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Intersection graph of subgroups of some non-abelian groups

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Abstract

The intersection graph of subgroups of a group G is a graph whose vertex set is the set of all proper subgroups of G and two distinct vertices are adjacent if and only if their intersection is non-trivial. In this paper, we obtain the clique number and degree of vertices of intersection graph of subgroups of dihedral group, quaternion group and quasi-dihedral group.

Keywords: Intersection graph, subgroups, non-abelian groups.

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1 Introduction

There are several graphs associated with algebraic structures to investigate some specific properties of algebraic structures. Among them the intersection graphs have its own importance, which have been studied in the literature over the past fifty years. In 1964, Bosak [1] initiated the study of the intersection graphs of semigroups. Later, Cs*á*k*á*ny and Poll*á*k [5] defined the intersection graph of subgroups of finite group. Followed by this, Zelinca investigated the intersection graph of subgroups of a finite abelian group [7]. In the recent years, several interesting properties of the intersection graphs of subgroups groups have been obtained in the literature, see for instance [2], [4], [5], [6] and the references therein.

Let G be a group. *The intersection graph of subgroups of* G, denoted by $\mathscr{I}(G)$, is a graph with all the proper subgroups of G as its vertices and two distinct vertices in $\mathscr{I}(G)$ are adjacent if and only if the corresponding subgroups have a non-trivial intersection in G.

Let G be a simple graph. The *degree* of a vertex v in G, denoted by $deg_G(v)$ is the number of vertices to which v is adjacent. A *clique* of G is a complete subgraph of G. The *clique number* of G is the is the cardinality of a largest clique in G and it is denoted by $\omega(G)$.

For a positive integer n, $\tau(n)$ denotes the number of positive divisor of n; $\sigma(n)$ denotes the sum of all the positive divisors of n.

The aim of this paper is to find the clique number and degree of vertices of the intersection graph of subgroups of dihedral group, quaternion group and quasi-dihedral group.

We will use the following result of Chakrabarty *et al.* in the subsequent section.

Theorem 1.1. *([3]) Let $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}$, where p_i's are distinct primes and $\alpha_i \geq 1$. If H is a proper subgroup of $\mathbb{Z}_n$ with $|H| = p_{i_1}^{\beta_{i_1}} p_{i_2}^{\beta_{i_2}} \dots p_{i_r}^{\beta_{i_r}}$, then* $\deg_{\mathscr{I}(\mathbb{Z}_n)}(H) = \tau(n) - \prod_{j \notin \{i_1, i_2, \dots i_r\}} (\alpha_j + 1) - 3.$

2 Properties of $\mathscr{I}(D_n)$, $\mathscr{I}(Q_n)$, $\mathscr{I}(QD_{2^\alpha})$

First, we start with the dihedral group. The dihedral group of order $2n$ $(n \geq 3)$ is defined by

$$D_n = \langle a, b \mid a^n = b^2 = 1, ab = ba^{-1} \rangle.$$

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The subgroups of D_n are listed below:

(i) cyclic groups $H_0^r := \langle a^{\frac{n}{r}} \rangle$ of order r, where r is a divisor of n;

(ii) cyclic groups $H_i^1 := \langle a^i b \rangle$ of order 2, where $i = 1, 2, \ldots, n$;

(iii) dihedral groups $H_i^r := \langle a^{\frac{n}{r}}, a^i b \rangle$ of order $2r$, where r is a divisor of n, $r \neq 1, n$ and $i = 1, 2, \ldots, \frac{n}{r}$.

The number of subgroups of D_n listed in (i), (ii), (iii) are $\tau(n) - 1$, n, $\sigma(n) - n - 1$ respectively and so the total number of proper subgroups of D_n is $\tau(n) + \sigma(n) - 2$.

Theorem 2.2. *Let $n \geq 3$ be an integer with $n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_k^{\alpha_k}$, where p_i's are distinct primes and $\alpha_i \geq 1$, and let $r = p_{i_1}^{\beta_{i_1}} p_{i_2}^{\beta_{i_2}} \ldots p_{i_r}^{\beta_{i_r}}$ be a divisor of n. Then*

(1) $\deg_{\mathscr{I}(D_n)}(H_0^r) = \tau(n) + \sigma(n) - n - 3 - \prod\limits_{j \notin \{i_1, i_2, \ldots, i_r\}} (\alpha_j + 1) - \sum\limits_{\substack{d \mid s \\ d \neq 1}} \frac{n}{d}$, *where* $s = \frac{n}{p_{i_1}^{\alpha_{i_1}} p_{i_2}^{\alpha_{i_2}} \ldots p_{i_k}^{\alpha_{i_k}}}$;

(2) *for each $r \neq 1, n$ and $i = 1, 2, \ldots, \frac{n}{r}$,* $\deg_{\mathscr{I}(D_n)}(H_i^r) = \tau(n) + \sigma(n) - n - 2 - \prod\limits_{j \notin \{i_1, i_2, \ldots i_r\}} (\alpha_j + 1)$;

(3) *for each $i = 1, 2, \ldots, n$,* $\deg_{\mathscr{I}(D_n)}(H_i^1) = \tau(n) - 2$.

Proof. (1): First we count the number of subgroups listed in (i) to which H_0^r is adjacent in $\mathscr{I}(D_n)$. Here $\langle a \rangle \cong \mathbb{Z}_n$, so by Theorem 1.1, H_0^r is adjacent with $\tau(n) - \prod\limits_{j \notin \{i_1, i_2, \ldots i_r\}} (\alpha_j + 1) - 2$ subgroups of $\mathbb{Z}_n$ including $\mathbb{Z}_n$. Clearly H_0^r is not adjacent with all the n subgroups of D_n listed in (ii). Finally, we count the number of subgroups listed in (iii) to which H_0^r is adjacent. For every divisor $d \neq 1$ of $s = \frac{n}{p_{i_1}^{\alpha_{i_1}} p_{i_2}^{\alpha_{i_2}} \ldots p_{i_k}^{\alpha_{i_k}}}$, H_0^r is not adjacent with H_i^d, $i = 1, 2, \ldots, \frac{n}{d}$; H_0^r is adjacent with each of the remaining proper subgroups of D_n listed in (iii). The total number of such subgroups is $\sigma(n) - n - 1 - \sum\limits_{\substack{d \mid s \\ d \neq 1}} \frac{n}{d}$. Summing up all these values gives the degree of H_0^r.
(2): For each $r \neq 1, n$ and $i = 1, 2, \ldots, \frac{n}{r}$, H_0^r is the maximal cyclic subgroup of H_i^r and so the number of subgroups listed in (i) to which H_i^r is adjacent is the same as the number of subgroups listed in (i) to which H_0^r is adjacent including H_0^r. The number of such subgroup is $\tau(n) - \prod\limits_{j \notin \{i_1, i_2, \ldots i_r\}} (\alpha_j + 1) - 1$. Among the subgroups of D_n listed in (ii), H_i^r has exactly r subgroups as its subgroups and so H_i^r is adjacent with only these subgroups in the list. Finally, we count the number of subgroups listed in (iii) to which H_i^r is adjacent. For every divisor l of r, H_i^r is intersect with H_i^l; for every divisor d of s, $(d, r) = 1$ and so by chinese remainder theorem there exist an integer, let it be t such that H_t^1 is a subgroup of both H_i^r and H_i^d. So H_i^r adjacent with all the subgroups of D_n listed in (iii). The total number of such subgroups is $\sigma(n) - n - 1$. The degree of is just the sum of these three values.
(3): For each $i = 1, 2, \ldots, n$, the order of H_i^1 is 2. The number of subgroups of D_n contains H_i^1 is $\tau(n) - 2$ and H_i^1 is not intersect with remaining proper subgroups of D_n, since order of H_i^1 is prime. This completes the proof. □

Theorem 2.3. *For $n \geq 3$, $\omega(\mathscr{I}(D_n)) = \sigma(n) - n - 1 + \prod\limits_{i=1}^{k} \alpha_i$.*

Proof. Take $\mathcal{A} := \mathcal{C}_1 \cup \mathcal{C}_2$, where $\mathcal{C}_1 := \{H_i^r \mid r \mid n, r \neq 1, n, i = 1, 2, \ldots, \frac{n}{r}\}$ and $\mathcal{C}_2 := \bigcup\{\langle a^{\frac{n}{r}} \rangle \mid r \mid n, r \neq 1$ with r has every prime divisor of n as a factor$\}$. Clearly $\mathcal{A}$ is a maximal clique and $|\mathcal{A}| = |\mathcal{C}_1| + |\mathcal{C}_2| = (\sigma(n) - n - 1) + \prod\limits_{i=1}^{k} \alpha_i$. Let $\mathcal{B}$ be another clique different from $\mathcal{A}$. Then $\mathcal{B}$ should contains either $\langle a^{\frac{n}{r}} \rangle$, for some $r \mid n$, $r \neq 1$ and r does not contains all the prime divisors of n or $\langle a^i b \rangle$, for some $i = 1, 2, \ldots, n$. If $\mathcal{B}$ contains the subgroup $\langle a^{\frac{n}{r}} \rangle$, for some $r \mid n$, $r \neq 1$ and r does not contains all the prime divisors of n, then let p_j be the prime divisor of n which is not a divisor of r. Here G has at least two subgroups of order $2p_j$ and so we cannot take the subgroups of order $2p_j$ in $\mathcal{B}$. It follows that $|\mathcal{B}| < |\mathcal{A}|$. If $\mathcal{B}$ contains the subgroup $\langle a^i b \rangle$, for some $i = 1, 2, \ldots, n$, then $\langle a^i b \rangle$ adjacent with $\tau(n) - 2$ and so we cannot take $\sigma(n) - \tau(n) + 1$ subgroups in $\mathcal{B}$. It follows that $|\mathcal{B}| < |\mathcal{A}|$. This completes the proof. □

Next, we consider the quaternion group. For any integer $n > 1$, the quaternion group of order $4n$, is defined by

$$Q_n = \langle a, b | a^{2n} = b^4 = 1, b^2 = a^n, ab = ba^{-1} \rangle.$$

The subgroups of Q_n are listed below:

(i) cyclic groups $H_{0,r} := \langle a^{\frac{2n}{r}} \rangle$, of order r, where r is a divisor of $2n$;

(ii) cyclic groups $H_{i,1} := \langle a^i b \rangle$ of order 4, where $i = 1, \ldots, n$;

(iii) quaternion groups $H_{i,r} := \langle a^{\frac{n}{r}}, a^i b \rangle$ of order $4r$, where r is a divisor of n, $i = 1, \ldots, \frac{n}{r}$.

The number of subgroups of Q_n listed in (i), (ii), (iii) are $\tau(2n) - 1$, n, $\sigma(n) - n - 1$ and so the total number of proper subgroups of Q_n is $\tau(2n) + \sigma(n) - 2$.

Theorem 2.4. *Let $n > 1$ be an integer with $n = p_1^{\alpha_1} p_2^{\alpha_2} \ldots p_k^{\alpha_k}$, where p_i's are distinct primes and $\alpha_i \geq 1$, and let $r = p_{i_1}^{\beta_{i_1}} p_{i_2}^{\beta_{i_2}} \ldots p_{i_r}^{\beta_{i_r}}$ be a divisor of n.*

(1) *If r is even, then* $\deg_{\mathscr{I}(Q_n)}(H_{0,r}) = \tau(2n) + \sigma(n) - 3 - \prod\limits_{j \notin \{i_1, i_2, \ldots i_r\}} (\alpha_j + 1)$;

(2) *If r is odd, then* $\deg_{\mathscr{I}(Q_n)}(H_{0,r}) = \tau(2n) + \sigma(n) - n - 3 - \prod\limits_{j \notin \{i_1, i_2, \ldots i_r\}} (\alpha_j + 1) - \sum\limits_{\substack{d|s \\ d \neq 1}} \frac{n}{d}$, *where* $s = \frac{n}{p_{i_1}^{\alpha_{i_1}} p_{i_2}^{\alpha_{i_2}} \ldots p_{i_k}^{\alpha_{i_k}}}$;

(3) *For each* $i = 1, \ldots, \frac{n}{r}$, $\deg_{\mathscr{I}(Q_n)}(H_{i,r}) = \tau(2n) + \sigma(n) - 3 - \prod\limits_{j \notin \{i_1, i_2, \ldots i_r\}} (\alpha_j + 1)$;

(4) *For each* $i = 1, \ldots, n$, $\deg_{\mathscr{I}(Q_n)}(H_{i,1}) = \tau(2n) + \sigma(n) - 3 - \prod\limits_{j \notin \{i_1, i_2, \ldots i_r\}} (\alpha_j + 1)$, *where α_j's are powers of odd prime factors of n.*

Proof. (1)-(2): First we count the number of subgroups listed in (i) to which $H_{0,r}$ is adjacent. Here $\langle a \rangle \cong \mathbb{Z}_{2n}$, by Theorem 1.1, $H_{0,r}$ adjacent with $\tau(2n) - \prod\limits_{j \notin \{i_1, i_2, \ldots i_r\}} (\alpha_j + 1) - 2$ subgroups of $\mathbb{Z}_{2n}$ including $\mathbb{Z}_{2n}$. Now we consider the following two cases:

Case a: r is even. Here Q_n has an unique subgroup of order 2 and so every subgroup of even order in Q_n are adjacent with each other, so $H_{0,r}$ is adjacent with $\sigma(n) - 1$ subgroups of Q_n excluding Q_n listed in (ii), (iii). This completes the proof of part (1).

Case b: r is odd. Clearly $H_{0,r}$ is not adjacent with all the n subgroups of Q_n listed in (ii), since order of $H_{1,r}$ is 4. Finally we count the number of subgroups listed in (iii) to which $H_{0,r}$ is adjacent. For every divisor $d \neq 1$ of $s = \frac{n}{p_{i_1}^{\alpha_{i_1}} p_{i_2}^{\alpha_{i_2}} \ldots p_{i_k}^{\alpha_{i_k}}}$, $H_{0,r}$ is not adjacent with $H_{i,d}$, $i = 1, 2, \ldots, \frac{n}{d}$; $H_{0,r}$ is adjacent with remaining proper subgroups of Q_n listed in (iii). The total number of such subgroups is $\sigma(n) - n - 1 - \sum\limits_{\substack{d|s \\ d \neq 1}} \frac{n}{d}$. This completes the proof of part (2).

(3): For each $i = 1, \ldots, \frac{n}{r}$, $H_{0,r}$ is the maximal cyclic subgroup of $H_{i,r}$ and so the number of subgroups listed in (i) to which $H_{i,r}$ is adjacent is the same as the number of subgroups listed in (i) to which $H_{0,r}$ is adjacent including $H_{0,r}$. The number of such subgroups is $\tau(n) - \prod\limits_{j \notin \{i_1, i_2, \ldots i_r\}} (\alpha_j + 1) - 1$. Also Q_n has a unique subgroup of order 2 and so $H_{i,r}$ is adjacent with all the subgroups listed in (ii), (iii), since order of subgroups of Q_n listed in (ii), (iii) is even. The total number of such subgroups is $\sigma(n) - 1$. This completes the proof of part (3).

(4): Since Q_n has an unique subgroup of order 2, so $H_{i,1}$ is adjacent with all the subgroups listed in (ii), (iii). Also $H_{i,1}$ is adjacent all the even order subgroups of Q_n listed in (i). But $H_{i,1}$ is not adjacent with an odd order subgroups of Q_n listed in (i). The number of such subgroups is $\tau(2n) + \sigma(n) - \prod\limits_{j \notin \{i_1, i_2, \ldots i_r\}} (\alpha_j + 1) - 2$, where α_j's are powers of odd prime factors of n. This completes the proof of part (4). □

Theorem 2.5. *Let $n > 1$ be an integer and $2n = 2^{\alpha_1} p_2^{\alpha_2} \ldots p_k^{\alpha_k}$, where p_i's are distinct primes and $\alpha_i \geq 1$. Then* $\omega(\mathscr{I}(Q_n)) = \sigma(n) + \alpha_1 \prod\limits_{i=2}^{k} (\alpha_i + 1) - 1$.

Proof. Let $\mathcal{A}$ be the set of all even order subgroups of Q_n. Then $|\mathcal{A}| = \sigma(n) + \alpha_1 \prod_{i=2}^{k}(\alpha_i + 1) - 1$ and $\mathcal{A}$ is a maximal clique in $\mathscr{I}(Q_n)$. Let $\mathcal{B}$ be another clique different from $\mathcal{A}$. Then $\mathcal{B}$ should contains $\langle a^{\frac{2n}{r}} \rangle$, for some an odd divisor r of n, $r \neq 1$. Then $\mathcal{B}$ cannot contain the subgroups of order 4. It follows that $|\mathcal{B}| < |\mathcal{A}|$. This completes the proof. □

Finally, we consider the quasi-dihedral group. For any positive integer $\alpha > 3$, the quasi-dihedral group of order 2^α, is defined by

$$QD_{2^\alpha} = \langle a, b \mid a^{2^{\alpha-1}} = b^2 = 1, bab^{-1} = a^{2^{\alpha-2}-1} \rangle.$$

The proper subgroups of QD_{2^α} are listed below:

(i) cyclic groups $H_0^r = \langle a^{\frac{2^{\alpha-1}}{r}} \rangle$, where r is a divisor of $2^{\alpha-1}$, $r \neq 1$;

(ii) the dihedral group $H_1^{2^{\alpha-2}} = \langle a^2, b \rangle \cong D_{2^{\alpha-2}}$ and the dihedral subgroups H_i^r of $H_1^{2^{\alpha-2}}$, where r is a divisor of $2^{\alpha-2}$, $r \neq 2^{\alpha-2}$, $i \in \{1, 2, \ldots, \frac{2^{\alpha-2}}{r}\}$;

(iii) the quaternion group $H_{2,2^{\alpha-3}} = \langle a^2, ab \rangle \cong Q_{2^{\alpha-3}}$ and the quaternion subgroups $H_{i,r}$ of $H_{2,2^{\alpha-3}}$, where r is a divisor of $2^{\alpha-3}$, $r \neq 2^{\alpha-3}$, $i \in \{1, 2, \ldots, \frac{2^{\alpha-3}}{r}\}$.

The number of subgroups of QD_{2^α} listed in (i), (ii), (iii) are $\tau(2^{\alpha-1}) - 1$, $2^{\alpha-1} - 1$, $2^{\alpha-2} - 1$ and so the total number of proper subgroups of QD_{2^α} is $\alpha + 3(2^{\alpha-2} - 1)$.

Theorem 2.6. *If* $\alpha \geq 4$, *then*

(1) *for each divisor* r *of* $2^{\alpha-1}$, $r \neq 1$, $\deg_{\mathscr{I}(QD_{2^\alpha})}(H_0^r) = \alpha + 2^{\alpha-1} - 4$;

(2) *for each divisor* r *of* $2^{\alpha-2}$, $r \neq 1$, $i = 1, 2, \ldots, \frac{2^{\alpha-2}}{r}$, $\deg_{\mathscr{I}(QD_{2^\alpha})}(H_i^r) = \alpha + 2^{\alpha-1} + r - 4$;

(3) *for each divisor* r *of* $2^{\alpha-3}$, $r \neq 1$, $i = 1, 2, \ldots, \frac{2^{\alpha-2}}{r}$, $\deg_{\mathscr{I}(QD_{2^\alpha})}(H_{i,r}) = \alpha + 2^{\alpha-1} - 4$;

(4) *for* $i = 2, 2^2, \ldots, 2^{\alpha-2}$, $\deg_{\mathscr{I}(QD_{2^\alpha})}(H_i^1) = \alpha - 2$;

(5) *for* $i = 1, 3, \ldots, 2^{\alpha-3}$, $\deg_{\mathscr{I}(QD_{2^\alpha})}(H_{i,1}) = \alpha + 2^{\alpha-1} - 4$.

Proof. The only maximal subgroups of QD_2^α are $H_0^{2^{\alpha-1}}$, the dihedral subgroup $H_1^{2^{\alpha-2}}$ and quaternion subgroup $H_{2,2^{\alpha-3}}$. Here H_0^2 is a subgroup of all the subgroup of QD_{2^α} other than H_i^1, $i = 2, 2^2, \ldots, 2^{\alpha-2}$; also no subgroups listed in (i), (iii) are adjacent with H_i^1, $i = 2, 2^2, \ldots, 2^{\alpha-2}$. It follows that $\deg_{\mathscr{I}(QD_{2^\alpha})}(H_0^r) = \alpha + 3(2^{\alpha-2} - 1) - 2^{\alpha-2} - 1 = \alpha + 2^{\alpha-1} - 4$. Proofs of parts (3) and (5) are similar to the above.

Next, we count the number of subgroups of QD_{2^α} to which H_i^r is adjacent. By the above argument H_i^r is adjacent with all the subgroups listed in (i), (iii) and the dihedral subgroups of $H_1^{2^{\alpha-2}}$; also H_i^r has r subgroups of order 2 as its subgroups and so H_i^r adjacent with these subgroups, so $\deg_{\mathscr{I}(QD_{2^\alpha})}(H_i^r) = \alpha + 3(2^{\alpha-2} - 1) - (2^{\alpha-2} - r) - 1 = \alpha + 2^{\alpha-1} + r - 4$.

Finally, we count the number of subgroups of QD_{2^α} to which H_i^1 is adjacent. Note that $\deg_{\mathscr{I}(QD_{2^\alpha})}(H_i^1) = \deg_{\mathscr{I}(D_n)}(H_i^1) + 1$, since order of H_i^1 is 2, and it is not a subgroup of any subgroups of $H_{i,2^{\alpha-3}}$; $H_1^{2^{\alpha-2}}$ is also a vertex of $\mathscr{I}(QD_{2^\alpha})$. So by Theorem 2.2(3), we have $\deg_{\mathscr{I}(QD_{2^\alpha})}(H_i^1) = \tau(2^{\alpha-2}) - 1 = \alpha - 2$. Hence the proof. □

Theorem 2.7. *For* $\alpha \geq 3$, $\omega(\mathscr{I}(QD_{2^\alpha})) = \alpha + 2^{\alpha-1} - 3$.

Proof. Let $\mathcal{A}$ be the set of all subgroups of QD_{2^α} other than $\langle ba^i \rangle$, $i = 2, 2^2, \ldots, 2^{\alpha-2}$. Clearly $\mathcal{A}$ is a maximal clique in $\mathscr{I}(QD_{2^\alpha})$ and $|\mathcal{A}| = \alpha + 3(2^{\alpha-2} - 1) - 2^{\alpha-2} = \alpha + 2^{\alpha-1} - 3$. Let $\mathcal{B}$ be another clique in $\mathscr{I}(QD_{2^\alpha})$. Then $\mathcal{B}$ contains exactly one subgroup of the form $\langle ba^i \rangle$, $i = 2, 2^2, \ldots, 2^{\alpha-2}$. It follow that $|\mathcal{B}| < |\mathcal{A}|$, since for one cyclic subgroup in $\mathcal{B}$, we take more than one quaternion subgroups in $\mathcal{A}$. This completes the proof. □

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