Malaya J. Mat. 4(3)(2016), 421–429

Malaya Journal of Matematik
www.malayajournal.org

MJM
an international journal of mathematical sciences with computer applications...

ISSN : 2319-3786

Semi-invariant submanifolds of a Kenmotsu manifold with a generalized almost r-contact structure admitting a semi-symmetric metric connection

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Abstract

We consider an almost r-contact Kenmotsu manifold admitting a semi-symmetric metric connection and study semi-invariant submanifolds of an almost r-contact Kenmotsu manifold endowed with a semi-symmetric meric connection. We obtain Gauss and Weingarten formuale for such a connection and also discuss the integrability conditions of the distributions on a generalized Kenmotsu manifold.

Keywords: Kenmotsu manifolds, almost r-contact structures, semi-invariant submanifolds, semi-symmetric metric connection, integrability conditions, parallel horizontal distribution.

2010 MSC: 53C40, 53C25.

1 Introduction

A Let M be an n-dimensional differentiable manifold. The torsion tensor T of a linear connection ∇ in M is given by

$$T(X,Y) = \nabla_X Y - \nabla_Y X - [X,Y].$$

The connection ∇ is symmetric if its torsion tensor T vanishes, otherwise it is non-symmetric. The connection ∇ is metric if there is a Riemannian metric g in M such that $\nabla g = 0$, otherwise it is non-metric. It is well known that a linear connection is symmetric and metric if it is the Levi-Civita connection. In 1924, A. friedmann and J. A. Schouten introduced the notion of semi-symmetric linear connection [8]. In 1932, H. A. Hayden [10] introduced semi-symmetric metric connection in a Riemannian manifold and this was studied systematically by K. Yano [14]. In 1975, S. Golab studied some properties of semi-symmetric and quarter-symmetric linear connections [9]. A linear connection ∇ is said to be semi-symmetric if its torsion tensor T is of the form

$$T(X,Y) = \eta(Y)X - \eta(X)Y,$$

where η is a 1-form.

On the other hand, A. Bejancu, introduced the notion of semi-invariant submanifolds [6] or contact CR-submanifolds [5], as a generalization of invariant and anti-invariant submanifolds of an almost contact metric manifold and was followed by several geometers in [1, 2, 4, 7, 11, 12]. Semi-invariant submanifolds of a Kenmotsu manifold immersed in a generalized almost r-contact metric structure was defined and studied by R. Nivas and S. Yadav [13]. The first author, M. D. Siddiqi and J. P. ojha studied some characteristic

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properties of semi-invariant submanifolds of a Kenmotsu manifold immersed in a generalized almost r-contact structure admitting a quarter-symmetric non-metric connection [3].

Semi-symmetric connections play an important role in the study of Riemannian manifolds. There are various physical problems involving the semi-symmetric metric connection. For example, if a man is moving on the surface of the earth always facing one definite point, say Jaruselam or Mekka or the North pole, then this displacement is semi-symmetric and metric [8]

Motivated by the above studies, in this paper we study semi-invariant submanifolds of a Kenmotsu manifold immersed in a generalized almost r-contact structure admitting a semi-symmetric metric connection. The paper is organized as follows : In Section 2, we give a brief account of a Kenmotsu manifold immersed in a generalized almost r-contact metric manifold. In Section 3, semi-invariant submanifolds, semi-symmetric metric connection are defined and also Gauss and Weingarten equations are obtained. In Section 4, some lemmas on semi-invariant submanifolds are proved and integrability conditions of certain distributions on semi-invariant submanifolds are discussed. In the last Section 5, semi-invariant submanifolds of a generalized Kenmotsu manifold with parallel horizontal distributions for semi-symmetric metric connection are investigated.

2 Preliminaries

Let $\bar{M}$ be a $(2n + r)$-dimensional Kenmotsu manifold with a generalized almost r-contact structure (ϕ, ξ_p, η_p, g), where ϕ is a tensor field of type $(1, 1)$, ξ_p are r-vector fields, η_p are r 1-forms and g is the associated Riemannian metric, satisfying

$$\phi^2 = a^2 I + \sum_{p=1}^{r} \eta_p \otimes \xi_p, \tag{2.1}$$

$$\eta_p(\xi_q) = \delta_{pq}, \qquad p, q \in (r) := 1, 2, 3......r, \tag{2.2}$$

$$\phi(\xi_p) = 0, \ \ p \in (r), \tag{2.3}$$

$$\eta_p(\phi X) = 0, \ \ p \in (r), \tag{2.4}$$

$$g(\phi X, \phi Y) + a^2 g(X, Y) + \sum_{p=1}^{r} \eta_p(X)\eta_p(Y) = 0, \tag{2.5}$$

$$\eta_p(X) = g(X, \xi_p), \tag{2.6}$$

$$(\bar{\bar{\nabla}}_X \phi)Y = -\sum_{p=1}^{r} \eta_p(Y)\phi X - g(X, \phi Y) \sum_{p=1}^{r} \xi_p, \tag{2.7}$$

$$\bar{\bar{\nabla}}_X \xi_p = X - \sum_{p=1}^{r} \eta_p(X)\xi_p, \tag{2.8}$$

where I is the identity tensor field and X, Y are vector fields on $\bar{M}$ and $\bar{\bar{\nabla}}$ denotes the Riemannian connection.

3 Semi-invariant Submanifolds

An n-dimensional Riemannian submanifold M of a Kenmotsu manifold $\bar{M}$ with an almost r-contact structure is called a semi-invariant submanifold, if ξ_p is tangent to M and there exists on M a pair of orthogonal distributions $(D, D^{\perp})$ such that
(i) $TM = D \oplus D^{\perp} + \{\xi_p\}$,
(ii) the distribution D is invariant under ϕ, that is, $\phi D_x = D_x$ for all $x \in M$,
(ii) the distribution $D^{\perp}$ is anti-invariant under ϕ, that is, $\phi D_x^{\perp} \subset T_x^{\perp} M$ for all $x \in M$,

where T_xM and $T_x^{\perp}M$ are respectively the tangent and normal space of M at x.
The distribution D $(resp., D^{\perp})$ can be defined by projection P $(resp., Q)$ which satisfies the conditions

$$P^2 = P, Q^2 = Q, PQ = QP = 0. \tag{3.9}$$

The pair of distributions $(D, D^{\perp})$ is called the ξ-horizontal (resp., ξ-vertical), if $\xi_x \in D_x$ (resp., $\xi_x \in D_x^{\perp}$). A semi-invariant submanifold M is said to be an invariant (resp., anti-invariant) submanifold if $D_x^{\perp} = 0$ (resp., $D_x = 0$) for each $x \in M$, we also call M proper, if neither D nor $D^{\perp}$ is null. It is easy to check that each hypersurface of M which is tangent to ξ_p inherits a structure of the semi-invariant submanifold of $\bar{M}$.
Owing due to the existence of 1-form η_p, we define a semi-symmetric metric connection $\bar{\nabla}$ in a Kenmotsu manifold with a generalized almost r-contact structure by

$$\bar{\nabla}_X Y = \bar{\bar{\nabla}}_X Y + \sum_{p=1}^{r} \eta_p(Y) X - g(X, Y) \sum_{p=1}^{r} \xi_p \tag{3.10}$$

for any $X, Y \in TM$, where $\bar{\bar{\nabla}}$ is the induced connection on M. From (2.7) and (3.10), we get

$$(\bar{\nabla}_X \phi) Y = -2 \sum_{p=1}^{r} \eta_p(Y) \phi X - g(X, \phi Y) \sum_{p=1}^{r} \xi_p. \tag{3.11}$$

We denote the metric tensor of $\bar{M}$ as well as that is induced on M by g. Let $\bar{\nabla}$ be the semi-symmetric metric connection on $\bar{M}$ and ∇ be the induced connection on M with respect to the unit normal N.

Theorem 3.1. *The connection induced on the semi-invariant submanifolds of a generalized Kenmotsu manifold with a semi-symmetric metric connection is also a semi-symmetric metric connection.*

Proof. Let ∇ be the induced connection with respect to the unit normal N on semi-invariant submanifolds of a generalized Kenmotsu manifold with a semi-symmetric metric connection $\bar{\nabla}$. Then

$$\bar{\nabla}_X Y = \nabla_X Y + m(X, Y), \tag{3.12}$$

where m is a tensor field of type $(0, 2)$ on semi-invariant submanifold M. If ∇^* is the induced connection on semi-invariant submanifolds from the Riemannian connection $\bar{\bar{\nabla}}$, then

$$\bar{\bar{\nabla}}_X Y = \nabla^*_X Y + h(X, Y), \tag{3.13}$$

where h is the second fundamental tensor. Now from (3.10), (3.12) and (3.13), we have

$$\nabla_X Y + m(X, Y) = \nabla^*_X Y + h(X, Y) + \eta_p(Y) \phi X - g(X, Y) \sum_{p=1}^{r} \xi_p.$$

Equating the tangential and normal components from both the sides of the above equation, we get

$$h(X, Y) = m(X, Y),$$

$$\nabla_X Y = \nabla^*_X Y + \eta_p(Y) \phi X - g(X, Y) \sum_{p=1}^{r} \xi_p.$$

Thus the connection ∇ is also a semi-symmetric metric connection. □

Now, the Gauss formula for semi-invariant submanifolds of a generalized Kenmotsu manifold with a semi-symmetric metric connection is

$$\bar{\nabla}_X Y = \nabla_X Y + h(X, Y) \tag{3.14}$$

and Weingarten formula for M is given by

$$\bar{\nabla}_X N = -A_N X + \nabla^{\perp}_X N \tag{3.15}$$

for $X, Y \in TM, N \in T^{\perp}M$, where h and A are called the second fundamental tensors of M and $\nabla^{\perp}$ denotes the operator of the normal connection. Moreover, we have

$$g(h(X, Y), N) = g(A_N X, Y). \tag{3.16}$$

Any vector field X tangent to M is given as

$$X = PX + QX + \eta_p(X)\xi_p, \tag{3.17}$$

where PX and QX belong to the distribution D and $D^\perp$ respectively. For any vector field N normal to M, we have

$$\phi N = BN + CN, \tag{3.18}$$

where BN (resp., CN) denotes the tangential (resp., normal) component of ϕN.

4 Integrability of distributions

Lemma 4.1. *Let M be a semi-invariant submanifolds of a generalized Kenmotsu manifold with a semi-symmetric metric connection. Then*

$$2(\bar{\nabla}_X\phi)Y = \nabla_X\phi Y - \nabla_Y\phi X + h(X,\phi Y) - h(Y,\phi X) - \phi[X,Y]$$

for each $X, Y \in D$.

Proof. Using Gauss formula, we have

$$\bar{\nabla}_X\phi Y - \bar{\nabla}_Y\phi X = \nabla_X\phi Y - \nabla_Y\phi X + h(X,\phi Y) - h(Y,\phi X). \tag{4.19}$$

Also the covariant differentiation yields

$$\bar{\nabla}_X\phi Y - \bar{\nabla}_Y\phi X = (\bar{\nabla}_X\phi)Y - (\bar{\nabla}_Y\phi)X + \phi[X,Y]. \tag{4.20}$$

From (4.19) and (4.20), we get

$$(\bar{\nabla}_X\phi)Y - (\bar{\nabla}_Y\phi)X = \nabla_X\phi Y - \nabla_Y\phi X + h(X,\phi Y) - h(Y,\phi X) - \phi[X,Y]. \tag{4.21}$$

Using $\eta_p(X) = 0$ for each $X \in D$ in (3.11), we get

$$(\bar{\nabla}_X\phi)Y + (\bar{\nabla}_Y\phi)X = 0. \tag{4.22}$$

On adding (4.21) and (4.22), we get the result. □

Similar computations also yields the following:

Lemma 4.2. *Let M be a semi-invariant submanifold of a generalized Kenmotsu manifold with a semi-symmetric metric connection. Then*

$$2(\bar{\nabla}_X\phi)Y = -A_{\phi Y}X + \nabla^\perp_X\phi Y - \nabla_Y\phi X - h(Y,\phi X) - \phi[X,Y]$$

for each $X \in D$, $Y \in D^\perp$.

Lemma 4.3. *Let M be a semi-invariant submanifold of a generalized Kenmotsu manifold with a semi-symmetric metric connection. Then*

$$P\nabla_X\phi PY - PA_{\phi QY}X = \phi P\nabla_XY - 2\sum_{p=1}^{r}\eta_p(Y)\phi PX, \tag{4.23}$$

$$Q\nabla_X\phi PY - QA_{\phi QY}X = Bh(X,Y), \tag{4.24}$$

$$h(X,\phi PY) + \nabla^\perp_X\phi QY = \phi Q\nabla_XY + Ch(X,Y) - 2\sum_{p=1}^{r}\eta_p(Y)\phi QX, \tag{4.25}$$

$$\eta_P(\nabla_X\phi PY) - \eta_P(A_{\phi QY}X) = -2g(X,\phi Y) \tag{4.26}$$

for all $X, Y \in TM$.

Proof. By the covariant differentiation of ϕY, we have

$$\bar{\nabla}_X\phi Y = (\bar{\nabla}_X\phi)Y + \phi(\bar{\nabla}_X Y).$$

Using (3.14) and (3.17) in the above equation, we get

$$(\bar{\nabla}_X\phi)Y = \bar{\nabla}_X\phi PY + \bar{\nabla}_X\phi QY - \phi\nabla_X Y - \phi h(X,Y). \tag{4.27}$$

By the use of Gauss and Weingarten formulae and (3.18) in (4.27), we have

$$(\bar{\nabla}_X\phi)Y = P\nabla_X\phi PY + Q\nabla_X\phi PY + \eta_P(\nabla_X\phi PY)\xi_p + h(X,\phi PY) - PA_{\phi QY}X \tag{4.28}$$

$$-QA_{\phi QY}X - \eta_P(A_{\phi QY}X)\xi_p + \nabla^{\perp}_X\phi QY - \phi P\nabla_X Y - \phi Q\nabla_X Y - Bh(X,Y) - Ch(X,Y).$$

On comparing (4.27) and (4.28) and equating horizontal, vertical and normal components, we get (4.23), (4.24), (4.25) and (4.26) respectively. □

Definition 4.1. *The horizontal distribution D is said to be parallel with respect to the connection ∇ on M, if $\nabla_X Y \in D$ for all vector fields $X, Y \in D$.*

Theorem 4.2. *Let M be semi-invariant submanifolds of a generalized Kenmotsu manifold $\bar{M}$ with a semi-symmetric metric connection. If M is ξ_p- horizontal, then the distribution D is integrable if and only if*

$$h(X,\phi Y) = h(\phi X, Y) \tag{4.29}$$

for all $X, Y \in D$.

Proof. Let M be ξ_p- horizontal and $X, Y \in D$, then (4.25) reduces to

$$h(X,\phi Y) = \phi Q\nabla_X Y + Ch(X,Y) \tag{4.30}$$

from which we get

$$h(X,\phi Y) - h(\phi X, Y) = \phi Q[X,Y].$$

Thus if M is ξ_p horizontal, then we have

$$h(X,\phi Y) = h(\phi X, Y).$$

Hence D is integrable. □

Theorem 4.3. *Let M be semi-invariant submanifolds of a generalized Kenmotsu manifold $\bar{M}$ with a semi-symmetric metric connection. If M is ξ_p-vertical, then the distribution $D^{\perp}$ is integrable if and only if $A_{\phi X}Y = A_{\phi Y}X$.*

Proof. Let M be ξ_p-vertical and $X, Y \in D^{\perp}$, then (4.25) reduces to

$$\nabla^{\perp}_X\phi Y = \phi Q\nabla_X Y + Ch(X,Y) - 2\sum_{p=1}^{r}\eta_p(Y)\phi QX. \tag{4.31}$$

By using (3.11), (3.15) and (4.31), we get

$$\bar{\nabla}_X\phi Y = -2\sum_{p=1}^{r}\eta_p(Y)\phi X - 2g(X,\phi Y)\sum_{p=1}^{r}\xi_p + \phi P\nabla_X Y \tag{4.32}$$

$$+\phi Q\nabla_X Y + Bh(X,Y) + Ch(X,Y).$$

Since M is ξ_p-verticle, Weingarten formula is given by

$$\nabla^{\perp}_X\phi Y = \bar{\nabla}_X\phi Y + A_{\phi Y}X$$

which by using (4.32) becomes

$$\nabla^{\perp}_X\phi Y = -2\sum_{p=1}^{r}\eta_p(Y)\phi X + \phi P\nabla_X Y + \phi Q\nabla_X Y + Bh(X,Y) \tag{4.33}$$

$$+Ch(X,Y)+A_{\phi Y}X.$$

From (4.31) and (4.33), we get

$$\phi P\nabla_X Y=-A_{\phi Y}X-Bh(X,Y).$$

Similarly, $\phi P\nabla_Y X=-A_{\phi X}Y-Bh(X,Y)$, which gives

$$\phi P[X,Y]=A_{\phi X}Y-A_{\phi Y}X.$$

Thus if M is ξ_p-verticle, we see that $[X,Y]\in D^{\perp}$, that is, $P[X,Y]=0$ if and only if $A_{\phi X}Y=A_{\phi Y}X$. □

5 Parallel horizontal distribution

Definition 5.2. *A non-zero normal vector field N is said to be D-parallel normal section if*

$$\nabla^{\perp}_X N=0 \quad for\ \ all \quad X\in D. \tag{5.34}$$

Definition 5.3. *A semi-invariant submanifold M is said to be totally r-contact umbilical if there exists a normal vector H on M such that*

$$h(X,Y)=g(\phi X,\phi Y)H+\sum_{p=1}^{r}\eta_p(X)h(Y,\xi_p)+\sum_{p=1}^{r}\eta_p(Y)h(X,\xi_p) \tag{5.35}$$

for all vector fields X, Y tangent to M.

If $H=0$, then the fundamental form is given by

$$h(X,Y)=\sum_{p=1}^{r}\eta_p(X)h(Y,\xi_p)+\sum_{p=1}^{r}\eta_p(Y)h(X,\xi_p), \tag{5.36}$$

then M is called totally r-contact geodesic.

Theorem 5.4. *If M is totally r-contact umbilical semi-invariant submanifolds of a generalized Kenmotsu manifold $\bar{M}$ with a semi-symmetric metric connection with parallel horizontal distribution, then M is totally r-contact geodesic.*

Proof. Let M be semi-invariant submanifolds of a generalized Kenmotsu manifold $\bar{M}$ with a semi-symmetric metric connection. Then from (4.23) and (4.24), we have

$$P\nabla_X\phi PY-PA_{\phi QY}X=\phi P\nabla_X Y-2\sum_{p=1}^{r}\eta_p(Y)\phi PX,$$

$$Q\nabla_X\phi PY-QA_{\phi QY}X=Bh(X,Y).$$

Adding the last two equations, we have

$$\nabla_X\phi PY-A_{\phi QY}X=\phi P\nabla_X Y+Bh(X,Y). \tag{5.37}$$

Interchanging X and Y in (5.37), we get

$$\nabla_Y\phi PX-A_{\phi QX}Y=\phi P\nabla_Y X+Bh(X,Y). \tag{5.38}$$

Adding (5.37) and (5.38), we get

$$\nabla_X\phi PY+\nabla_Y\phi PX-A_{\phi QY}X-A_{\phi QX}Y=\phi P\nabla_X Y+\phi P\nabla_Y X+2Bh(X,Y).$$

Taking inner product with Z, we get

$$g(\nabla_X\phi PY+\nabla_Y\phi PX-A_{\phi QY}X-A_{\phi QX}Y,Z)=g(\phi P\nabla_X Y+\phi P\nabla_Y X+2Bh(X,Y),Z).$$

Splitting the above equation, we get

$$g(\nabla_X\phi PY,Z)+g(\nabla_Y\phi PX,Z)-g(A_{\phi QY}X,Z)-g(A_{\phi QX}Y,Z)=g(\phi P\nabla_X Y,Z)$$

$$+g(\phi P\nabla_Y X,Z)+g[2B(g(\phi X,\phi Y)H+\sum_{p=1}^{r}\eta_p(X)h(Y,\xi_p)+\sum_{p=1}^{r}\eta_p(Y)h(X,\xi_p),Z)].$$

$$g(\nabla_X\phi PY,Z)+g(\nabla_Y\phi PX,Z)-g(h(X,Z),\phi QY)-g(h(Y,Z),\phi QX)=g(\phi P\nabla_X Y,Z)$$

$$+g(\phi P\nabla_Y X,Z)+2g(\phi X,\phi Y)g(BH,Z)+2\sum_{p=1}^{r}\eta_p(X)g(Bh(Y,\xi_p),Z)+2\sum_{p=1}^{r}\eta_p(Y)g(Bh(X,\xi_p),Z).$$

$$=g(\phi P\nabla_X Y,Z)+g(\phi P\nabla_Y X,Z)-2a^2g(X,Y)g(BH,Z)-2\sum_{p=1}^{r}\eta_p(X)\eta_p(Y)g(BH,Z)$$

$$+2\sum_{p=1}^{r}\eta_p(X)g(h(Y,\xi_p),\phi Z)+2\sum_{p=1}^{r}\eta_p(Y)g(h(X,\xi_p),\phi Z)$$

which by replacing Y by BH and Z by X and then using (5.35), we get

$$g(\nabla_X\phi PBH,X)+g(\nabla_{BH}\phi PX,X)-g(X,X)g(H,\phi QBH)-g(BH,X)g(H,\phi QX) \quad (5.39)$$

$$=g(\phi P\nabla_X BH,X)+g(\phi P\nabla_{BH}X,X)-2a^2g(X,BH)g(BH,X)-2\sum_{p=1}^{r}\eta_p(X)\eta_p(BH)g(BH,X)$$

$$+2\sum_{p=1}^{r}\eta_p(X)g(h(BH,\xi_p),\phi X)+2\sum_{p=1}^{r}\eta_p(BH)g(h(X,\xi_p),\phi X).$$

For any $X\in D$, we have

$$g(X,BH)=g(\phi X,BH)=0.$$

Taking covariant differentiation along vector X, we get

$$g(\nabla_X\phi X,BH)+g(\phi X,\nabla_X BH)=0.$$

As the horizontal distribution D is parallel, so we have

$$g(\phi X,\nabla_X BH)=0. \quad (5.40)$$

From (5.39) and (5.40), we get

$$g(\nabla_{BH}\phi PX,X)-g(H,\phi QBH)=g(\phi P\nabla_{BH}X,X).$$

For any unit vector $X\in D$, we have

$$g((\nabla_{BH}\phi P)X,X)+g(\phi P\nabla_{BH}X,X)-g(H,\phi QBH)=g(\phi P\nabla_{BH}X,X).$$

$$g((\nabla_{BH}\phi P)X,X)-g(H,\phi QBH)=0. \quad (5.41)$$

From (5.41), we have

$$g(BH,QBH)+\sum_{p=1}^{r}\eta_p(PH)g(\phi X,X)=0.$$

Thus we have

$$g((\nabla_{BH}\phi P)X,X)=g(H,\phi QBH)=-g(\phi H,QBH)=-g(BH,QBH)=0.$$

provided $BH=0$.
Since $\phi H\in D^{\perp}$, we have $CH=0$, hence $\phi H=0$, thus $H=0$.
Hence M is totally r-contact geodesic. □

Remark 5.1. *For a generalized Kenmotsu manifold with a semi-symmetric metric connection, we have*

$$\bar{\nabla}_X\xi_p = \bar{\bar{\nabla}}_X\xi_p + \sum_{p=1}^{r}\eta_p(\xi_p)X - g(X,\xi_p)\sum_{p=1}^{r}\xi_p \tag{5.42}$$
$$= 2PX + 2QX.$$

Equating the tangential and normal components, we have

$$\bar{\nabla}_X\xi_p = 2PX + 2QX = 2X, \tag{5.43}$$

$$h(X,\xi_p) = 0, \tag{5.44}$$

$$\eta_p(X)\xi_p = 0. \tag{5.45}$$

Also for any $X \in D$, we have

$$g(A_N\xi_p, X) = g(h(X,\xi_p), N) = 0. \tag{5.46}$$

Thus if $X \in D$, then $A_N\xi_p \in D^{\perp}$ and if $X \in D^{\perp}$, then $A_N\xi_p \in D$.

Theorem 5.5. *Let M be D-umbilic (resp., $D^{\perp}$-umbilic) semi-invariant submanifolds of a generalized Kenmotsu manifold $\bar{M}$ with a semi-symmetric metric connection. If M is ξ_p-horizontal (resp., ξ_p-verticle) , then it is D-totally geodesic (resp., $D^{\perp}$-totally geodesic).*

Proof. If M is D-umbilic semi-invariant submanifolds of a generalized Kenmotsu manifold $\bar{M}$ with a semi-symmetric metric connection with ξ_p-horizontal, then we have

$$h(X,\xi_p) = g(X,\xi_p)L \tag{5.47}$$

which means that $L = 0$, from which we get $h(X,\xi_p) = 0$. Hence M is D-totally geodesic.
Similarly, we can prove that if M is a $D^{\perp}$-umbilic semi-invariant submanifold with ξ_p-verticle, then M is $D^{\perp}$-totally geodesic. □

6 Acknowledgment

The authors are thankful to the referee for his valuable suggestions for the improvement of the paper.

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Received: February 6, 2016; *Accepted*: June 5, 2016

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