Malaya J. Mat. 4(2)(2016) 297–304

Malaya Journal of Matematik

MJM
an international journal of mathematical sciences with computer applications...

www.malayajournal.org

ISSN : 2319-3786

On the Spectral Expansion Formula for a Class of Dirac Operators

O. Akcay[a,*] and Kh. R. Mamedov[b]

[a,b] Department of Mathematics, Mersin University, 33343, Mersin, Turkey.

Abstract

This paper deals with a problem for the canonical Dirac differential equations system with piecewise continuous coefficient and spectral parameter dependent in boundary conditions. The resolvent operator is constructed. The completeness theorem for eigenvector functions is proved. The spectral expansion formula with respect to eigenvector functions is obtained and Parseval equality is given.

Keywords: Dirac operator, completeness theorem, expansion formula.

2010 MSC: 34L10, 34L40.

1 Introduction

Consider the following boundary value problem generated by Dirac differential equations system

$$By' + \Omega(x)y = \lambda\rho(x)y, \quad 0 < x < \pi \tag{1.1}$$

with boundary conditions

$$\begin{aligned} U_1(y) &:= b_1y_2(0) + b_2y_1(0) - \lambda\left(b_3y_2(0) + b_4y_1(0)\right) = 0, \\ U_2(y) &:= c_1y_2(\pi) + c_2y_1(\pi) + \lambda\left(c_3y_2(\pi) + c_4y_1(\pi)\right) = 0, \end{aligned} \tag{1.2}$$

where

$$B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \Omega(x) = \begin{pmatrix} p(x) & q(x) \\ q(x) & -p(x) \end{pmatrix}, \quad y = \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix},$$

$p(x)$, $q(x)$ are real measurable functions, $p(x) \in L_2(0,\pi)$, $q(x) \in L_2(0,\pi)$, λ is a spectral parameter,

$$\rho(x) = \begin{cases} 1, & 0 \le x \le a, \\ \alpha, & a < x \le \pi, \end{cases}$$

and $1 \neq \alpha > 0$. Let us define $k_1 = b_1b_4 - b_2b_3 > 0$, $k_2 = c_1c_4 - c_2c_3 > 0$.

In the finite interval, the spectral properties of Dirac operators by different aspects are examined by many authors, for example [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13] and etc. In the case of $\rho(x) \neq 1$, the solution of Dirac system was investigated in [5], in this work the representation of this solution has not operator transformation. The asymptotic formulas of eigenvalues and eigenfunctions of Dirac operator with discontinuous coefficient $\rho(x)$ were studied in [1]. Numerical computation of eigenvalues of Dirac system was worked in [11], [12]. Moreover, the theory of Dirac operators was comprehensively given in [6], [10].

This paper is organized as follows: in section 2, the operator formulation of the boundary value problem (1.1),(1.2) and the asymptotic formula of eigenvalues of the problem (1.1),(1.2) are given. In section 3, we prove completeness theorem of eigenfunctions. The expansion formula with respect to eigenfunctions and Parseval equality are obtained.

*Corresponding author.

E-mail address: ozge.akcy@gmail.com (O. Akcay), hanlar@mersin.edu.tr(Kh. R. Mamedov).

2 Preliminaries

An inner product in Hilbert space $H_\rho = L_{2,\rho}(0,\pi;\mathbb{C}^2)\oplus\mathbb{C}^2$ is given by

$$\langle Y,Z\rangle = \int_0^\pi \left\{ y_1(x)\overline{z_1(x)} + y_2(x)\overline{z_2(x)} \right\}\rho(x)\,dx + \frac{1}{k_1}y_3\overline{z_3} + \frac{1}{k_2}y_4\overline{z_4}, \tag{2.3}$$

where

$$Y = \begin{pmatrix} y_1(x) \\ y_2(x) \\ y_3 \\ y_4 \end{pmatrix} \in H_\rho, \quad Z = \begin{pmatrix} z_1(x) \\ z_2(x) \\ z_3 \\ z_4 \end{pmatrix} \in H_\rho.$$

Let us define the operator L:

$$L(Y) := \begin{pmatrix} l(y) \\ b_1y_2(0) + b_2y_1(0) \\ -(c_1y_2(\pi) + c_2y_1(\pi)) \end{pmatrix}$$

with domain

$$D(L) := \Big\{ Y \mid Y = (y_1(x), y_2(x), y_3, y_4)^T \in H_\rho,\ y_1(x), y_2(x) \in AC[0,\pi],$$

$$y_3 = b_3y_2(0) + b_4y_1(0),\ y_4 = c_3y_2(\pi) + c_2y_1(\pi), l(y) \in L_{2,\rho}(0,\pi;\mathbb{C}^2) \Big\}$$

where

$$l(y) = \frac{1}{\rho(x)}\left\{By' + \Omega(x)y\right\}.$$

Consequently, the boundary value problem (1.1),(1.2) is equivalent to the operator equation $LY = \lambda Y$.

Lemma 2.1. *The following properties for the operator L are valid:*

- *The eigenvector functions corresponding to different eigenvalues are orthogonal,*
- *The eigenvalues are real valued.*

Let $\varphi(x,\lambda) = \begin{pmatrix} \varphi_1(x,\lambda) \\ \varphi_2(x,\lambda) \end{pmatrix}$ and $\psi(x,\lambda) = \begin{pmatrix} \psi_1(x,\lambda) \\ \psi_2(x,\lambda) \end{pmatrix}$ be solutions of the system (1.1) satisfying the initial conditions

$$\varphi(0,\lambda) = \begin{pmatrix} \lambda b_3 - b_1 \\ b_2 - \lambda b_4 \end{pmatrix}, \quad \psi(\pi,\lambda) = \begin{pmatrix} -c_1 - \lambda c_3 \\ c_2 + \lambda c_4 \end{pmatrix}.$$

The characteristic function of the problem (1.1),(1.2) is defined by

$$\Delta(\lambda) = W[\varphi(x,\lambda), \psi(x,\lambda)] = \varphi_2(x,\lambda)\psi_1(x,\lambda) - \varphi_1(x,\lambda)\psi_2(x,\lambda), \tag{2.4}$$

where $W[\varphi(x,\lambda), \psi(x,\lambda)]$ is Wronskian of the vector solutions $\varphi(x,\lambda)$ and $\psi(x,\lambda)$. The Wronskian does not depend on x. It follows from (2.4) that

$$\Delta(\lambda) = b_2\psi_1(0,\lambda) + b_1\psi_2(0,\lambda) - \lambda\left(b_4\psi_1(0,\lambda) + b_3\psi_2(0,\lambda)\right) = U_1(\psi)$$

or

$$\Delta(\lambda) = -c_1\varphi_2(\pi,\lambda) - c_2\varphi_1(\pi,\lambda) - \lambda\left(c_3\varphi_2(\pi,\lambda) + c_4\varphi_1(\pi,\lambda)\right) = -U_2(\varphi).$$

Moreover, the zeros λ_n of characteristic function coincide with the eigenvalues of the boundary value problem (1.1),(1.2). The function $\varphi(x,\lambda_n)$ and $\psi(x,\lambda_n)$ are eigenfunctions and there exist a sequence β_n such that

$$\psi(x,\lambda_n) = \beta_n\varphi(x,\lambda_n), \quad \beta_n \neq 0. \tag{2.5}$$

Definition 2.1. *Norming constants of the boundary value problem (1.1), (1.2) are defined as follows:*

$$\alpha_n := \int_0^\pi \left\{ \varphi_1^2(x,\lambda_n) + \varphi_2^2(x,\lambda_n) \right\}\rho(x)dx + \frac{1}{k_1}\left[b_3\varphi_2(0,\lambda_n) + b_4\varphi_1(0,\lambda_n)\right]^2 +$$

$$+ \frac{1}{k_2}\left[c_3\varphi_2(\pi,\lambda_n) + c_4\varphi_1(\pi,\lambda_n)\right]^2. \tag{2.6}$$

Lemma 2.2. *[7] The following relation is valid:*

$$\alpha_n \beta_n = \dot{\Delta}(\lambda_n), \tag{2.7}$$

where $\dot{\Delta}(\lambda) = \frac{d}{d\lambda}\Delta(\lambda)$.

Now, as a different from other works, the system (1.1) has $\rho(x)$ discontinuous coefficient. This coefficient influences the form of the solution of the equation (1.1). Therefore, the solution of the equation (1.1) has the integral representation (not operator transformation) as follows (detail in [5]): Assume that

$$\int_0^{\pi} \|\Omega(x)\|\, dx < +\infty$$

is satisfied for Euclidean norm of matrix function $\Omega(x)$. Then the integral representation of the solution of equation (1.1) satisfying the initial condition $E(0,\lambda) = I$, (I is unite matrix) can be represented

$$E(x,\lambda) = e^{-\lambda B\mu(x)} + \int_{-\mu(x)}^{\mu(x)} K(x,t) e^{-\lambda B t} dt,$$

where

$$\mu(x) = \begin{cases} x, & 0 \le x \le a, \\ \alpha x - \alpha a + a, & a < x \le \pi, \end{cases}$$

and for a kernel $K(x,t)$ the inequality

$$\int_{-\mu(x)}^{\mu(x)} \|K(x,t)\|\, dt \le e^{\sigma(x)} - 1,$$

$$\sigma(x) = \int_0^x \|\Omega(s)\|\, ds$$

holds.

Using this integral representation, the following lemma is proved:

Lemma 2.3. *[7] The solution* $\varphi(x,\lambda) = \begin{pmatrix} \varphi_1(x,\lambda) \\ \varphi_2(x,\lambda) \end{pmatrix}$ *has the following integral representation*

$$\begin{aligned} \varphi_1(x,\lambda) = {} & (\lambda b_3 - b_1)\cos\lambda\mu(x) + (\lambda b_4 - b_2)\sin\lambda\mu(x) + \\ & + (\lambda b_3 - b_1)\int_0^{\mu(x)} \left[\tilde{A}_{11}(x,t)\cos\lambda t + \approx{A}_{12}(x,t)\sin\lambda t\right] dt + \\ & + (\lambda b_4 - b_2)\int_0^{\mu(x)} \left[\approx{A}_{11}(x,t)\sin\lambda t - \tilde{A}_{12}(x,t)\cos\lambda t\right] dt, \end{aligned} \tag{2.8}$$

$$\begin{aligned} \varphi_2(x,\lambda) = {} & (\lambda b_3 - b_1)\sin\lambda\mu(x) + (b_2 - \lambda b_4)\cos\lambda\mu(x) + \\ & + (\lambda b_3 - b_1)\int_0^{\mu(x)} \left[\tilde{A}_{21}(x,t)\cos\lambda t + \approx{A}_{22}(x,t)\sin\lambda t\right] dt + \\ & + (\lambda b_4 - b_2)\int_0^{\mu(x)} \left[\approx{A}_{21}(x,t)\sin\lambda t - \tilde{A}_{22}(x,t)\cos\lambda t\right] dt, \end{aligned} \tag{2.9}$$

where

$$\begin{aligned} \tilde{A}_{1j}(x,t) &= K_{1j}(x,-t) + K_{1j}(x,t), \\ \approx{A}_{1j}(x,t) &= K_{1j}(x,t) - K_{1j}(x,-t), \\ \tilde{A}_{2j}(x,t) &= K_{2j}(x,-t) + K_{2j}(x,t), \\ \approx{A}_{2j}(x,t) &= K_{2j}(x,t) - K_{2j}(x,-t), \end{aligned}$$

and $\tilde{A}_{1j}(x,.) \in L_2(0,\pi)$, $\approx{A}_{1j}(x,.) \in L_2(0,\pi)$, $\tilde{A}_{2j}(x,.) \in L_2(0,\pi)$, $\approx{A}_{2j}(x,.) \in L_2(0,\pi)$, $j = 1,2$.

Moreover, using (2.8) and (2.9), as $|\lambda| \to \infty$ uniformly in $x \in [0, \pi]$, the following asymptotic formulas hold:

$$\varphi_1(x,\lambda) = \lambda\left(b_3 \cos\lambda\mu(x) + b_4 \sin\lambda\mu(x)\right) + O\left(e^{|Im\lambda|\mu(x)}\right), \tag{2.10}$$

$$\varphi_2(x,\lambda) = \lambda\left(b_3 \sin\lambda\mu(x) - b_4 \cos\lambda\mu(x)\right) + O\left(e^{|Im\lambda|\mu(x)}\right). \tag{2.11}$$

Lemma 2.4. *[7] The eigenvalues λ_n, $(n \in \mathbb{Z})$ of the boundary value problem (1.1),(1.2) are in the form*

$$\lambda_n = \tilde{\lambda}_n + \epsilon_n,$$

where

$$\tilde{\lambda}_n = \left[n + \frac{1}{\pi}\arctan\left(\frac{c_3 b_4 - c_4 b_3}{b_3 c_3 + c_4 b_4}\right)\right]\frac{\pi}{\mu(\pi)}$$

and $\{\epsilon_n\} \in l_2$. Moreover, the eigenvalues are simple.

3 Completeness Theorem

Firstly, we construct the resolvent operator and then we prove the completeness theorem of the eigenfunctions of the problem (1.1), (1.2). The expansion formula respect to eigenfunctions is obtained and Parseval equality is given.

Lemma 3.5. *If λ is not a spectrum point of operator L, then the resolvent operator exists and has the following form*

$$y(x,\lambda) = \int_0^\pi R_\lambda(x,t) f(t)\rho(t)dt + \frac{f_4}{\Delta(\lambda)}\varphi(x,\lambda) + \frac{f_3}{\Delta(\lambda)}\psi(x,\lambda), \tag{3.12}$$

where

$$R_\lambda(x,t) = -\frac{1}{\Delta(\lambda)}\begin{cases} \psi(x,\lambda)\widetilde{\varphi}(t,\lambda), & t \le x, \\ \varphi(x,\lambda)\widetilde{\psi}(t,\lambda), & t \ge x, \end{cases} \tag{3.13}$$

here $\widetilde{\varphi}(t,\lambda)$ denotes the transposed vector function of $\varphi(t,\lambda)$.

Proof. Let $F(x) = \begin{pmatrix} f(x) \\ f_3 \\ f_4 \end{pmatrix} \in D(L)$, $f(x) = \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix}$. To construct the resolvent operator of L, we solve the following problem

$$By' + \Omega(x)y = \lambda\rho(x)y + \rho(x)f(x) \tag{3.14}$$

$$\begin{gathered} b_1 y_2(0) + b_2 y_1(0) - \lambda\left(b_3 y_2(0) + b_4 y_1(0)\right) = f_3, \\ c_1 y_2(\pi) + c_2 y_1(\pi) + \lambda\left(c_3 y_2(\pi) + c_4 y_1(\pi)\right) = -f_4. \end{gathered} \tag{3.15}$$

By applying the method of variation of parameters, we want to find the solution of problem (1.1), (1.2) which has a form

$$y(x,\lambda) = c_1(x,\lambda)\varphi(x,\lambda) + C_2(x,\lambda)\psi(x,\lambda). \tag{3.16}$$

Then, we get the equations system

$$c_1'(x,\lambda)\tilde{\psi}(x,\lambda)B\varphi(x,\lambda) = \tilde{\psi}(x,\lambda)\rho(x)f(x),$$

$$c_2'(x,\lambda)\tilde{\varphi}(x,\lambda)B\psi(x,\lambda) = \tilde{\varphi}(x,\lambda)\rho(x)f(x).$$

Using this system, we have

$$c_1(x,\lambda) = c_1(\pi,\lambda) - \frac{1}{\Delta(\lambda)}\int_x^\pi \tilde{\psi}(t,\lambda)f(t)\rho(t)dt, \tag{3.17}$$

$$c_2(x,\lambda) = c_2(0,\lambda) - \frac{1}{\Delta(\lambda)}\int_0^x \tilde{\varphi}(t,\lambda)f(t)\rho(t)dt. \tag{3.18}$$

Substituting the expression (3.17) and (3.18) into (3.16), we find

$$y(x,\lambda) = c_1(\pi,\lambda)\varphi(x,\lambda) + c_2(0,\lambda)\psi(x,\lambda) + \int_0^\pi R_\lambda(x,\lambda)f(t)\rho(t)dt,$$

where

$$R_\lambda(x,t) = -\frac{1}{\Delta(\lambda)}\begin{cases} \psi(x,\lambda)\widetilde{\varphi}(t,\lambda), & t \le x, \\ \varphi(x,\lambda)\widetilde{\psi}(t,\lambda), & t \ge x. \end{cases}$$

Taking the boundary conditions (3.15), we have

$$c_1(\pi,\lambda) = \frac{f_4}{\Delta(\lambda)}, \qquad c_2(0,\lambda) = \frac{f_3}{\Delta(\lambda)}.$$

Consequently,

$$y(x,\lambda) = \int_0^\pi R_\lambda(x,t)f(t)\rho(t)dt + \frac{f_4}{\Delta(\lambda)}\varphi(x,\lambda) + \frac{f_3}{\Delta(\lambda)}\psi(x,\lambda)$$

is obtained. □

Theorem 3.1. *The system of the eigenfunctions* $\{\varphi(x,\lambda_n)\}, (n \in \mathbb{Z})$ *of boundary value problem (1.1),(1.2) is complete in* $L_{2,\rho}(0,\pi;\mathbb{C}^2) \oplus \mathbb{C}^2$.

Proof. Taking into account (2.5) and (2.7) that $\psi(x,\lambda_n) = \frac{\dot{\Delta}(\lambda_n)}{\alpha_n}\varphi(x,\lambda_n)$. Using (3.12), (3.13) and this equality, we get

$$\underset{\lambda=\lambda_n}{Res}\; y(x,\lambda) = -\frac{1}{\alpha_n}\varphi(x,\lambda_n)\left\{\int_0^\pi \widetilde{\varphi}(x,\lambda_n)f(t)\rho(t)\,dt - \frac{f_4}{\beta_n} - f_3\right\}. \tag{3.19}$$

Let $F(x) \in L_{2,\rho}(0,\pi;\mathbb{C}^2) \oplus \mathbb{C}^2$ be such that

$$\langle F(x), \varphi(x,\lambda_n)\rangle = \int_0^\pi \widetilde{\varphi}(t,\lambda_n)f(t)\rho(t)dt + \frac{1}{k_1}f_3\left[b_3\varphi_2(0,\lambda_n) + b_4\varphi_1(0,\lambda_n)\right] +$$

$$+\frac{1}{k_2}f_4\left[c_3\varphi_2(\pi,\lambda_n) + c_4\varphi_1(\pi,\lambda_n)\right] = 0.$$

It follows from the boundary conditions (1.2) and (2.5) that

$$b_3\varphi_2(0,\lambda_n) + b_4\varphi_1(0,\lambda_n) = -k_1$$

and

$$c_3\varphi_2(\pi,\lambda_n) + c_4\varphi_1(\pi,\lambda_n) = -\frac{k_2}{\beta_n}.$$

Thus,

$$\langle F(x), \varphi(x,\lambda_n)\rangle = \int_0^\pi \widetilde{\varphi}(t,\lambda_n)f(t)\rho(t)dt - f_3 - \frac{f_4}{\beta_n} = 0$$

is found. From here and (3.19), $Res_{\lambda=\lambda_n} y(x,\lambda) = 0$ is obtained. Hence, $y(x,\lambda)$ is entire function with respect to λ for each fixed $x \in [0,\pi]$. The following inequality is similarly obtained as in ([8], Lemma 1.3.2)

$$|\Delta(\lambda)| \ge |\lambda|^2 C_\delta \exp(|Im\lambda|\,\mu(\pi)) \tag{3.20}$$

which is valid in the domain

$$G_\delta := \left\{\lambda : \left|\lambda - \tilde{\lambda}_n\right| \ge \delta,\;\; n = 0, \pm1, \pm2...\right\},$$

where δ is a sufficiently small positive number. Taking into account the inequality (3.20) and the following equalities (see [1])

$$\lim_{|\lambda|\to\infty}\max_{0\le x\le\pi} \exp\left(-|Im\lambda|\,\mu(x)\right)\left|\int_0^x \widetilde{\varphi}(t,\lambda)f(t)\rho(t)dt\right| = 0, \tag{3.21}$$

$$\lim_{|\lambda|\to\infty}\max_{0\le x\le\pi} \exp\left(-|Im\lambda|\left(\mu(\pi) - \mu(x)\right)\right)\left|\int_x^\pi \widetilde{\psi}(t,\lambda)f(t)\rho(t)dt\right| = 0, \tag{3.22}$$

we have

$$\lim_{|\lambda|\to\infty}\max_{0\le x\le\pi}|y(x,\lambda)| = 0.$$

Consequently, $y(x,\lambda) \equiv 0$. From (3.14) and (3.15), $F(x) = 0$ a.e. on $(0,\pi)$ is obtained. □

Theorem 3.2. *Let $F(x) \in D(L)$. Then the following expansion formula holds:*

$$f(x) = \sum_{n=-\infty}^{\infty} a_n \varphi(x, \lambda_n), \tag{3.23}$$

$$f_3 = \sum_{n=-\infty}^{\infty} a_n \left[b_3 \varphi_2 (0, \lambda_n) + b_4 \varphi_1 (0, \lambda_n) \right], \tag{3.24}$$

$$f_4 = \sum_{n=-\infty}^{\infty} a_n \left[b_3 \varphi_2 (\pi, \lambda_n) + b_4 \varphi_1 (\pi, \lambda_n) \right], \tag{3.25}$$

where

$$a_n = \frac{1}{\alpha_n} \left\langle f(x), \varphi(x, \lambda_n) \right\rangle .$$

The series converges uniformly with respect to $x \in [0, \pi]$. The series (3.23)-(3.25) converges in $L_{2,\rho}(0, \pi; \mathbb{C}^2) \oplus \mathbb{C}^2$ for $F(x) \in L_{2,\rho}(0, \pi; \mathbb{C}^2) \oplus \mathbb{C}^2$ and Parseval equality

$$\|F\|^2 = \sum_{n=-\infty}^{\infty} \alpha_n |a_n|^2 \tag{3.26}$$

is valid.

Proof. Since $\varphi(x, \lambda)$ and $\psi(x, \lambda)$ are solution of the problem (1.1),(1.2),

$$\begin{aligned} y(x, \lambda) &= -\frac{1}{\lambda \Delta(\lambda)} \psi(x, \lambda) \int_0^x \left\{ -\frac{\partial}{\partial t} \widetilde{\varphi}(t, \lambda) B + \widetilde{\varphi}(t, \lambda) \Omega(t) \right\} f(t) dt \\ &\quad - \frac{1}{\lambda \Delta(\lambda)} \varphi(x, \lambda) \int_x^\pi \left\{ -\frac{\partial}{\partial t} \widetilde{\psi}(t, \lambda) B + \widetilde{\psi}(t, \lambda) \Omega(t) \right\} f(t) dt + \frac{f_4}{\Delta(\lambda)} \varphi(x, \lambda) + \frac{f_3}{\Delta(\lambda)} \psi(x, \lambda) \end{aligned}$$

can be written. Integrating by parts and using the expression of Wronskian

$$y(x, \lambda) = -\frac{1}{\lambda} f(x) - \frac{1}{\lambda} z(x, \lambda) + \frac{f_4}{\Delta(\lambda)} \varphi(x, \lambda) + \frac{f_3}{\Delta(\lambda)} \psi(x, \lambda) \tag{3.27}$$

is obtained, where

$$\begin{aligned} z(x, \lambda) = \frac{1}{\Delta(\lambda)} \Big\{ &\psi(x, \lambda) \int_0^x \widetilde{\varphi}(t, \lambda) B f'(t) dt + \varphi(x, \lambda) \int_x^\pi \widetilde{\psi}(t, \lambda) B f'(t) dt + \\ &+ \psi(x, \lambda) \int_0^x \widetilde{\varphi}(t, \lambda) \Omega(t) f(t) dt + \varphi(x, \lambda) \int_x^\pi \widetilde{\psi}(t, \lambda) \Omega(t) f(t) dt \Big\}. \end{aligned}$$

It follows from (3.21) and (3.22) that

$$\lim_{|\lambda| \to \infty} \max_{0 \le x \le \pi} |z(x, \lambda)| = 0, \quad \lambda \in G_\delta. \tag{3.28}$$

Now, we integrate $y(x, \lambda)$ with respect to λ over the contour Γ_N with oriented counter clockwise as follows:

$$I_N(x) = \frac{1}{2\pi i} \oint_{\Gamma_N} y(x, \lambda) d\lambda,$$

where

$$\Gamma_N = \left\{ \lambda : |\lambda| = \left(N + \frac{1}{\pi} \arctan \left(\frac{c_3 b_4 - c_4 b_3}{b_3 c_3 + c_4 b_4} \right) \right) \frac{\pi}{\mu(\pi)} + \frac{\pi}{2\mu(\pi)} \right\},$$

N is sufficiently large natural number. Applying residue theorem, we have

$$\begin{aligned} I_N(x) &= \sum_{n=-N}^{N} \underset{\lambda = \lambda_n}{Res}\, y(x, \lambda) \\ &= -\sum_{n=-N}^{N} \frac{1}{\alpha_n} \varphi(x, \lambda_n) \int_0^\pi \widetilde{\varphi}(t, \lambda_n) f(t) \rho(t) dt + \sum_{n=-N}^{N} \frac{f_4}{\dot{\Delta}(\lambda_n)} \varphi(x, \lambda_n) + \sum_{n=-N}^{N} \frac{f_3}{\dot{\Delta}(\lambda_n)} \psi(x, \lambda_n). \end{aligned}$$

On the other hand, taking into account the equation (3.27)

$$f(x) = \sum_{n=-N}^{N} a_n \varphi(x, \lambda_n) + \epsilon_N(x) \tag{3.29}$$

is found, where

$$\epsilon_N(x) = -\frac{1}{2\pi i} \oint_{\Gamma_N} \frac{1}{\lambda} z(x, \lambda) d\lambda$$

and

$$a_n = \frac{1}{\alpha_n} \int_0^{\pi} \widetilde{\varphi}(t, \lambda_n) f(t) \rho(t) dt.$$

From (3.28), $\lim_{N\to\infty} \max_{0\leq x\leq\pi} |\epsilon_N(x)| = 0$. Thus, by going over in (3.29) to the limit as $N \to \infty$ the expansion formula (3.23) with respect to eigenfunction is obtained. Since the system of $\{\varphi(x, \lambda_n)\}, (n \in \mathbb{Z})$ is complete and orthogonal in $L_{2,\rho}(0, \pi; \mathbb{C}^2) \oplus \mathbb{C}^2$, Parseval equality (3.26) is valid. □

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Received: March 17, 2016; *Accepted*: April 14, 2016

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Website: http://www.malayajournal.org/