50	8.19672	10798.38013
	+25	8.10729	10201.07300
	-25	8.01492	9842.09215
	-50	8.00710	9062.00921
b_1	+50	8.20573	12801.13780
	+25	8.07819	12017.49001
	-25	8.04162	10038.29410
	-50	7.96502	9056.19042
b_2	+50	8.19063	10961.80562
	+25	8.05721	10208.83041
	-25	8.00300	9072.90521
	-50	7.98402	9004.00461
A	+50	8.20963	13042.07521
	+25	8.07491	11562.06531
	-25	8.01962	10281.64081
	-50	7.92081	8042.90513
R	+50	9.04810	19302.98032
	+25	8.70831	10571.64812
	-25	8.08941	10049.64192
	-50	7.49802	9834.94210

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