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Compactons solutions for the fractional nonlinear dispersive K(2,2) equations by the homotopy perturbation method

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Abstract

In this paper, the homotopy perturbation is successively used to obtain approximate analytical solutions of the nonlinear dispersive $K(2,2)$ equation with time and space derivative. Comparison between the numerical and the exact solutions revealed that HPM is an alternative analytical method for solving fractional differential equations.

Keywords: Caputo fractional derivative, homotopy perturbation method, K(2,2) equation, fractional differential equations.

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1 Introduction

Fractional differential systems have recently been proved to be useful in physics, engineering and control processing in various fields of sciences such as viscoelasticity, diffusion, control, relaxation processes. Many contributions have been made to both the theory and applications of the fractional differential equations during the past decades ([7],[15],[18]). As in general, there exists no method that yields an exact solution, approximate solutions are then derived using the linearization ([7],[15]) or Adomian decomposition method (ADM) [1] . The variational iteration method (VIM) was first proposed by He [11] for solving non-linear problems and it is found to be an effective way to approximate the solutions of the fractional differential equations, both linear and nonlinear [14]. Momani and Odibat [17] and Yıldırım [20] applied the homotopy perturbation method (HPM) to fractional differential equations and revealed that HPM is an alternative analytical method for solving fractional differential equations.
Our concern in this work is to consider the numerical solution of the nonlinear dispersive $K(2,2)$ equation with time and space fractional derivatives of the form

$$^{c}D_t^{\alpha}u + (2u + 6u_{xx})^{c}\, D_x^{\beta}u + 2uu_{xxx} = 0, \;\; 0 < \alpha, \beta \leq 1, \tag{1.1}$$

with the initial condition

$$u(x,0) = g(x). \tag{1.2}$$

When $\alpha = \beta = 1$, this equation turns to the classical $K(2,2)$ equation

$$u_t + (u^2)_x + (u^2)_{xxx} = 0, \tag{1.3}$$

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developed in [19] for describing the compacton solution; i.e., a compact wave that preserves its shape after the interaction with another compact wave.
For the case $g(x) = (\frac{4}{3})\cos^2(\frac{x}{4})$, Eq. (1.3) has an exact solution $u(x,t) = (\frac{4}{3})\cos^2(\frac{x-t}{4})$ and it is developed in [8] for describing the compacton solution [14]. This example is solved numerically by the HPM method and by the variational homotopy method in [2]. It wil be used for comparing the exact and the numerical approximation in section 3.1.
We will extend the application of the HPM method in order to derive analytical approximate solutions to nonlinear time and space fractional $K(2,2)$ equations (1.1). Precisely, we use the new homotopy described in [17] for handling an iterative formula easy-to-use for computation. Observing the numerical results, and comparing with the exact solution, the proposed method reveals to be very close to the exact solution and consequently, an efficient way to solve the nonlinear fractional $K(2,2)$ equation (1.1)-(1.2). This method can take the advantages of the conventional perturbation method while eliminating its restrictions. HPM has been applied by many authors ([3],[4],[6],[9],[20]) and used for many types of linear and non-linear equations in science and engineering. This is the raison why we try to use it in this work.

2 Basic definitions

There are several definitions of a fractional derivative of order $\alpha > 0$ (see [7], [15], [16], [18]). The most commonly used definitions are the Riemann–Liouville and Caputo. We give some basic definitions and properties of the fractional calculus theory which are used further in this paper .

Definition 2.1. *A real function $f(x)$, $x > 0$, is said to be in the space C_μ, $\mu \in \mathbb{R}$ if there exists a real number $p > \mu$, such that $f(x) = x^p f_1(x)$, where $f_1(x) \in C\,[0,\infty)$, and it is said to be in the space C_μ^m, if $f^{(m)} \in C_\mu$, $m \in \mathbb{N}$.*

Definition 2.2. *The left sided Riemann–Liouville fractional integral of order $\alpha \geq 0$ of a function $f \in C_\mu$, $\mu \geq -1$, is defined as*

$$I^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)dt, \quad x > 0, \alpha > 0, \tag{2.4}$$

$$I^0 f(x) = f(x).$$

Definition 2.3. *Let $f \in C_{-1}^m$, $m \in \mathbb{N}$. Then the (left sided) Caputo fractional derivative of f is defined as*

$$^cD^\alpha f(x) = I^{m-\alpha} D^m f(x) = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_0^x (x-t)^{m-\alpha-1} f^{(m)}(t)dt, & m-1 < \alpha < m, \\ \frac{d^m f(t)}{dt^m}, & \alpha = m. \end{cases} \tag{2.5}$$

According to (2.5), we can obtain

$$^cD^\alpha C = 0, \ \text{C is constant}$$

and

$$^cD^\alpha t^\beta = \begin{cases} \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)} t^{\beta-\alpha}, & \beta > \alpha - 1, \\ 0, & \beta \leq \alpha - 1. \end{cases} \tag{2.6}$$

Remark 2.1. *In this paper, we consider equation (1.1) with time-and space-fractional derivative in the Caputo sens. When $\alpha \in \mathbb{R}^+$, the time fractional derivative is defined as*

$$^cD_t^\alpha u(x,t) = \frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_0^t (x-t)^{m-\alpha-1} \frac{\partial^m u(x,t)}{\partial \tau^m}(\tau)d\tau, & m-1 < \alpha < m, \\ \frac{\partial^m u(x,t)}{\partial t^m}, & \alpha = m. \end{cases} \tag{2.7}$$

The form of the space fractional derivative is similar to the above and we just omit it here.

3 The Homotopy Perturbation Method

To illustrate the basic ideas of this method, we consider the following non-linear differential equation

$$A(u) - f(r) = 0, \quad r \in \Omega, \tag{3.8}$$

with the following boundary conditions

$$B(u, \frac{\partial u}{\partial n}) = 0, \quad r \in \Gamma, \tag{3.9}$$

where A is a general differential operator, $f(r)$ is a known analytic function, B is a boundary operator, is the unknown function, and Γ is the boundary of the domain Ω. The operator A can be generally divided into two operators, L and N, where L is a linear, and N a nonlinear operator. Equation (3.8) can be, therefore, written as follows

$$L(u) + N(u) - f(r) = 0. \tag{3.10}$$

Using the homotopy technique, we construct a homotopy $v(r, p) : \Omega \times [0, 1] \longrightarrow \mathbb{R}$, which satisfies

$$H(v, p) = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \tag{3.11}$$

or

$$H(v, p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0, \tag{3.12}$$

where $p \in [0, 1]$ is an embedding parameter, and $u_0$is the initial approximation of equation (3.8) which satisfies the boundary conditions. Clearly, from Eq. (3.11) and (3.12) we will have

$$H(v, 0) = L(v) - L(u_0) = 0, \tag{3.13}$$
$$H(v, 1) = A(v) - f(r) = 0. \tag{3.14}$$

The changing process of p from zero to unity is just that of $v(r, p)$ changing from $u_0(r)$ to $u(r)$. In topology, this is called deformation and $L(v) - L(u_0)$ and $A(v) - f(r)$ are called homotopic. If the embedding parameter p $(0 \leq p \leq 1)$ is considered as a "small parameter", applying the classical perturbation technique, we can assume that the solution of equation (3.11) or (3.12) can be given as a power series in p

$$v = v_0 + pv_1 + p^2 v_2 + \cdots . \tag{3.15}$$

Setting $p = 1$, results in the approximate solution of Eq. (3.8)

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots . \tag{3.16}$$

The convergence of the series (3.16) has been proved in ([5],[6]).

3.1 New modification of the HPM

Momani and al. [17] introduce an algorithm to handle in a realistic and efficient way the nonlinear PDEs of fractional order. They consider the nonlinear partial differential equations with time fractional derivative of the form

$$\begin{cases} {}^cD_t^\alpha u(x, t) = f(u, u_x, u_{xx}) = L(u, u_x, u_{xx}) + N(u, u_x, u_{xx}) + h(x, t), t > 0 \\ u^k(x, 0) = g_k(x), \quad k = 0, 1, 2, ...m - 1, \end{cases} \tag{3.17}$$

where L is a linear operator, N is a nonlinear operator which also might include other fractional derivatives of order less than α. The function h is considered to be a known analytic function and ${}^cD^\alpha$, $m - 1 < \alpha \leq m$, is the Caputo fractional derivative of order α.
In view of the homotopy technique, we can construct the following homotopy

$$\frac{\partial u^m}{\partial t^m} - L(u, u_x, u_{xx}) - h(x, t) = p[\frac{\partial u^m}{\partial t^m} + N(u, u_x, u_{xx}) -^c D_t^\alpha u], \tag{3.18}$$

or

$$\frac{\partial u^m}{\partial t^m} - h(x, t) = p[\frac{\partial u^m}{\partial t^m} + L(u, u_x, u_{xx}) + N(u, u_x, u_{xx}) -^c D_t^\alpha u], \tag{3.19}$$

where $p \in [0,1]$. The homotopy parameter p always changes from zero to unity. In case $p = 0$, Eq. (3.18) becomes the linearized equation

$$\frac{\partial u^m}{\partial t^m} = L(u, u_x, u_{xx}) + h(x,t), \tag{3.20}$$

or in the second form, Eq. (3.19) becomes the linearized equation

$$\frac{\partial u^m}{\partial t^m} = h(x,t). \tag{3.21}$$

When it is one, Eq. (3.18) or Eq. (3.19) turns out to be the original fractional differential equation (3.17). The basic assumption is that the solution of Eq. (3.18) or Eq. (3.19) can be written as a power series in p

$$u = u_0 + pu_1 + p^2 u_2 + p^3 u_3 \cdots . \tag{3.22}$$

Finally, we approximate the solution by

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{3.23}$$

4 HPM method for the fractional K(2,2) equations

In this section, we apply the New modification of the HPM 3.1 for solving the K(2,2) equation with time-fractional derivative and we use the classical HPM to obtain analytical solution for K(2,2) equation with space-fractional derivative.

4.1 Numerical solutions of time-fractional K(2,2) equation

If one fixes $\beta = 1$ and considers the following form of the time-fractional $K(2,2)$ equation

$$^cD_t^\alpha u + (2\,u + 6\,u_{xx})\,u_x + 2\,uu_{xxx} = 0, \quad 0 < \alpha \leq 1, \tag{4.24}$$

with the initial condition

$$u(x,0) = (\frac{4}{3})\cos^2(\frac{x}{4}), \tag{4.25}$$

the exact solution of (4.24)-(4.25) for the special case $\alpha = 1$ is

$$u(x,t) = (\frac{4}{3})\cos^2(\frac{x-t}{4}). \tag{4.26}$$

In view of Eq. (3.19), the homotopy for Eq. (4.24) can be constructed as

$$\frac{\partial u}{\partial t} = p[\frac{\partial u}{\partial t} - (2\,u + 6\,u_{xx})\,u_x - 2\,uu_{xxx} - \,^cD_t^\alpha u]. \tag{4.27}$$

Substituting (3.22) into (4.27) and equating the terms with identical powers of p, one obtains the following set of linear partial differential equations

$$\begin{aligned}
&p^0 : \tfrac{\partial u_0}{\partial t} = 0,\\
&p^1 : \tfrac{\partial u_1}{\partial t} = \tfrac{\partial u_0}{\partial t} - (2u_0 + 6u_{0xx})\,u_{0x} - 2u_0u_{0xxx} -^c D_t^\alpha u_0,\\
&p^2 : \tfrac{\partial u_2}{\partial t} = \tfrac{\partial u_1}{\partial t} - (2u_0 + 6u_{0xx})\,u_{1x} - (2u_1 + 6u_{1xx})\,u_{0x} - 2u_0u_{1xxx} - 2u_1u_{0xxx} - \,^cD_t^\alpha u_1,\\
&p^3 : \tfrac{\partial u_3}{\partial t} = \tfrac{\partial u_2}{\partial t} - (2u_0 + 6u_{0xx})\,u_{2x} - (2u_1 + 6u_{1xx})\,u_{1x} - (2u_2 + 6u_{2xx})\,u_{0x} - 2(u_0u_{2xxx} + 2u_2u_{0xxx} + 2u_1u_{1xxx}) -^c D_t^\alpha u_2,\\
&\vdots
\end{aligned} \tag{4.28}$$

with the following conditions

$$u_0(x,0) = (\frac{4}{3})\cos^2(\frac{x}{4}), \tag{4.29}$$

$$u_i(x,0) = 0 \quad for \quad i = 1,2,.... \tag{4.30}$$

Selecting the initial value $u(x,0) = (\frac{4}{3})\cos^2(\frac{x}{4})$ for $u_0(x,t)$ and using equations (4.28) one obtains the following successive approximations

$$\begin{aligned}
&u_0\,(x,t) = \tfrac{4}{3}\cos^2(\tfrac{x}{4}),\\
&u_1\,(x,t) = \tfrac{1}{3}\sin(\tfrac{x}{2})t,\\
&u_2\,(x,t) = \tfrac{1}{3}\sin(\tfrac{x}{2})t - \tfrac{1}{12}\cos(\tfrac{x}{2})t^2 - \tfrac{1}{3}\sin(\tfrac{x}{2})\tfrac{t^{2-\alpha}}{\Gamma(3-\alpha)},\\
&u_3\,(x,t) = \tfrac{1}{3}\sin(\tfrac{x}{2})t - \tfrac{1}{6}\cos(\tfrac{x}{2})t^2 - \tfrac{1}{72}\sin(\tfrac{x}{2})t^3 - \tfrac{2}{3}\sin(\tfrac{x}{2})\tfrac{t^{2-\alpha}}{\Gamma(3-\alpha)} + \tfrac{1}{3}\sin(\tfrac{x}{2})\tfrac{t^{3-2\alpha}}{\Gamma(4-2\alpha)} + \tfrac{1}{3}\cos(\tfrac{x}{2})\tfrac{t^{3-\alpha}}{\Gamma(4-\alpha)},\\
&\vdots
\end{aligned} \tag{4.31}$$

The first four terms of the decomposition series solution for Eq. (4.24) is given as

$$u(x,t) = \frac{4}{3}\cos^2(\frac{x}{4}) + \sin(\frac{x}{2})t - \frac{1}{4}\cos(\frac{x}{2})t^2 - \frac{1}{72}\sin(\frac{x}{2})t^3 - \sin(\frac{x}{2})\frac{t^{2-\alpha}}{\Gamma(3-\alpha)} + \frac{1}{3}\sin(\frac{x}{2})\frac{t^{3-2\alpha}}{\Gamma(4-2\alpha)} + \frac{1}{3}\cos(\frac{x}{2})\frac{t^{3-\alpha}}{\Gamma(4-\alpha)} \tag{4.32}$$

Case 1: substituting $\alpha = 1$ into (4.32), we obtain the following (four-terms) approximation of the IVP (4.24)-(4.25).

$$u(x,t) = (\frac{4}{3})\cos^2(\frac{x}{4}) + \frac{2}{3}(\cos(\frac{x}{4})\sin(\frac{x}{4}))t - \frac{1}{12}(-1+\cos^2(\frac{x}{4}))t^2 - \frac{1}{36}(\cos(\frac{x}{4})\sin(\frac{x}{4}))t^3. \tag{4.33}$$

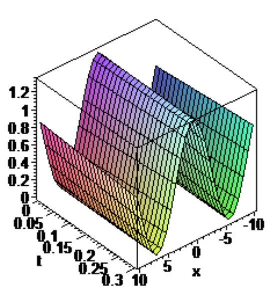
Note that this result is the same obtained by the variational homotopy perturbation method (VHPM) in [2].
On the other hand, an expansion of the exact solution (4.26) in Taylor series over $t = 0$ to order 3 gives:

$$u(x,t) = (\frac{4}{3})\cos^2(\frac{x-t}{4}) = (4/3)\cos^2(\frac{x}{4}) + \frac{2}{3}(\cos(\frac{x}{4})\sin(\frac{x}{4}))t - \frac{1}{12}(-1+\cos^2(\frac{x}{4}))t^2 - \frac{1}{36}(\cos(\frac{x}{4})\sin(\frac{x}{4}))t^3 + O(t^4).$$

This confirms our result.
Case 2: substituting $\alpha = \frac{1}{2}$ into (4.32), one obtains

$$u(x,t) = (\frac{4}{3})\cos^2(\frac{x}{4}) + \sin(\frac{x}{2})t - \frac{4}{3\sqrt{\pi}}\sin(\frac{x}{2})t^{\frac{3}{2}} - \frac{1}{12}\cos(\frac{x}{2})t^2 + \frac{8}{45\sqrt{\pi}}\cos(\frac{x}{2})t^{\frac{5}{2}} - \frac{1}{72}\sin(\frac{x}{2})t^3.$$

In the same manner, the rest of components can be obtained using the iteration formula (4.37) and the Maple package.



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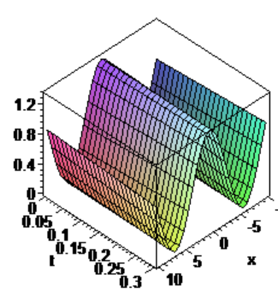

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Figure 1: (Left): Exact solution for Eq. (4.24) with the initial condition (4.25); (Right): Approximative solution of Eq. (4.24) by HPM method for $\alpha = 1$ with four terms.

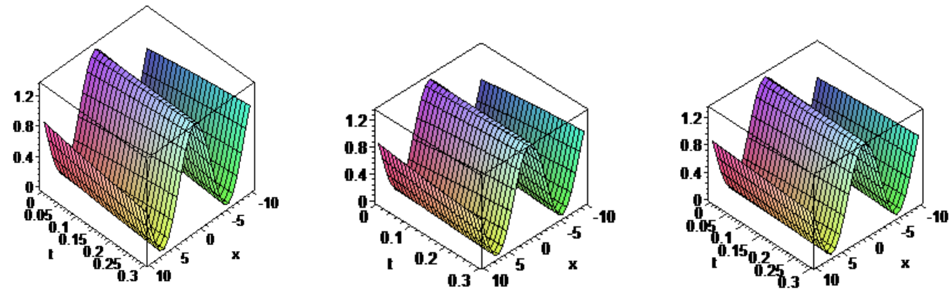
solution.png 

Figure 2: Series solution of Eq. (4.24) by HPM method with four terms for $\alpha = 1/4$ (left), $\alpha = 1/2$ (middle), and $\alpha = 0.9$ (right).

In Fig. 1, one has represented the graph of the exact solution (Left) of the initial value problem (4.24)-(4.25) and its series approximation (Right) with four terms and $\alpha = 1$. As it is shown, there is a similarity between the exact and the approximate solution by HPM method. In addition, it is clear that if one wants more accuracy, it is sufficient to increase the order of p.

The Fig. 2, represents the series solution obtained by the HPM method with four terms for respectively, $\alpha = 1/4$, $\alpha = 1/2$, and $\alpha = 0.9$.

4.2 Numerical solutions of space-fractional K(2,2) equation

We next consider the following space-fractional $K(2,2)$ equation with initial condition

$$u_t + (2u + 6u_{xx})^c\, D_x^{\beta} u + 2uu_{xxx} = 0, \quad 0 < \beta \le 1, \tag{4.34}$$

with initial condition

$$u(x,0) = x^2. \tag{4.35}$$

This initial condition is taken as polynomial to avoid heavy calculations of fractional differentiation.

According to the HPM, we construct the following homotopy

$$u_t - v_{0t} + p[(2\,u + 6u_{xx})^c\, D_x^{\beta} u + 2uu_{xxx} + v_{0t}] = 0, \quad 0 < \beta \le 1, \tag{4.36}$$

where $p \in [0,1]$ and $v_0 = u(x,0) = x^2$.
In view of the HPM, substituting equation (3.22) into equation (4.36) and equating the coefficients of like powers of p, we get the following set of differential equations

$$\begin{aligned}
p^0 &: \tfrac{\partial u_0}{\partial t} = v_{0t}, \quad u_0(x,0) = x^2,\\
p^1 &: \tfrac{\partial u_1}{\partial t} = -\,(2u_0 + 6u_{0xx})^c\, D_x^{\beta} u_0 - 2u_0u_{0xxx} - v_{0t},\\
p^2 &: \tfrac{\partial u_2}{\partial t} = -\,(2u_0 + 6u_{0xx})^c\, D_x^{\beta} u_1 - (2u_1 + 6u_{1xx})^c\, D_x^{\beta} u_0 - 2(u_0u_{1xxx} + u_1u_{0xxx}),\\
&\vdots
\end{aligned} \tag{4.37}$$

with the following conditions

$$u(x,0) = x^2, \tag{4.38}$$

$$u_i(x,0) = 0 \quad for \quad i = 1,2,.... \tag{4.39}$$

Using the initial conditions (4.38) and solving the above equations (4.37) yields

$$\begin{aligned}
&u_0\,(x,t) = x^2,\\
&u_1\,(x,t) = (a_1x^{2-\beta} + a_2x^{4-\beta})t,\\
&u_2\,(x,t) = (a_3x^{1-\beta} + a_4x^{2-2\beta} + a_5x^{4-2\beta} + a_6x^{3-\beta} + a_7x^{6-4\beta})\tfrac{t^2}{2},\\
&\vdots
\end{aligned} \tag{4.40}$$

where
$a_1 = \frac{-24}{\Gamma(3-\beta)}$, $a_2 = \frac{-4}{\Gamma(3-\beta)}$, $a_3 = 2(2-\beta)(1-\beta)(\beta)a_1$, $a_4 = \frac{288}{\Gamma(3-\beta)} + 18(2-\beta)(1-\beta)a_2^2$, $a_5 = \frac{48}{\Gamma(3-2\beta)} - 2\frac{\Gamma(5-\beta)}{\Gamma(5-2\beta)}a_1 + [6+3(4-\beta)(3-\beta)]a_2^2$, $a_6 = -2(4-\beta)(3-\beta)(2-\beta)a_2$, $a_7 = -2\frac{\Gamma(5-\beta)}{\Gamma(5-2\beta)}a_2 + a_2^2$.
Setting $p = 1$ and adding the iteratives terms (4.40), yields the following general approximate solution

$$u(x,t) = x^2 + (a_1 x^{2-\beta} + a_2 x^{4-\beta})t + (a_3 x^{1-\beta} + a_4 x^{2-2\beta} + a_5 x^{4-2\beta} + a_6 x^{3-\beta} + a_7 x^{6-4\beta})\frac{t^2}{2}. \tag{4.41}$$

While substituting $\beta = 1$ into (4.41), one obtains

$$u(x,t) = x^2 + (-24x - 4x^3)t + (288 + 664x^2)\frac{t^2}{2}, \tag{4.42}$$

and in the same manner, for $\beta = \frac{1}{2}$, it gives the following solution

$$u(x,t) = x^2 + \left(\frac{-32}{\sqrt{\pi}}x^{\frac{3}{2}} + \frac{-16}{3\sqrt{\pi}}x^{\frac{7}{2}}\right)t + \begin{pmatrix} \frac{-24}{\sqrt{\pi}}x^{\frac{1}{2}} + 384\frac{\sqrt{\pi}+1}{\pi}x \\ +\frac{140}{\sqrt{\pi}}x^{\frac{5}{2}} + \left(118 + \frac{2752}{3\pi}\right)x^3 \\ +\left(\frac{35}{3} + \frac{256}{9\pi}\right)x^4 \end{pmatrix}\frac{t^2}{2}.$$

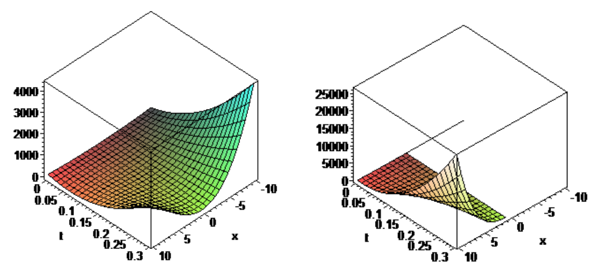


Figure 3: Series solution of Eq. (4.34-4.35) by HPM method with four terms for $\beta = 1$ (left) and $\beta = 1/2$ (right).

In Fig. 3, one has represented the graphs of the series solutions of Eq. (4.34-4.35) for $\beta = 1$ (left) and $\beta = 1/2$ (right).

5 Conclusion

In this work, homotopy perturbation method has been used for solving $K(2,2)$ equation with time and space fractional derivative. The final results obtained from HPM and compared with the exact solution shown that there is a similarity between the exact and the approximate solutions. In addition, it is obvious that; considering more power of p lead us to the more accurate results. This is the raison why one can say that HPM is an alternative analytical method for solving the general nonlinear dispersive $K(m,n)$ equation.

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