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On the Stability of $\alpha-$Cauchy-Jensen type functional equation in Banach Algebras

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Abstract

Using fixed point methods, we prove the generalized Hyers-Ulam stability of homomorphisms in Banach algebras for the following $\alpha-$Cauchy-Jensen functional equation:

$$f(\frac{x+y}{\alpha}+z)+f(\frac{x-y}{\alpha}+z)=\frac{2}{\alpha}f(x)+2f(z),$$

where $\alpha \in \mathbb{N}_{\geq 2}$.

Keywords: Cauchy-Jensen type functional equation, fixed point, homomorphism in Banach algebra, generalized Hyers-Ulam stability.

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1 Introduction

The study of stability problems for functional equations is related to a question of S. M. Ulam [25] concerning the stability of group homomorphisms.

Let $(G,.)$ be a group and let $(H,.,d)$ be a metric group with the metric d. Given $\delta > 0$, does there exist $\epsilon > 0$ such that if a mapping $h : G \to H$ satisfies the inequality

$$d(h(xy),h(x)h(y)) \leq \delta$$

for all $x,y \in G$, then there is a homomorphism $a : G \to H$ with

$$d(h(x),a(x)) \leq \epsilon$$

for all $x \in G$?

In 1941, Hyers [7] considered the case of approximately additive mappings $f : E \to F$, where E and F are Banach spaces and f satisfies Hyers inequality

$$\|f(x+y)-f(x)-f(y)\| \leq \epsilon$$

for all $x,y \in E$ and $\epsilon > 0$. He proved that then there exists a unique additive mapping $T : E \to F$ satisfying

$$\|f(x)-T(x)\| \leq \epsilon$$

for all $x \in E$.

In 1950, T. Aoki [1] was the second author to study this problem for additive mappings.

In 1978, Th. M. Rassias [18] generalized the result of Hyers by considering the stability problem for unbounded Cauchy differences. This phenomenon of stability introduced by Th. M. Rassias [18] is called the Hyers-Ulam-Rassias stability.

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Theorem 1.1 ([18, Th. M. Rassias]). *Let $f : E \to F$ be a mapping from a real normed vector space E into a Banach space F satisfying the inequality*

$$\|f(x+y) - f(x) - f(y)\| \leq \theta(\|x\|^p + \|y\|^p), \tag{1.1}$$

for all $x, y \in E$, where θ and p are constants with $\theta > 0$ and $p < 1$. Then there exists a unique additive mapping $T : E \to F$ such that

$$\|f(x) - T(x)\| \leq \frac{2\theta}{2-2^p} \|x\|^p, \tag{1.2}$$

for all $x \in E$. If $p < 0$ then inequality (1.1) holds for all $x, y \neq 0$, and (1.3) for $x \neq 0$. Also, if the function $t \to f(tx)$ from $\mathbb{R}$ into F is continuous for each fixed $x \in E$, then T is linear.

The above inequality (1.3) has produced a lot of influence on the development of what we now call the Hyers-Ulam-Rassias stability of functional equations. Beginning around the year 1980 the topic of approximate homomorphisms, or the stability of the equation of homomorphism, was studied by a number of mathematicians. Rassias [20], following the spirit of the innovative approach of Rassias [18] for the unbounded Cauchy difference, proved a similar stability theorem in which he replaced the factor $\|x\|^p + \|y\|^p$ by $\|x\|^p.\|y\|^q$ for $p, q \in \mathbb{R}$ with $p + q \neq 1$ (see also [21] for a number of other new results).

Theorem 1.2 ([19, 20]). *Let E be a real normed linear space and F a real complete normed linear space. Assume that $f : E \to F$ is an approximately additive mapping for which there exist constants $\theta \geq 0$ and $p \in \mathbb{R}\backslash\{1\}$ such that f satisfies the inequality*

$$\|f(x+y) - f(x) - f(y)\| \leq \theta. \|x\|^{p/2} . \|y\|^{p/2},$$

for all $x, y \in E$. Then there exists a unique additive mapping $T : E \to F$ such that

$$\|f(x) - T(x)\| \leq \frac{\theta}{|2^p - 2|} \|x\|^p,$$

for all $x \in E$. If, in addition, $f : E \to F$ is a mapping such that the transformation $t \to f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in E$, then T is $\mathbb{R}-$linear.

Găvruta [6] generalized Rassias' result. The stability problems of several functional equations have been extensively investigated by a number of authors and there are many interesting results concerning this problem (see [2, 3, 8–16, 22–24]).

We now recall one of fundamental results of fixed point theory. For the proof, we refer to [5] : Let X a set, a function $d : X \times X \to [0, \infty]$ is called a generalized metric on X if d satisfies

1. $d(x, y) = 0$ if and only if $x = y$.

2. $d(x, y) = d(y, x)$ for all $x, y \in X$.

3. $d(x, z) \leq d(x, y) + d(y, z)$ for all $x, y, z \in X$.

Theorem 1.3 ([5, The alternative of fixed point]). *Let (X, d) be a complete generalized metric space and let $J : X \to X$ be a strict contractive mapping with a Lipschitz constant $L < 1$. Then for each given element $x \in X$, either*

$$d(J^n x, J^{n+1} x) = \infty$$

for all nonnegative integer n or there exists a positive integer n_0 such that

1. *$d(J^n x, J^{n+1} x) < \infty \ \forall n \geq n_0$;*

2. *the sequence $J^n x$ converge to a fixed y^* for J;*

3. *y^* is the unique fixed point of J in the set $Y = \{y \in X, d(J^{n_0} x, y) < \infty\}$;*

4. *$d(y, y^*) \leq (1/(1-L))d(y, Jy)$.*

In [17], Park proved the generalized Hyers-Ulam stability of the Cauchy-Jensen functional equation

$$f(\frac{x+y}{2}+z)+f(\frac{x-y}{2}+z)=f(x)+2f(z) \tag{1.3}$$

in Banach Algebras, by using the fixed theorem.

In the present paper, using the fixed point method, we prove the Hyers-Ulam-Rassias stability of homomorphisms in real Banach algebras for the α-Cauchy-Jensen functional equation

$$f(\frac{x+y}{\alpha}+z)+f(\frac{x-y}{\alpha}+z)=\frac{2}{\alpha}f(x)+2f(z), \tag{1.4}$$

where $\alpha \in \mathbb{N}_{\geq 2}$.

2 Stability of homomorphisms in real Banach algebras

Throughout this section, assume that $\mathbb{N}$ the set of all positive integers, $\mathbb{N}_{\geq 2}=\mathbb{N}\backslash\{0,1\}$, A is a real Banach algebra with norm $\|.\|_A$ and that B is a real Banach algebra with norm $\|.\|_B$.

For a given mapping $f: A \to B$,we define

$$D_\alpha f(x,y,z)=f(\frac{x+y}{\alpha}+z)+f(\frac{x-y}{\alpha}+z)-\frac{2}{\alpha}f(x)-2f(z),$$

for all $x,y,z \in A$ and $\alpha \in \mathbb{N}_{\geq 2}$.

Lemma 2.1. *If a mapping $f: A \to B$ satisfies (1.4), then f is a Jensen type additive-additive mapping.*

Proof. Letting $x=y=z=0$ in (1.4), we get $f(0)=0$. Setting $y=z=0$ in (1.4), we obtain

$$2f(\frac{x}{\alpha})=\frac{2}{\alpha}f(x), \tag{2.5}$$

for all $x \in A$. Replacing y by 0 in (1.4) and by (2.5), we get

$$2f(\frac{x}{\alpha}+z)=\frac{2}{\alpha}f(x)+2f(z)=2f(\frac{x}{\alpha})+2f(z),$$

for all $x,z \in A$. Then

$$f(t+z)=f(t)+f(z), \tag{2.6}$$

for all $t,z \in A$, with $t=x/\alpha$. This implies that f is an additive mapping.
Now, we substitute $t=\frac{u+v}{2}$ and $z=\frac{u-v}{2}$ in (2.6), we obtain

$$f(\frac{u+v}{2})+f(\frac{u-v}{2})=f(u), \tag{2.7}$$

for all $u,v \in A$. Therefore f is a Jensen type additive mapping. □

Using the fixed point method, we establish the Hyers-Ulam-Rassias stability of homomorphisms in real Banach algebras for the functional equation $D_\alpha f(x,y,z)=0$.

Theorem 2.4. *Let $f: A \to B$ be a mapping for which there exists a function $\varphi: A^3 \to [0,\infty)$ such that*

$$\|D_\alpha f(x,y,z)\|_B \leq \varphi(x,y,z); \tag{2.8}$$

$$\|f(xy)-f(x)f(y)\|_B \leq \varphi(x,y,0); \tag{2.9}$$

$$\sum_{n\geq 0} a^{-n}\varphi(a^n x, a^n y, a^n z) < \infty \tag{2.10}$$

and

$$\varphi(ax,ax,ax) \leq aL\varphi(x,x,x) \tag{2.11}$$

for all $x,y,z \in A$, with $L<1$ and $a=1+2/\alpha$. If $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in A$, then there exists a unique homomorphism $h: A \to B$ such that

$$\|f(x)-h(x)\|_B \leq \frac{1}{a-aL}\varphi(x,x,x), \tag{2.12}$$

for all $x \in A$.

Proof. Letting $x = y = z$ in (2.8), we get

$$\left\|\frac{f(ax)}{a} - f(x)\right\|_B \leq \frac{1}{a}\varphi(x,x,x); \tag{2.13}$$

for all $x \in A$, with $a = 1 + 2/\alpha$.

Consider the set

$$S := \{g : A \to B\}$$

and introduce the generalized metric on S:

$$d(g,k) = \inf\{C \in \mathbb{R}_+ : \|g(x) - k(x)\| \leq C\varphi(x,x,x), \forall x \in A\},$$

with the convention $\inf \emptyset := +\infty$. As in [4], one can prove that the generalized metric space (S,d) is complete.

Now, we consider the linear mapping $J : S \to S$ such that

$$Jg(x) := \frac{1}{a}g(ax) \tag{2.14}$$

for all $x \in A$. Hence $d(Jf, f) \leq \frac{1}{a} < \infty$.

First, we start by proving that J is strictly contractive on the (S,d). To this end, let $f, g \in S$ be given. Without loss of generality, we may suppose that $d(g,k)$ is finite. In this case, let $C \in \mathbb{R}_+$ be an arbitrary constant such that

$$\|g(x) - k(x)\| \leq C\varphi(x,x,x)$$

for all $x \in A$. Then

$$\begin{aligned}
\|Jg(x) - Jk(x)\|_B &= \|\frac{g(ax)}{a} - \frac{k(ax)}{a}\|_B \\
&\leq \frac{1}{a}\varphi(ax,ax,ax) \qquad (2.15)\\
&\leq CL\varphi(x,x,x) \qquad (2.16)
\end{aligned}$$

for every $x \in A$, i.e, $d(Jg, Jk) \leq CL$. This implies that

$$d(Jg, Jk) \leq Ld(f,g)$$

for all $g, k \in S$. As $L < 1$, then operator J is strictly contractive.

By Theorem 1.3, there exists a mapping $h : A \to B$ satisfying the following

(1) h is a fixed point of J, that is,

$$h(ax) = ah(x)$$

for all $x \in A$. The mapping h is a unique fixed point of J in the set

$$S^* = \{g \in S : d(g,k) < \infty\}.$$

(2) $\lim_{n\to\infty} d(J^n f, h) = 0$. This implies the equality

$$\lim_{n\to\infty} J^n f(x) = \lim_{n\to\infty} \frac{f(a^n x)}{a^n} = h(x) \tag{2.17}$$

for all $x \in A$.

(3) $d(f,h) \leq \frac{1}{1-L}d(Jf,f)$, which implies the inequality

$$d(f,g) \leq \frac{1}{a - aL}.$$

This implies that the inequality (2.12) holds.

It follows from (2.8), (2.10) and (2.17) that

$$\|D_\alpha h(x,y,z)\|_B = \lim_{n\to\infty} \frac{1}{a^n} \|D_\alpha f(a^n x, a^n y, a^n z)\|_B \tag{2.18}$$

$$\leq \lim_{n\to\infty} \frac{1}{a^n} \varphi(a^n x, a^n y, a^n z) = 0 \tag{2.19}$$

for all $x,y,z \in A$ and $n \in \mathbb{N}$. So

$$h(\frac{x+y}{\alpha} + z) + h(\frac{x-y}{\alpha} + z) = \frac{2}{\alpha} h(x) + 2h(z),$$

for all $x,y,z \in A$. By Lemma 2.1, the mapping $h : A \to B$ is Cauchy additive.

By the same reasoning as the proof of theorem of [18], the mapping $h : A \to B$ is $\mathbb{R}$-linear.

It follow from (2.9) that

$$\begin{aligned} \|h(xy) - h(x)h(y)\| &= \lim_{n\to\infty} \frac{1}{a^{2n}} \|f(a^n x a^n y) - f(a^n x) f(a^n y)\|_B \\ &\leq \lim_{n\to\infty} \frac{1}{a^{2n}} \varphi(a^n x, a^n y, 0) \\ &\leq \lim_{n\to\infty} \frac{1}{a^n} \varphi(a^n x, a^n y, 0) = 0 \end{aligned}$$

for all $x,y \in A$. So $h(xy) = h(x)h(y)$ for all $x,y \in A$. Thus $h : A \to B$ is a homomorphism satisfying (2.12), as desired. □

Corollary 2.1 ([17])**.** *Let $f : A \to B$ be a mapping for which there exists a function $\varphi : A^3 \to [0,\infty)$ such that*

$$\|D_2 f(x,y,z)\|_B \leq \varphi(x,y,z);$$

$$\|f(xy) - f(x)f(y)\|_B \leq \varphi(x,y,0);$$

$$\sum_{n\geq 0} 2^{-n} \varphi(2^n x, 2^n y, 2^n z) < \infty$$

and

$$\varphi(2x,2x,2x) \leq 2L\varphi(x,x,x)$$

for all $x,y,z \in A$, with $L < 1$. If $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in A$, then there exists a unique homomorphism $h : A \to B$ such that

$$\|f(x) - h(x)\|_B \leq \frac{1}{2-2L} \varphi(x,x,x),$$

for all $x \in A$.

Corollary 2.2. *Let $p < 1$ and δ be nonnegative real numbers and let $f : A \to B$ be a mapping satisfying*

$$\|D_\alpha f(x,y,z)\|_B \leq \delta(\|x\|_A^p + \|y\|_A^p + \|z\|_A^p);$$

and

$$\|f(xy) - f(x)f(y)\|_B \leq \delta(\|x\|_A^p + \|y\|_A^p);$$

for all $x,y,z \in A$, with $a = 1 + 2/\alpha$. If $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in A$, then there exists a unique homomorphism $h : A \to B$ such that

$$\|f(x) - h(x)\|_B \leq \frac{3\delta}{a - a^p} \|x\|_A^p,$$

for all $x \in A$.

Proof. The proof follows from Theorem 2.4 by taking

$$\varphi(x,y,z) := \delta(\|x\|_A^p + \|y\|_A^p + \|z\|_A^p)$$

for all $x,y,z \in A$. Then, $L = a^{p-1}$ (with $a = 1 + 2/\alpha$) and we get the desired result. □

Theorem 2.5. *Let $f : A \to B$ be a mapping for which there exists a function $\varphi : A^3 \to [0, \infty)$ such that*

$$\|D_\alpha f(x,y,z)\|_B \leq \varphi(x,y,z); \tag{2.20}$$

$$\|f(xy) - f(x)f(y)\|_B \leq \varphi(x,y,0); \tag{2.21}$$

$$\sum_{n\geq 0} a^{2n}\varphi(a^{-n}x, a^{-n}y, a^{-n}z) < \infty \tag{2.22}$$

and

$$a\varphi(x,x,x) \leq L\varphi(ax,ax,ax) \tag{2.23}$$

for all $x, y, z \in A$, with $L < 1$ and $a = 1 + 2/\alpha$. If $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in A$, then there exists a unique homomorphism $h : A \to B$ such that

$$\|f(x) - h(x)\|_B \leq \frac{L}{a - aL}\varphi(x,x,x), \tag{2.24}$$

for all $x \in A$.

Proof. We consider the linear mapping $J : S \to S$ such that

$$Jg(x) := ag(\frac{x}{a})$$

for all $x \in A$ with $a = 1 + 2/\alpha$.

It follows from (2.13) that

$$\|f(x) - af(x/a)\|_B \leq \varphi(x/a, x/a, x/a) \leq \frac{L}{a}\varphi(x,x,x) \tag{2.25}$$

for all $x \in A$. Hence $d(f, Jf) \leq \frac{L}{a}$.

Let $g, k \in S$ and $C \in \mathbb{R}_+$ be an arbitrary constant such that

$$\|g(x) - k(x)\| \leq C\varphi(x,x,x)$$

for all $x \in A$. Then

$$\begin{aligned} \|Jg(x) - Jk(x)\|_B &= \|ag(\frac{x}{a}) - ak(\frac{x}{a})\|_B \\ &\leq aC\varphi(\frac{x}{a}, \frac{x}{a}, \frac{x}{a}) \\ &\leq CL\varphi(x,x,x) \end{aligned} \tag{2.26}$$

for all $x \in A$, i.e, $d(Jg, Jk) \leq CL$. We hence conclude that

$$d(Jg, Jk) \leq Ld(g,k)$$

for all $g, k \in S$. As $L < 1$, then operator J is strictly contractive.

By Theorem 1.3, there exists a mapping $h : A \to B$ satisfying the following

(1) h is a fixed point of J such that $\lim_{n\to\infty} d(J^n f, h) = 0$. This implies the equality

$$h(x) = \lim_{n\to\infty} J^n f(x) = \lim_{n\to\infty} a^n f(\frac{x}{a^n}) \tag{2.27}$$

and $ah(\frac{x}{a}) = h(x)$ for all $x \in A$. The mapping h is a unique fixed point of J in the set

$$S^* = \{g \in S : d(f,g) < \infty\}.$$

(2) $d(f,h) \leq \frac{1}{1-L}d(Jf, f)$, which implies the inequality

$$d(f,g) \leq \frac{L}{a - aL}.$$

This implies that the inequality (2.24) holds.

It follows from (2.20), (2.22) and (2.27) that

$$\begin{aligned} \|D_\alpha h(x,y,z)\|_B &= \lim_{n\to\infty} a^n \left\| D_\alpha f(\frac{1}{a^n}x, \frac{1}{a^n}y, \frac{1}{a^n}z) \right\|_B \\ &\leq \lim_{n\to\infty} a^n \varphi(\frac{1}{a^n}x, \frac{1}{a^n}y, \frac{1}{a^n}z) \quad (2.28) \\ &\leq \lim_{n\to\infty} a^{2n} \varphi(\frac{1}{a^n}x, \frac{1}{a^n}y, \frac{1}{a^n}z) = 0 \quad (2.29) \end{aligned}$$

for all $x, y, z \in A$ and $n \in \mathbb{N}$. So

$$h(\frac{x+y}{\alpha} + z) + h(\frac{x-y}{\alpha} + z) = \frac{2}{\alpha}h(x) + 2h(z),$$

for all $x, y, z \in A$. By Lemma 2.1, the mapping $h : A \to B$ is Cauchy additive.

By the same reasoning as the proof of theorem of [18], the mapping $h : A \to B$ is $\mathbb{R}$-linear.

It follow from (2.21) that

$$\begin{aligned} \|h(xy) - h(x)h(y)\| &= \lim_{n\to\infty} a^{2n} \|f(\frac{xy}{a^{2n}}) - f(\frac{x}{a^n})f(\frac{y}{a^n})\|_B \\ &\leq \lim_{n\to\infty} a^{2n} \varphi(a^{-n}x, a^{-n}y, 0) = 0 \end{aligned}$$

for all $x, y \in A$. So $h(xy) = h(x)h(y)$ for all $x, y \in A$.
Thus $h : A \to B$ is a homomorphism satisfying (2.24), as desired. □

Corollary 2.3 ([17]). *Let $f : A \to B$ be a mapping for which there exists a function $\varphi : A^3 \to [0, \infty)$ such that*

$$\|D_2 f(x,y,z)\|_B \leq \varphi(x,y,z);$$

$$\|f(xy) - f(x)f(y)\|_B \leq \varphi(x,y,0);$$

$$\sum_{n\geq 0} 2^{2n} \varphi(2^{-n}x, 2^{-n}y, 2^{-n}z) < \infty$$

and

$$2\varphi(x,x,x) \leq L\varphi(2x,2x,2x)$$

for all $x, y, z \in A$, with $L < 1$. If $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in A$, then there exists a unique homomorphism $h : A \to B$ such that

$$\|f(x) - h(x)\|_B \leq \frac{L}{2-2L}\varphi(x,x,x),$$

for all $x \in A$.

Corollary 2.4. *Let $p > 1$ and δ be nonnegative real numbers and let $f : A \to B$ be a mapping satisfying*

$$\|D_\alpha f(x,y,z)\|_B \leq \delta(\|x\|_A^p + \|y\|_A^p + \|z\|_A^p);$$

and

$$\|f(xy) - f(x)f(y)\|_B \leq \delta(\|x\|_A^p + \|y\|_A^p);$$

for all $x, y, z \in A$, with $a = 1 + 2/\alpha$. If $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in A$, then there exists a unique homomorphism $h : A \to B$ such that

$$\|f(x) - h(x)\|_B \leq \frac{3\delta}{a^p - a}\|x\|_A^p,$$

for all $x \in A$.

Proof. The proof follows from Theorem 2.5 by taking

$$\varphi(x,y,z) := \delta(\|x\|_A^p + \|y\|_A^p + \|z\|_A^p)$$

for all $x, y, z \in A$. Then, $L = a^{1-p}$ (with $a = 1 + 2/\alpha$) and we get the desired result. □

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