

Intuitionistic filter

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Abstract

The aim of this paper is to introduce a intuitionistic filter and study some of its properties.

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1 Introduction and Preliminaries

In the philosophy of mathematics intuitionism is an approach where mathematics considered to be purely the result of the constructive mental activity of human rather than the discovery of fundamental principles claimed to exist in an objective reality. Intuitionistic sets and Intuitionistic points are introduced by D.Coker [3] in 1996. This concept is originated from the study of zadeh [7] who introduced Intuitionistic fuzzy set in the year 1965. This concept is the discrete form of Intuitionistic fuzzy set and it is also one of several ways of introducing vagueness in mathematical objects. After coker introduced Intuitionistic set and Intuitionistic topology several papers were published in intuitionistic fuzzy topology. It is known that filters are used to define convergence and hence limits. In this paper, we defined filters based on intuitionistic sets and derived various properties of intuitionistic filter.

Definition 1.1. [3]: Let X be a nonempty fixed set. An intuitionistic set (IS for short) A is an object having the form $A = \langle X, A^1, A^2 \rangle$ where A^1 and A^2 are subsets of X satisfying $A^1 \cap A^2 = \phi$. The set A^1 is called the set of members of A , while A^2 is called the set of non members of A .

Definition 1.2. [3]: Let X be a nonempty set. Let $A = \langle X, A^1, A^2 \rangle$ and $B = \langle X, B^1, B^2 \rangle$ be an intuitionistic sets on X and let $\{A_i : i \in J\}$ be an arbitrary family of IS's in X , where $A^i = \langle X, A_i^1, A_i^2 \rangle$. Then

$$(1) A \subseteq B \text{ iff } A^1 \subseteq B^1 \text{ and } B^2 \subseteq A^2.$$

$$(2) A = B \text{ iff } A \subseteq B \text{ and } B \subseteq A.$$

$$(3) \cup A_i = \langle X, \cup A_i^1, \cap A_i^2 \rangle.$$

$$(4) \cap A_i = \langle X, \cap A_i^1, \cup A_i^2 \rangle$$

$$(5) \tilde{X} = \langle X, X, \phi \rangle$$

$$(6) \tilde{\phi} = \langle X, \phi, X \rangle.$$

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2 Intuitionistic filter

In this chapter we introduced intuitionistic filters and study some of its basic properties.

Definition 2.3. An intuitionistic filter ($\mathcal{I}_{\mathcal{F}}$ for short)on a nonempty set X is a family of IS's in X satisfying the following axioms :

$$(\mathcal{I}_{\mathcal{F}1}) : \tilde{\phi} \notin \mathcal{I}_{\mathcal{F}}$$

$$(\mathcal{I}_{\mathcal{F}2}) : \text{If } F \in \mathcal{I}_{\mathcal{F}} \text{ and } H \supset F, \text{ then } H \in \mathcal{I}_{\mathcal{F}}.$$

$$(\mathcal{I}_{\mathcal{F}3}) : \text{If } F \in \mathcal{I}_{\mathcal{F}} \text{ and } H \in \mathcal{I}_{\mathcal{F}}, \text{ then } F \cap H \in \mathcal{I}_{\mathcal{F}}.$$

In this case the pair $(X, \mathcal{I}_{\mathcal{F}})$ is called an intuitionistic filter.

Example 2.1. Let $X = \{a, b\}$ and consider the family $\mathcal{I}_{\mathcal{F}} = \{\tilde{X}, A_1, A_2\}$ where $A_1 = \prec X, \{a\}, \{b\} \succ$, $A_2 = \prec X, \{a\}, \phi \succ$. Then $(X, \mathcal{I}_{\mathcal{F}})$ is an intuitionistic filter on X .

Example 2.2. Let $X = \{a, b, c\}$ and consider the family $\mathcal{I}_{\mathcal{F}} = \{\tilde{X}, A_1, A_2\}$ where $A_1 = \prec X, \{a, b\}, \phi \succ$, $A_2 = \prec X, \{b, c\}, \phi \succ$. It is not an intuitionistic filter on X as $\prec X, \{a, b\}, \phi \succ \cap \prec X, \{b, c\}, \phi \succ = \prec X, \{b\}, \phi \succ$ which does not belong to $\mathcal{I}_{\mathcal{F}}$ and hence axiom $(\mathcal{I}_{\mathcal{F}3})$ is not satisfied.

Result 2.1. Let $\{F_i : i \in J\}$ be a family of intuitionistic filters on X . Then $\bigcap_{i \in J} F_i$ is an intuitionistic filter on X .

Proof. Let $F_i = \{\prec X, F_i^1, F_i^2 \succ : i \in J\}$ be any nonempty collection of intuitionistic filters on X .

$$\text{Let } \mathcal{F} = \bigcap \{F_i : i \in J\}.$$

To prove that \mathcal{F} is an intuitionistic filter on X .

Since each F_i is an intuitionistic filter on X , $\prec X, X, \phi \succ \in F_i$ for all i .

Hence $\prec X, X, \phi \succ \in \bigcap F_i$.

$$\Rightarrow \prec X, X, \phi \succ \in \mathcal{F}.$$

Therefore \mathcal{F} is nonempty.

(i) Since $\prec X, \phi, X \succ \notin F_i$ for all $i \in J$.

Therefore $\prec X, \phi, X \succ \notin \bigcap_{i \in J} F_i = \mathcal{F}$.

(ii) Let $\prec X, A^1, A^2 \succ \in F_i$ for all i and $\prec X, B^1, B^2 \succ \supset \prec X, A^1, A^2 \succ$.

Since each F_i is an intuitionistic filter on X .

$$\Rightarrow \prec X, B^1, B^2 \succ \in F_i \text{ for all } i.$$

$$\Rightarrow \prec X, B^1, B^2 \succ \in \bigcap_{i \in J} F_i = \mathcal{F}.$$

(iii) Let $A = \prec X, A^1, A^2 \succ \in \mathcal{F}$ and $B = \prec X, B^1, B^2 \succ \in \mathcal{F}$.

$$\Rightarrow \prec X, A^1, A^2 \succ \in F_i \text{ for all } i \text{ and } \prec X, B^1, B^2 \succ \in F_i \text{ for all } i.$$

As each F_i is an intuitionistic filter on X .

Therefore by Axiom $(\mathcal{I}_{\mathcal{F}3})$ $\prec X, A^1 \cap B^1, A^2 \cup B^2 \succ \in F_i$ for all i .

Hence $A \cap B \in \mathcal{F}$.

Therefore $\bigcap_{i \in J} F_i$ is an intuitionistic filter on X . □

Corollary 2.1. Union of intuitionistic filters need not be an intuitionistic filter and it is Justified by the following example.

Example 2.3. Let $X = \{a, b\}$. $\mathcal{I}_{\mathcal{F}1} = \{\prec X, \phi, \{a\} \succ, \prec X, \{a\}, \phi \succ, \prec X, \{b\}, \phi \succ, \prec X, \{b\}, \{a\} \succ, \prec X, X, \phi \succ, \prec X, \phi, \phi \succ\}$ and $\mathcal{I}_{\mathcal{F}2} = \{\prec X, \phi, \{b\} \succ, \prec X, \{a\}, \phi \succ, \prec X, \{b\}, \phi \succ, \prec X, \{a\}, \{b\} \succ, \prec X, X, \phi \succ, \prec X, \phi, \phi \succ\}$. $\mathcal{I}_{\mathcal{F}1} \cup \mathcal{I}_{\mathcal{F}2} = \{\prec X, \phi, \{a\} \succ, \prec X, \phi, \{b\} \succ, \prec X, \{a\}, \phi \succ, \prec X, \{a\}, \{b\} \succ, \prec X, X, \phi \succ, \prec X, \phi, \phi \succ, \prec X, \{b\}, \phi \succ, \prec X, \{b\}, \{a\} \succ\}$ is not a intuitionistic filter as it does not satisfy the Axiom IF3.

Definition 2.4. A family $F_i = \{\prec X, F_i^1, F_i^2 \succ : i \in J\}$ of intuitionistic sets in X satisfies the finite intersection property (FIP for short) if every finite subfamily $\{F_i : i = 1, 2, \dots, n\}$ of $F_i = \{\prec X, F_i^1, F_i^2 \succ : i \in J\}$ satisfies the condition $\bigcap_{i=1}^n F_i \neq \tilde{\phi}$

Theorem 2.1. Let X be a nonempty set. Let $C = \{\prec X, K_i^1, K_i^2 \succ : i = 1, 2, 3, \dots, n\}$ be a nonempty family of intuitionistic sets of X . Then there exists a intuitionistic filter on X containing C iff C has finite intersection property.

Proof. Suppose that $C = \{\prec X, K_i^1, K_i^2 \succ : i = 1, 2, 3, \dots, n\}$ has finite intersection property.

Let $G = \{B : B \text{ is the intersection of finite subfamily of } C\}$

As C has finite intersection property, it follows from definition, $\prec X, \phi, X \succ \notin G$.

Consider the collection $\mathcal{I}_{\mathcal{F}} = \{A_i = \prec X, A_i^1, A_i^2 \succ : A_i \text{ contains a member of } G\}$.

By the construction of $\mathcal{I}_{\mathcal{F}}$, we have $\prec X, \cap K_i^1, \cup K_i^2 \succ \subseteq \prec X, A_i^1, A_i^2 \succ$

$\Rightarrow \cap K_i^1 \subseteq A_i^1$ and $A_i^2 \subseteq \cup K_i^2$.

So $\cap K_i^1 \subseteq K_i^1 \subseteq A_i^1$ and $A_i^2 \subseteq K_i^2 \subseteq \cup K_i^2$,

That is $K_i^1 \subseteq A_i^1$ and $A_i^2 \subseteq K_i^2$.

Hence $\prec X, K_i^1, K_i^2 \succ \subseteq \prec X, A_i^1, A_i^2 \succ$

Therefore $C \subseteq \mathcal{I}_{\mathcal{F}}$.

To prove that $\mathcal{I}_{\mathcal{F}}$ is an intuitionistic filter on X .

Axiom ($\mathcal{I}_{\mathcal{F}1}$): By the construction of $\mathcal{I}_{\mathcal{F}}$, we have $\prec X, \cap K_i^1, \cup K_i^2 \succ \subseteq \prec X, A_i^1, A_i^2 \succ$

$\Rightarrow \cap K_i^1 \subseteq A_i^1$ and $A_i^2 \subseteq \cup K_i^2$ and $\prec X, \phi, X \succ \notin G$. (by the finite intersection property)

Hence $\prec X, \phi, X \succ \notin \mathcal{I}_{\mathcal{F}}$.

Axiom ($\mathcal{I}_{\mathcal{F}2}$): Let $\prec X, A_1^1, A_1^2 \succ \in \mathcal{I}_{\mathcal{F}}$.

$\Rightarrow \prec X, A_1^1, A_1^2 \succ \supseteq \prec X, \cap K_1^1, \cup K_1^2 \succ$.

If $\prec X, A_2^1, A_2^2 \succ \supseteq \prec X, A_1^1, A_1^2 \succ$ then $\prec X, A_2^1, A_2^2 \succ \supseteq \prec X, \cap K_2^1, \cup K_2^2 \succ$

Therefore $\prec X, A_2^1, A_2^2 \succ \in \mathcal{I}_{\mathcal{F}}$.

Axiom ($\mathcal{I}_{\mathcal{F}3}$): Let $\prec X, A_1^1, A_1^2 \succ \in \mathcal{I}_{\mathcal{F}}$ and $\prec X, A_2^1, A_2^2 \succ \in \mathcal{I}_{\mathcal{F}}$

To prove that $\prec X, A_1^1 \cap A_2^1, A_1^2 \cup A_2^2 \succ \in \mathcal{I}_{\mathcal{F}}$.

Since $\prec X, A_1^1, A_1^2 \succ \in \mathcal{I}_{\mathcal{F}}$ and $\prec X, A_2^1, A_2^2 \succ \in \mathcal{I}_{\mathcal{F}}$.

So that both $\prec X, A_1^1, A_1^2 \succ$ and $\prec X, A_2^1, A_2^2 \succ$ contains some members of G say

$\prec X, A_1^1, A_1^2 \succ \supseteq \prec X, \cap K_1^1, \cup K_1^2 \succ$, $\prec X, A_2^1, A_2^2 \supseteq \prec X, \cap K_2^1, \cup K_2^2 \succ$ Where $\prec X, \cap K_1^1, \cup K_1^2 \succ$

and $\prec X, \cap K_2^1, \cup K_2^2 \succ \in G$.

Since $\prec X, \cap K_1^1, \cup K_1^2 \succ$ and $\prec X, \cap K_2^1, \cup K_2^2 \succ \in C$.

$\Rightarrow \prec X, \cap K_1^1, \cup K_1^2 \succ \cap \prec X, \cap K_2^1, \cup K_2^2 \succ \in C$

$\Rightarrow \prec X, (\cap K_1^1) \cap (\cap K_2^1), (\cup K_1^2) \cup (\cup K_2^2) \succ \in C$,

But $\prec X, A_1^1, A_1^2 \succ \supseteq \prec X, \cap K_1^1, \cup K_1^2 \succ$ and $\prec X, A_2^1, A_2^2 \supseteq \prec X, \cap K_2^1, \cup K_2^2 \succ$

$\Rightarrow \prec X, A_1^1, A_1^2 \succ \cap \prec X, A_2^1, A_2^2 \succ \supseteq \prec X, \cap K_1^1, \cup K_1^2 \succ \cap \prec X, \cap K_2^1, \cup K_2^2 \succ$

$\Rightarrow \prec X, A_1^1 \cap A_2^1, A_1^2 \cup A_2^2 \succ \supseteq \prec X, (\cap K_1^1) \cap (\cap K_2^1), (\cup K_1^2) \cup (\cup K_2^2) \succ$

$\Rightarrow \prec X, (\cap K_1^1) \cap (\cap K_2^1), (\cup K_1^2) \cup (\cup K_2^2) \succ \subseteq \prec X, A_1^1 \cap A_2^1, A_1^2 \cup A_2^2 \succ$.

$\Rightarrow (\cap K_1^1) \cap (\cap K_2^1) \subseteq A_1^1 \cap A_2^1$ and $A_1^2 \cup A_2^2 \subseteq (\cup K_1^2) \cup (\cup K_2^2) \succ$

Thus $\prec X, A_1^1 \cap A_2^1, A_1^2 \cup A_2^2 \succ \in \mathcal{I}_{\mathcal{F}}$.

Therefore $\mathcal{I}_{\mathcal{F}}$ is an intuitionistic filter on X containing C .

Conversely, Let $\mathcal{I}_{\mathcal{F}}$ be an intuitionistic filter on X containing C .

Then $\mathcal{I}_{\mathcal{F}} \supseteq C \supseteq G$.

Now $\mathcal{I}_{\mathcal{F}}$ being an intuitionistic filter on X , $\prec X, \phi, X \succ \notin \mathcal{I}_{\mathcal{F}}$.

So $\prec X, \phi, X \succ \notin G$.

Again $\prec X, \cap_{i=1}^n K_i^1, \cup K_i^2 \succ \neq \prec X, \phi, X \succ$

Therefore C must have finite intersection property. □

Remark 2.1. The intuitionistic filter $\mathcal{I}_{\mathcal{F}}$ as defined in Theorem 2.1, is said to be generated by C and C is said to be a sub base of $\mathcal{I}_{\mathcal{F}}$.

By Theorem 2.1, we have C is a sub base for $\mathcal{I}_{\mathcal{F}} \Leftrightarrow C$ has F.I.P.

Also the intuitionistic filter $\mathcal{I}_{\mathcal{F}}$ obtained above is the coarsest intuitionistic filter which contains C .

Because, if $\mathcal{I}_{\mathcal{F}1}$ is any other intuitionistic filter containing C , then $\mathcal{I}_{\mathcal{F}1}$ must contain all finite intersections of members of C and their supersets.

Hence $\mathcal{I}_{\mathcal{F}1} \supset \mathcal{I}_{\mathcal{F}}$.

This implies $\mathcal{I}_{\mathcal{F}}$ is coarsest of all intuitionistic filters on X which contains C .

Theorem 2.2. Let $\mathcal{I}_{\mathcal{F}}$ be an intuitionistic filter on a nonempty set X and $A = \prec X, A^1, A^2 \succ$ be an intuitionistic set in X . Then there exists a intuitionistic filter $\mathcal{I}_{\mathcal{F}1}$ finer than $\mathcal{I}_{\mathcal{F}}$ such that $\prec X, A^1, A^2 \succ \in \mathcal{I}_{\mathcal{F}1}$ if and only if $\prec X, A^1, A^2 \succ \cap \prec X, G^1, G^2 \succ \neq \check{\phi}$ for every $G = \prec X, G^1, G^2 \succ \in \mathcal{I}_{\mathcal{F}}$.

Proof. Let $A \cap G \neq \check{\phi}$ for every $G \in \mathcal{I}_{\mathcal{F}}$.

Let $C = \{A \cap G : G \in \mathcal{I}_{\mathcal{F}}\}$.

We need to show that C has F.I.P.

Let $\{A \cap (G_i = \prec X, G_i^1, G_i^2 \succ) : i = 1, 2, 3, \dots, n\}$ be a collection of finite number of members of C .

Then $\cap \{A \cap (G_i = \prec X, G_i^1, G_i^2 \succ) : i = 1, 2, 3, \dots, n\} = A \cap \{\cap G_i : i = 1, 2, 3, \dots, n\}$.

But by Axiom IF3 $\cap \{G_i : i = 1, 2, 3, \dots, n\} \in \mathcal{I}_{\mathcal{F}}$.

Therefore $\cap \{A \cap G_i : i = 1, 2, 3, \dots, n\} = A \cap G$ where $G = \cap \{G_i : i = 1, 2, 3, \dots, n\} \neq \tilde{\phi}$ by hypothesis.

Thus C has finite intersection property and hence by Theorem 2.1, there exists an intuitionistic filter say $\mathcal{I}_{\mathcal{F}_1}$ on X which contains C.

Let G be any member of $\mathcal{I}_{\mathcal{F}}$ so that $A \cap G \neq \tilde{\phi}$ is a member of C.

Also as shown above $\mathcal{I}_{\mathcal{F}_1}$ is an intuitionistic filter on X which contains C.

Hence $A \cap G$ is also a member of $\mathcal{I}_{\mathcal{F}_1}$.

But $G \supset A \cap G \in \mathcal{I}_{\mathcal{F}_1}$.

Therefore by Axiom ($\mathcal{I}_{\mathcal{F}_2}$), $G \in \mathcal{I}_{\mathcal{F}_1}$.

Since $G \in \mathcal{I}_{\mathcal{F}} \Rightarrow G \in \mathcal{I}_{\mathcal{F}_1}$.

Therefore $\mathcal{I}_{\mathcal{F}} \subset \mathcal{I}_{\mathcal{F}_1}$.

i.e $\mathcal{I}_{\mathcal{F}_1}$ is finer than $\mathcal{I}_{\mathcal{F}}$.

Conversely, let $\mathcal{I}_{\mathcal{F}_1}$ be an intuitionistic filter on X and $A \in \mathcal{I}_{\mathcal{F}_1}$ and $\mathcal{I}_{\mathcal{F}} \subset \mathcal{I}_{\mathcal{F}_1}$.

Let G be any arbitrary member of $\mathcal{I}_{\mathcal{F}}$.

Since $\mathcal{I}_{\mathcal{F}} \subset \mathcal{I}_{\mathcal{F}_1}$, we have $G \in \mathcal{I}_{\mathcal{F}_1}$.

Also it is given that $A \in \mathcal{I}_{\mathcal{F}_1}$.

Hence $A \cap G \in \mathcal{I}_{\mathcal{F}_1}$.

Further $A \cap G \neq \tilde{\phi}$. □

3 Supremum and infimum of intuitionistic set of intuitionistic filters

Definition 3.5. Let $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$ be a nonempty collection of intuitionistic filters on a nonempty set X such that

$\mathcal{M}_{\mathcal{I}_{\mathcal{F}}} = \{\mathcal{I}_{\mathcal{F}_i} = \prec X, F_i^1, F_i^2 \succ \text{ and } \mathcal{I}_{\mathcal{F}_i} \text{ is an intuitionistic filter on } X\}$. Then $\mathcal{I}_{\mathcal{F}_i}$ is said to be the supremum of $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$ if and only if

(a) $\mathcal{I}_{\mathcal{F}_i}$ is finer than every other intuitionistic filter in $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$.

(b) If $\mathcal{I}_{\mathcal{F}_i}^1$ is any other intuitionistic filter on X, which is finer than every other intuitionistic filter in $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$, then $\mathcal{I}_{\mathcal{F}_i}$ is coarser than $\mathcal{I}_{\mathcal{F}_i}^1$.

Definition 3.6. : Let $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$ be a nonempty collection of intuitionistic filters on a nonempty set X such that

$\mathcal{M}_{\mathcal{I}_{\mathcal{F}}} = \{\mathcal{I}_{\mathcal{F}_i} = \prec X, F_i^1, F_i^2 \succ \text{ and } \mathcal{I}_{\mathcal{F}_i} \text{ is an intuitionistic filter on } X\}$. Then $\mathcal{I}_{\mathcal{F}_i}$ is said to be the infimum of $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$ if and only if

(a) $\mathcal{I}_{\mathcal{F}_i}$ is coarser than every other intuitionistic filter in $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$.

(b) If $\mathcal{I}_{\mathcal{F}_i}^1$ is any other intuitionistic filter on X, which is coarser than every other intuitionistic filter in $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$, then $\mathcal{I}_{\mathcal{F}_i}$ is finer than $\mathcal{I}_{\mathcal{F}_i}^1$.

Remark 3.2. : If $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$ is any nonempty class of intuitionistic filters on X, then infimum of $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$ always exists because we know that there is at least one intuitionistic filter $\{\prec X, X, \phi \succ\}$ on X which is coarser than every member of $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$. Also supremum of $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$ may or may not exist as will be clear from example given below.

Let $X = \{a, b, c\}$ on which we have the following intuitionistic filters. $\mathcal{I}_{\mathcal{F}_1} = \{\prec X, X, \phi \succ\}$,

$\mathcal{I}_{\mathcal{F}_2} = \{\prec X, X, \phi \succ, \prec X, \{a, b\}, \phi \succ\}$, $\mathcal{I}_{\mathcal{F}_3} = \{\prec X, X, \phi \succ, \prec X, \{b, c\}, \phi \succ\}$,

$\mathcal{I}_{\mathcal{F}_4} = \{\prec X, X, \phi \succ, \prec X, \{a, c\}, \phi \succ\}$,

$\mathcal{I}_{\mathcal{F}_5} = \{\prec X, X, \phi \succ, \prec X, \{a\}, \phi \succ, \prec X, \{a, b\}, \phi \succ, \prec X, \{a, c\}, \phi \succ\}$ and

$\mathcal{I}_{\mathcal{F}_6} = \{\prec X, X, \phi \succ, \prec X, \{b\}, \phi \succ, \prec X, \{a, b\}, \phi \succ, \prec X, \{b, c\}, \phi \succ\}$.

Let $\mathcal{M}_{1\mathcal{I}_{\mathcal{F}}} = \{\mathcal{I}_{\mathcal{F}_1}, \mathcal{I}_{\mathcal{F}_2}, \mathcal{I}_{\mathcal{F}_3}, \mathcal{I}_{\mathcal{F}_4}\}$ clearly $\mathcal{I}_{\mathcal{F}_1}$ is the infimum of $\mathcal{M}_{1\mathcal{I}_{\mathcal{F}}}$, as it is the only intuitionistic filter on X which is coarser than every member of $\mathcal{M}_{1\mathcal{I}_{\mathcal{F}}}$. But on the other hand $\mathcal{M}_{1\mathcal{I}_{\mathcal{F}}}$ has no supremum as there is no intuitionistic filter in $\mathcal{M}_{1\mathcal{I}_{\mathcal{F}}}$ which is finer than each member of $\mathcal{M}_{1\mathcal{I}_{\mathcal{F}}}$.

Again let $\mathcal{M}_{2\mathcal{I}_{\mathcal{F}}} = \{\mathcal{I}_{\mathcal{F}_2}, \mathcal{I}_{\mathcal{F}_3}, \mathcal{I}_{\mathcal{F}_6}\}$.

Clearly $\mathcal{I}_{\mathcal{F}_6}$ is the finest of all intuitionistic filters in $\mathcal{M}_{2\mathcal{I}_{\mathcal{F}}}$ and it is coarsest of all intuitionistic filters on X which are finer than every member of $\mathcal{M}_{2\mathcal{I}_{\mathcal{F}}}$. Therefore $\mathcal{I}_{\mathcal{F}_6}$ is supremum of $\mathcal{M}_{2\mathcal{I}_{\mathcal{F}}}$ and it is a member of $\mathcal{M}_{2\mathcal{I}_{\mathcal{F}}}$.

Theorem 3.3. : Let $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}} = \{F_i : i \in J\}$ be a nonempty collection of intuitionistic filters on a nonempty set X. Then $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$ has a supremum if and only if the collection of all Intuitionistic subsets of X in the union of members of $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$ has the finite intersection property.

Proof. Let $\mathcal{I}_{\mathcal{F}} = \cup \{F_i : i \in J\}$ have the finite intersection property.

By Remark 2.1, there exists the coarsest intuitionistic filter on X containing $\mathcal{I}_{\mathcal{F}}$ and let that intuitionistic filter as $\mathcal{I}_{\mathcal{F}_1}$.

But $\mathcal{I}_{\mathcal{F}_1}$ is finer than every member of $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$.

Thus $\mathcal{I}_{\mathcal{F}_1}$ is the coarsest intuitionistic filter on X which is finer than every member of $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$.

Hence by Definition 3.5, $\mathcal{I}_{\mathcal{F}_1}$ is a supremum of $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$.

Conversely, Let $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$ has a supremum say $\prec X, F^1, F^2 \succ$.

By Definition 3.5, $\prec X, F^1, F^2 \succ$ is the coarsest of all intuitionistic filters on X which are finer than every member of $\mathcal{M}_{\mathcal{I}_{\mathcal{F}}}$.

That is $\mathcal{I}_{\mathcal{F}}$ is the coarsest of all intuitionistic filters on X such that $\mathcal{I}_{\mathcal{F}} \supset \cup \{F_i : i \in J\}$.

Therefore $\cup \{F_i : i \in J\}$ must have finite intersection property. \square

4 Intuitionistic filter base

Definition 4.7. Let X be a any nonempty set. An intuitionistic filter base ($\mathcal{I}_{\mathcal{F}_B}$ for short) on X is a nonempty family $\mathcal{I}_{\mathcal{F}_B}$ of intuitionistic subsets of X satisfying the following axioms :

(a) $\prec X, \phi, X \succ \notin \mathcal{I}_{\mathcal{F}_B}$.

(b) If $A \in \mathcal{I}_{\mathcal{F}_B}$ and $B \in \mathcal{I}_{\mathcal{F}_B}$, then there exists $C \in \mathcal{I}_{\mathcal{F}_B}$ such that $A \cap B \supset C$ or $C \subset A \cap B$

Example 4.4. Let $X = \{a, b, c, d\}$. Then $\{\prec X, \{a\}, \phi \succ, \prec X, \{a, b\}, \phi \succ, \prec X, \{a, c\}, \phi \succ, \prec X, \{a, b, c\}, \{d\} \succ, \prec X, \{a, b, d\}, \phi \succ\}$ is an intuitionistic filter base in X .

Remark 4.3. : $\mathcal{I}_{\mathcal{F}_B}$ has finite intersection property.

Remark 4.4. Every intuitionistic filter is an intuitionistic filter base.

Theorem 4.4. : Let $\mathcal{I}_{\mathcal{F}_B} = \{\prec X, G_i^1, G_i^2 \succ : i \in J\}$ be a family of intuitionistic subsets of a set X . Then $\mathcal{I}_{\mathcal{F}_B}$ is an intuitionistic filter base on X if and only if the family $\mathcal{I}_{\mathcal{F}}$ consisting of all those intuitionistic subsets of X which contain a member of $\mathcal{I}_{\mathcal{F}_B}$ is an intuitionistic filter on X .

Proof. By definition of $\mathcal{I}_{\mathcal{F}}$, each member of $\mathcal{I}_{\mathcal{F}_B}$ is also a member of $\mathcal{I}_{\mathcal{F}}$.

So that $\mathcal{I}_{\mathcal{F}_B} \subseteq \mathcal{I}_{\mathcal{F}}$ and as $\mathcal{I}_{\mathcal{F}_B}$ is an intuitionistic filter base i.e $\prec X, \phi, X \succ \notin \mathcal{I}_{\mathcal{F}_B}$.

Therefore $\prec X, \phi, X \succ \notin \mathcal{I}_{\mathcal{F}}$.

Let $\mathcal{I}_{\mathcal{F}} = \{\prec X, F_i^1, F_i^2 \succ : i \in J\}$ be an intuitionistic filter on X .

We need to show that $\mathcal{I}_{\mathcal{F}_B}$ is an intuitionistic filter base on X .

By Axiom ($\mathcal{I}_{\mathcal{F}_1}$), we have $\prec X, \phi, X \succ \notin \mathcal{I}_{\mathcal{F}}$ and $\mathcal{I}_{\mathcal{F}_B} \subset \mathcal{I}_{\mathcal{F}}$.

Hence $\prec X, \phi, X \succ \notin \mathcal{I}_{\mathcal{F}_B}$.

Thus condition (a) for $\mathcal{I}_{\mathcal{F}_B}$ is satisfied.

Now let $\prec X, F_1^1, F_1^2 \succ$ and $\prec X, F_2^1, F_2^2 \succ \in \mathcal{I}_{\mathcal{F}_B}$

then as $\{\prec X, G_i^1, G_i^2 \succ : i \in J\} \subset \{\prec X, F_i^1, F_i^2 \succ : i \in J\}$

It follows that $\prec X, F_1^1, F_1^2 \succ$ and $\prec X, F_2^1, F_2^2 \succ \in \mathcal{I}_{\mathcal{F}} \Rightarrow \prec X, F_1^1 \cap F_2^1, F_1^2 \cup F_2^2 \succ \in \mathcal{I}_{\mathcal{F}}$ by Axiom IF3

and hence by the definition of $\mathcal{I}_{\mathcal{F}}$, there exist a $\prec X, G^1, G^2 \succ \in \mathcal{I}_{\mathcal{F}_B}$

such that $\prec X, G^1, G^2 \succ \subset \prec X, F_1^1 \cap F_2^1, F_1^2 \cup F_2^2 \succ$

Thus corresponding to $\prec X, F_1^1, F_1^2 \succ$ and $\prec X, F_2^1, F_2^2 \succ \in \mathcal{I}_{\mathcal{F}_B}$

there exists a $\prec X, G^1, G^2 \succ \in \mathcal{I}_{\mathcal{F}_B}$

such that $\prec X, G^1, G^2 \succ \subset \prec X, F_1^1 \cap F_2^1, F_1^2 \cup F_2^2 \succ$.

Hence condition (b) for $\mathcal{I}_{\mathcal{F}_B}$ to be an intuitionistic filter base is also satisfied.

Conversely, Let $\mathcal{I}_{\mathcal{F}_B}$ be an intuitionistic filter base on X .

We need to show that $\mathcal{I}_{\mathcal{F}} = \{\prec X, F_i^1, F_i^2 \succ\}$ is an intuitionistic filter on X .

By condition (a) of intuitionistic filter base $\prec X, \phi, X \succ \notin \mathcal{I}_{\mathcal{F}_B}$.

Hence $\prec X, \phi, X \succ \notin \mathcal{I}_{\mathcal{F}}$ as $\mathcal{I}_{\mathcal{F}}$ is the collection of all those intuitionistic subsets of X which contains a member of $\mathcal{I}_{\mathcal{F}_B}$.

Again let $\prec X, A^1, A^2 \succ \in \mathcal{I}_{\mathcal{F}}$ and $\prec X, B^1, B^2 \succ \supset \prec X, A^1, A^2 \succ$.

Then by definition of $\mathcal{I}_{\mathcal{F}}$, A contains a member of $\mathcal{I}_{\mathcal{F}_B}$ say $\prec X, G^1, G^2 \succ$.

Therefore $\prec X, G^1, G^2 \succ \subset \prec X, A^1, A^2 \succ$ and $\prec X, A^1, A^2 \succ \subset \prec X, B^1, B^2 \succ$.

Hence $\prec X, G^1, G^2 \succ \subset \prec X, B^1, B^2 \succ$ and $\prec X, B^1, B^2 \succ \in \mathcal{I}_{\mathcal{F}}$.

Hence Axiom ($\mathcal{I}_{\mathcal{F}2}$) is satisfied.

let $\prec X, F_1^1, F_1^2 \succ$ and $\prec X, F_2^1, F_2^2 \succ \in \mathcal{I}_{\mathcal{F}}$ so that there exist members $\prec X, G_1^1, G_1^2 \succ \in \mathcal{I}_{\mathcal{F}B}$ and $\prec X, G_2^1, G_2^2 \succ \in \text{IFB}$ such that $\prec X, G_1^1, G_1^2 \succ \subset \prec X, F_1^1, F_1^2 \succ$ and

$\prec X, G_2^1, G_2^2 \succ \subset \prec X, F_2^1, F_2^2 \succ$.

Hence $\prec X, G_1^1 \cap G_2^1, G_1^2 \cup G_2^2 \succ \subset \prec X, F_1^1 \cap F_2^1, F_1^2 \cup F_2^2 \succ$.

Since $\prec X, G_1^1, G_1^2 \succ$ and $\prec X, G_2^1, G_2^2 \succ \in \mathcal{I}_{\mathcal{F}B}$ and $\mathcal{I}_{\mathcal{F}B}$ is an intuitionistic filter base on X ,

so by condition (b) of intuitionistic filter base $\prec X, G_1^1, G_1^2 \succ \cap \prec X, G_2^1, G_2^2 \succ =$

$\prec X, G_1^1 \cap G_2^1, G_1^2 \cup G_2^2 \succ = \prec X, G^1, G^2 \succ$ say also belongs to $\mathcal{I}_{\mathcal{F}B}$.

Hence $\prec X, G^1, G^2 \succ \subset \prec X, F_1^1 \cap F_2^1, F_1^2 \cup F_2^2 \succ$ or $\prec X, F_1^1 \cap F_2^1, F_1^2 \cup F_2^2 \succ$

contains a member of $\mathcal{I}_{\mathcal{F}B}$.

So that $\prec X, F_1^1 \cap F_2^1, F_1^2 \cup F_2^2 \succ \in \mathcal{I}_{\mathcal{F}}$ whenever $\prec X, F_1^1, F_1^2 \succ$ and $\prec X, F_2^1, F_2^2 \succ \in \mathcal{I}_{\mathcal{F}}$.

Thus Axiom ($\mathcal{I}_{\mathcal{F}3}$) is satisfied.

Hence $\mathcal{I}_{\mathcal{F}}$ is an intuitionistic filter on X and is known as the intuitionistic filter generated

by the intuitionistic filter base $\mathcal{I}_{\mathcal{F}B}$ and $\mathcal{I}_{\mathcal{F}B}$ is a subfamily of $\mathcal{I}_{\mathcal{F}}$. □

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