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Zagreb Indices of a Graph and its Common Neighborhood Graph

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Abstract

A complete set of relations is established between the first and second Zagreb index of a graph and of its congraph. Formulas for the Zagreb indices of several derived graphs are also obtained.

Keywords: Vertex degree, Zagreb indices, Common neighborhood graph.

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1 Introduction

The graphs considered in this paper are assumed to be simple, i.e., to possess no directed or weighted edges and no self–loops. Let G be such a graph with vertex set $V(G)$ and edge set $E(G)$. If $|V(G)| = p$ and $|E(G)| = q$, then we say that G is a (p,q)-graph. The edge connecting the vertices x and y will be denoted by xy.

The set of vertices of G, adjacent to a vertex v will be denoted by $N_G(v)$. The degree of the vertex v, denoted by $d(v) = d_G(v)$, is the number of first neighbors of v, that is $d_G(v) = |N_G(v)|$.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The *common neighborhood graph* (*congraph*) of G, denoted by $con(G)$, is the graph with vertex set $V(con(G)) = V(G)$, in which two vertices are adjacent if and only if they have a common neighbor in G. In other words, for every $x,y \in V(G)$,

$$xy \in E(con(G)) \Longleftrightarrow N_G(x) \cap N_G(y) \neq \emptyset\,.$$

The concept of common neighborhood graphs originates from the study of a special kind of graph energy [2]. The basic properties of these derived graphs were established soon after that [1, 3]. Also, various mathematical properties of congraphs have been discovered [8, 13, 14].

Two old and most studied degree–based graph invariants are the so-called *first and second Zagreb indices*, defined as

$$M_1(G) = \sum_{v\in V(G)} d(v)^2 \qquad \text{and} \qquad M_2(G) = \sum_{uv\in E(G)} d(u)\,d(v)\,.$$

For details on their history, mathematical properties and chemical applications, we refer to [4, 5, 9–12] and the references cited therein.

The so-called *forgotten topological index* is defined as [6, 7]

$$F = F(G) = \sum_{v\in V(G)} d(v)^3\,.$$

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In [15], Li and Zheng introduced the first general Zagreb index as

$$M_1^k(G) = \sum_{v \in V(G)} d(v)^k$$

where $k \in \mathbb{N} \cup \{0\}$. Obviously, $M_1^0(G) = |V(G)|$, $M_1^1(G) = 2|E(G)|$, $M_1^2(G) = M_1(G)$, and $M_1^3(G) = F(G)$. Also, in [16], the the second general Zagreb index was defined as

$$M_2^k(G) = \sum_{uv \in E(G)} \big[d(u)\, d(v)\big]^k$$

where $k \in \mathbb{N} \cup \{0\}$. Obviously $M_2^0(G) = |E(G)|$ and $M_2^1(G) = M_2(G)$.

We now define two new degree–based graph invariants, pertaining to congraphs:

$$\Xi_1(G) = \sum_{v \in V(G)} d_G(v)\, d_{con(G)}(v) \qquad \text{and} \qquad \Xi_2(G) = \sum_{uv \in E(con(G))} d_G(u)\, d_G(v)\,.$$

Throughout this paper, we use standard graph–theoretical notation. $\overline{G}$ denoted the complement of the graph G. As usual, P_n, C_n, and K_n, are, respectively, the n-vertex path, cycle, and complete graph. In addition, $K_{n,m}$ is the complete bipartite graph with $n+m$ vertices. Recall that $K_{1,n-1}$ is called the star and often denoted by S_n.

In this paper, we investigate some properties of congraphs and the Zagreb indices of congraphs and establish relations between the Zagreb indices of congraphs and several degree–based invariants of the parent graphs.

2 Degree–related properties of common neighborhood graph

Lemma 2.1. *Let G be a simple (p,q)-graph and let $con(G)$ be a (p,q')-graph. Then, for every $v \in V(G)$ the following holds.*

(1) $d_{con(G)}(v) = \left| \bigcup_{u \in N_G(v)} N_G(u) \setminus \{v\} \right| = |N_{con(G)}(v)|\,.$

(2) *If G has no cycles of size 4, then* $d_{con(G)}(v) + d_G(v) = \sum_{u \in N_G(v)} d_G(u)$.

(3) *If $d_G(u) + d_G(v) > p$ holds for every $u, v \in V(G)$, then $con(G) \cong K_p$.*

(4) *If G has no cycles of size 3, then $con(G)$ is a subgraph of $\overline{G}$.*

Proof.
(1) From the definition of a congraph we have

$$u \in N_{con(G)}(v) \iff uv \in E(con(G)) \iff N_G(u) \cap N_G(v) \neq \emptyset\,.$$

Then there exists $a \in N_G(v)$ and $a \in N_G(u)$ such that

$$a \in N_G(v) \text{ and } u \in N_G(a)$$

implies

$$N_{con(G)}(v) = \bigcup_{u \in N_G(v)} N_G(u) \setminus \{v\}\,.$$

(2) For every $u, w \in N_G(v)$, we have $v \in N_G(u) \cap N_G(w)$. We can easily see that $N_G(u) \cap N_G(w) = \{v\}$, since, if there exist $a \in N_G(u) \cap N_G(w)$ such that $a \neq v$, it would follow that $au, vu, aw, vw \in E(G)$, that is we would

have a cycle of size 4, which is a contradiction. Also, by

$$\begin{aligned} d_{con(G)}(v) &= \left| \bigcup_{u \in N_G(v)} N_G(u) \setminus \{v\} \right| = \left| \bigcup_{u \in N_G(v)} (N_G(u) \setminus \{v\}) \right| \\ &= \sum_{u \in N_G(v)} |N_G(u) \setminus \{v\}| = \sum_{u \in N_G(v)} (|N_G(u)| - 1) \\ &= \left(\sum_{u \in N_G(v)} d(u) \right) - |N_G(v)| = \left(\sum_{u \in N_G(v)} d(u) \right) - d(v) \end{aligned}$$

the claim (2) in Lemma 2.1 follows.

(3) It suffices to show that $N_G(u) \cap N_G(v) \neq \emptyset$ for every $u, v \in V(G)$. Otherwise, we would have

$$p \geq |N_G(u) \cup N_G(v)| = |N_G(u)| + |N_G(v)| = d(u) + d(v) > p$$

which is a contradiction. Hence, it follows that $uv \in E(con(G))$ that is $con(G) \cong K_p$.

(4) It is enough to show that $E(con(G)) \subseteq E(\overline{G})$. Hence, for every $uv \in E(con(G))$, we have $N_G(u) \cap N_G(v) \neq \emptyset$. That is there exist $a \in N_G(u) \cap N_G(v)$. Then $au, av \in E(G)$, but $uv \notin E(G)$, otherwise G would have a cycle of size 3. Hence, $uv \in E(\overline{G})$. □

Theorem 2.1. *Let G be a (p,q)-graph. In the congraph of G, for every $u, v \in V(G)$, if $d(u) + d(v) > p$ then:*

(1) $\Xi_1(G) = 2q\,(p-1)$

(2) $\Xi_2(G) = 2q^2 - \frac{1}{2} M_1(G)$.

Proof. By Lemma 2.1, $con(G) \cong K_p$.

(1)

$$\begin{aligned} \Xi_1(G) &= \sum_{v \in V(G)} d_G(v)\, d_{con(G)}(v) = \sum_{v \in V(G)} d_G(v)\,(p-1) \\ &= (p-1) \sum_{v \in V(G)} d_G(v) = 2q(p-1). \end{aligned}$$

(2)

$$\begin{aligned} \Xi_2(G) &= \sum_{uv \in E(con(G))} d(u)\, d(v) = \sum_{uv \in E(K_p)} d(u)\, d(v) = \frac{1}{2} \sum_{u,v \in V(G),\, u \neq v} d(u)\, d(v) \\ &= \frac{1}{2} \left[\sum_{u \in V(G)} d(u) \sum_{v \in V(G)} d(v) - \sum_{v \in V(G)} d(v)^2 \right] = \frac{1}{2} [2q \cdot 2q - M_1(G)] \\ &= 2q^2 - \frac{1}{2} M_1(G). \end{aligned}$$

□

Theorem 2.2. *Let G be a (p,q)-graph and have no cycles of size 4. Also, let $con(G)$ be a (p,q')-graph. Then,*

$$q' = \frac{1}{2} \sum_{v \in V(G)} d_G(v)^2 - q = \frac{1}{2} M_1(G) - q. \tag{2.1}$$

Proof. First we show that $N_G(u) \cap N_G(w) = \{v\}$ holds for every $u, w \in N_G(v)$. Otherwise, if there would exist $a \in N_G(u) \cap N_G(v)$, then it is easy to see that G has a cycle of size 4, which is a contradiction. Hence, by Lemma 2.1 we get $d_{con(G)}(v) + d_G(v) = \sum\limits_{u \in N_G(v)} d_G(u)$. Thus,

$$\sum_{v \in V(G)} d_{con(G)}(v) + \sum_{v \in V(G)} d_G(v) = \sum_{v \in V(G)} \sum_{u \in N_G(v)} d_G(u)$$

and

$$2q' + 2q = \sum_{v \in V(G)} d_G(v)^2$$

from which Eq. (2.1) follows. □

Theorem 2.3. *Let G be a (p,q)-graph having no cycles of size 4. Also, let $con(G)$ be a (p,q')-graph. Then,*

(1) $M_1(con(G)) = F + 2\,\Xi_2(G) - 4M_2(G) + M_1(G)$;

(2) $M_2(G) = \frac{1}{2}\left[\Xi_1(G) + M_1(G)\right]$.

Proof. By Lemma 2.1, we have:
(1)

$$\begin{aligned} M_1(con(G)) &= \sum_{v \in V(con(G))} d_{con(G)}(v)^2 = \sum_{v \in V(G)} \left(\sum_{u \in N_G(v)} d(u) - d(v) \right)^2 \\ &= \sum_{v \in V(G)} \left(\sum_{u \in N_G(v)} d(u) \right)^2 - 2 \sum_{v \in V(G)} \left(\sum_{u \in N_G(v)} d(u) \right) d(v) + \sum_{v \in V(G)} d(v)^2 \\ &= F + 2\,\Xi_2(G) - 4M_2(G) + M_1(G)\,. \end{aligned}$$

(2)

$$\begin{aligned} \Xi_1(G) &= \sum_{v \in V(G)} d(v)\, d_{con(G)}(v) = \sum_{v \in V(G)} d(v) \left(\sum_{u \in N_G(v)} d(u) - d(v) \right) \\ &= \sum_{v \in V(G)} d(v) \left(\sum_{u \in N_G(v)} d(u) \right) - \sum_{v \in V(G)} d(v)^2 \\ &= 2 \sum_{uv \in E(G)} d(v)\, d(u) - \sum_{v \in V(G)} d(v)^2 = 2M_2(G) - M_1(G) \end{aligned}$$

□

If there is a cycle of size 4, then we can change it into a square. Two cycles of order 4 in a graph are said to be disjoint, if they have no common diagonals in their corresponding squares.

Definition 2.1. *A graph G is called type S, if any two cycles of size 4 are disjoint.*

Example 2.1. (1) *Every graph which has at most one cycle of size 4 is a graph of type S.*

(2) *Every graph, such that every two cycles of order 4 have at most one common edge in their corresponding squares, is a graph of type S.*

(3) *K_4 is a graph of type S.*

(4) *$K_{2,3}$ is not a graph of type S.*

Theorem 2.4. *Let G be a (p,q)-graph and s be the number corresponding squares of cycles of size 4. Also, let $con(G)$ be a (p,q')-graph. Then,*

(1) *If G is a graph of type S, then $M_1(G) = 2q + 2q' + 4s$.*

(2) *If G is a any graph, $M_1(G) \leq 2q + 2q' + 4s$.*

(3) *If G has no cycles of size 4, then $M_1(G) = 2q + 2q'$.*

Proof. (1) Let $V(G) = \{v_1, v_2, \ldots, v_p\}$ and $A = [a_{ij}]_{p\times p}$ be the adjacency matrix of graph G. Since $d(v_i) = \sum_{k=1}^{p} a_{ik}$, we get

$$\begin{aligned} M_1(G) &= \sum_{v_i \in V(G)} d(v_i)^2 = \sum_{v_i\in V(G)} \left(\sum_{k=1}^{p} a_{ik}\right)^2 \\ &= \sum_{v_i\in V(G)} \sum_{k=1}^{p} a_{ik}^2 + 2 \sum_{v_i\in V(G)} \sum_{1\leq k\leq k'\leq p} a_{ik}\, a_{ik'} \\ &= \sum_{v_i\in V(G)} \sum_{k=1}^{p} a_{ik} + 2 \sum_{v_i\in V(G)} \sum_{1\leq k\leq k'\leq p} a_{ik}\, a_{ik'} \\ &= \sum_{v_i\in V(G)} d(v_i) + 2 \sum_{v_i\in V(G)} \sum_{1\leq k\leq k'\leq p} a_{ik}\, a_{ik'}\,. \end{aligned}$$

Since $a_{ik}\, a_{ik'} = 0 \ \ or \ \ 1$. Hence it is equal with one if $a_{ik} = 1$ and $a_{ik'} = 1$. Therefore, for some $k \neq k'$ there exist $v_k, v_{k'} \in V(G)$ such that $v_i\, v_k \in E(G)$ and $v_i\, v_{k'} \in E(G)$. Hence $v_k\, v_{k'} \in E(con(G))$ and this edge appears only once, since G has no cycles of size 4. But, if G has any cycle of size 4, then this edge is appear only twice. Since every cycle of size 4 corresponds to a square and every square, have two diagonals. Thus $\sum_{v\in V(G)} \sum_{1\leq k\leq k'\leq p} a_{ik}\, a_{ik'} = q' + 2s$. Therefore, $M_1(G) = 2q + 2q' + 4s$.

(2) The proof of this part is similar to part (1) but since edge $v_i\, v_k \in E(G)$ appears at most twice, hence $M_1(G) \leq 2q + 2q' + 4s$.

(3) It directly follows from part (1).

□

Corollary 2.1. *Let G be a tree. Then,*

$$M_1(G) = 2q + 2q'\,.$$

Corollary 2.2. *Let G be a (p,q)-graph and s be the number corresponding squares of cycles of size 4. Also, let $con(G)$ be a (p,q')-graph. In this case, if G is graph of type S, then $q' = \frac{1}{2}M_1(G) - q - 2s$.*

The following theorem is well known.

Theorem 2.5. *Let G be a graph with vertices labeled $V(G) = \{v_1, v_2, \ldots, v_n\}$ and let A be its corresponding adjacency matrix. For any positive integer k, the (i, j) entry $a_{ij}^{(k)}$ of $A^k = [a_{ij}^{(k)}]$ is equal to the number of walks from v_i to v_j that use exactly k edges.*

Remark 2.1. *For a simple (p,q)-graph, we have*

(1) *For every $i \neq j$ entry $a_{ij}^{(2)}$ of $A^2 = [a_{ij}^{(2)}]$ is equal to the number paths of order 2 from v_i to v_j.*

(2) $tr A^2 = \sum_{i=1}^{p} a_{ii}^{(2)} = 2q$.

(3) $\sum_{\substack{1\leq i,j\leq p \\ i\neq j}} a_{ij}^{(2)}$ *is equal to the number paths of order 2 from u to v for every disjoint $u, v \in V(G)$.*

Lemma 2.2. *Let $A = [a_{ij}]$ be the adjacency matrix of the graph G. Define $B = [b_{ij}]$ such that*
$$b_{ij} = \begin{cases} 1 & a_{ij}^{(2)} \neq 0 \; for \; i \neq j \\ 0 & otherwise \end{cases}$$

Then B is the adjacency matrix of $con(G)$. In particular, if G has no cycles of size 4, then $B = A^2 - C$ where C is degree matrix of G.

Proof. For every $v_iv_j \in E(con(G))$, it is enough that $b_{ij} = 1$ and otherwise it is equal zero. By definition from b_{ij} we have b_{ij} is equal one if $a_{ij}^{(2)} \neq 0$ for $i \neq j$. This implies that $a_{ij}^{(2)} = |N_G(v_i) \cap N_G(v_j)| \neq 0$, that is $N_G(v_i) \cap N_G(v_j) \neq \emptyset$. Hence $v_iv_j \in E(con(G))$. In particular, if G has no cycle of size 4, then $a_{ij}^{(2)} = 1 \;\, or \;\, 0$ for $i \neq j$. Otherwise, we get $|N_G(v_i) \cap N_G(v_j)| \geq 2$. Then G has a cycle of size 4, which is a contradiction. Thus, $B = A^2 - C$. □

Remark 2.2. *For a (p,q)-graph, let r be the number paths of order 3 from u to v for every $\{u,v\} \subseteq V(G)$, and t_i the number of cycles of size 3 containing the vertex v_i . Then,*

(1) *For every $i \neq j$, the entry $a_{ij}^{(3)}$ of $A^3 = [a_{ij}^{(3)}]$ is equal to the number of walks from v_i to v_j of order 3.*

(2) $tr A^3 = \sum_{i=1}^{p} a_{ii}^{(3)} = \sum_{i=1}^{p} 2t_i = 6\,\ell$, *where ℓ is the number of triangle.*

(3) *Let r_{ij} be the number of paths from v_i to v_j of order 3, then*

$$a_{ij}^{(3)} = \begin{cases} d(v_i) + d(v_j) - 1 + r_{ij} & v_iv_j \in E(G) \\ r_{ij} & v_iv_j \notin E(G) \\ 2t_i & i = j \end{cases}$$

(4)

$$\begin{aligned} \sum_{1 \leq i,j \leq p} a_{ij}^{(3)} &= 6\,\ell + 2\left(\sum_{v_iv_j \in E(G)} (d(v_i) + d(v_j) - 1 + r_{ij})\right) + 2\left(\sum_{v_iv_j \notin E(G)} r_{ij}\right) \\ &= 6\,\ell + 2M_1(G) - 2q + 2r\,. \end{aligned}$$

Theorem 2.6. *Let G be a (p,q)-graph and $con(G)$ a (p,q')-graph. Also, let $A = [a_{ij}]_{p \times p}$ and $B = [b_{ij}]_{p \times p}$ be the adjacency matrices of G and $con(G)$, respectively.*

Then,

(1) $\Xi_1(G) = \sum_{1 \leq i,j \leq p} c_{ij}$ *where $A\,B = [c_{ij}]_{p \times p}$.*

(2) *If G has no cycle of size 4, then $\Xi_1(G)$ is equal to the number of paths of order 2 or 3 from u to v for every $u,v \in V(G)$.*

(3) *If G has no cycle of size 3 and 4, then $\Xi_1(G) = 2|L| + 2|L'|$, where $L = \{\{u,v\} \subseteq V(G) \,|\, d(u,v) = 2\}$ and $L' = \{\{u,v\} \subseteq V(G) |\; d(u,v) = 3\}$.*

Proof. (1) Let $V(G) = \{v_1, v_2, \ldots, v_p\}$. Since $d_G(v_k) = \sum_{i=1}^{p} a_{ik}$ and $d_{con(G)}(v_k) = \sum_{j=1}^{p} b_{kj}$, we have

$$\begin{aligned} \sum_{1 \leq i,j \leq p} c_{ij} &= \sum_{1 \leq i,j \leq p} \sum_{k=1}^{p} a_{ik}\, b_{kj} = \sum_{k=1}^{p} \sum_{1 \leq i,j \leq p} a_{ik}\, b_{kj} \\ &= \sum_{k=1}^{p} \left(\sum_{i=1}^{p} a_{ik}\right) \left(\sum_{j=1}^{p} b_{kj}\right) = \sum_{k=1}^{p} d(v_k)\, d_{con(G)}(v_k) \\ &= \sum_{v \in V(G)} d_G(v)\, d_{con(G)}(v) = \Xi_1(G)\,. \end{aligned}$$

(2)

$$\begin{aligned}\sum_{1\le i,j\le p} c_{ij} = \Xi_1(G) &= \sum_{v\in V(G)} d_G(v)\, d_{con(G)}(v) = \sum_{v_i\in V(G)} \sum_{k=1}^{p} a_{ik} \sum_{k'=1}^{p} b_{ik'} \\ &= \sum_{v_i\in V(G)} \sum_{1\le k,k'\le p} a_{ik}\, b_{ik'}\,.\end{aligned}$$

For $a_{ik} = 1$ and $b_{ik'} = 1$ we have $v_iv_k \in E(G)$ and $v_iv_{k'} \in E(con(G))$, respectively. Thus we have three cases:

case(1): For $k = k'$ and $i \neq j$, if $v_iv_j, v_iv_k \in E(G)$, then $a_{ik}\, b_{ik'} = 1$.

case(2): For $k = k'$ and $i = j$, if $av_i, av_k, v_iv_k \in E(G)$, then $a_{ik}\, b_{ik'} = 1$.

case(3): For $k \neq k'$ and $i \neq j$ if $v_iv_k, v_iv_j, v_jv_{k'} \in E(G)$,, then $a_{ik}\, b_{ik'} = 1$.

Since the graph G has no cycles of size 4, in every of the above cases only once appear. Thus, $\sum_{1\le i,j\le p} c_{ij} = \Xi_1(G)$ is the number all of paths of order 2 or 3 from u to v for every $u, v \in V(G)$.

(3) This part can be obtained easily from part (2).

□

Theorem 2.7. *Let G be a (p,q)-graph. Then, $2\,M_2(G) - 2\,M_1(G) + 2q = r + 6\,\ell$ where $r =$ the number of all paths of order 3 from u to v for every $\{u,v\} \subseteq V(G)$ and ℓ is the number of triangles.*

Proof. Let $V(G) = \{v_1, v_2, \ldots, v_p\}$ then

$$\begin{aligned}M_2(G) &= \sum_{v_iv_j\in E(G)} d(v_i)\, d(v_j) = \sum_{v_iv_j\in E(G)} \sum_{k=1}^{p} a_{ik} \sum_{k'=1}^{p} a_{k'j} \\ &= \sum_{k=1}^{p} \sum_{k'=1}^{p} \sum_{v_iv_j\in E(G)} a_{ik}\, a_{k'j} = \frac{1}{2} \sum_{\{k,k'\}\subseteq V(G)} \left(\sum_{v_iv_j\in E(G)} a_{ik}\, a_{k'j} \right).\end{aligned}$$

Since $v_iv_j \in E(G)$, if $a_{ik}a_{k'j} = 1$, then $v_iv_j \in E(G)$, $a_{ik} = 1$, and $a_{k'j} = 1$. In this case, there exist vertices v_k and $v_{k'}$ such that we have following four cases:

case(1): If $k' = i$ and $v_iv_j, v_iv_k \in E(G)$, then $a_{ik}\, a_{k'j} = 1$.

case(2): If $k = j$ and $v_iv_j, v_jv_{k'} \in E(G)$, then $a_{ik}\, a_{k'j} = 1$.

case(3): If $k = k'$ and $v_iv_k, v_iv_j \in E(G)$, then $a_{ik}\, a_{k'j} = 1$.

case(4): If $k \neq k'$ and $v_iv_k, v_iv_j, v_jv_{k'} \in E(G)$, then $a_{ik}\, a_{k'j} = 1$.

Thus, in every above cases determine all of the number of walks of order 3. Thus, by Remark 2.2,

$$M_2(G) = \frac{1}{2} \sum_{1\le i,j\le p} a_{ij}^{(3)} = \frac{1}{2}(6\,\ell + 2\,M_1(G) - 2q + 2\,r) = 3\ell + M_1(G) - q + r\,.$$

□

Example 2.2.

Let G be a $(4,4)$-graph with $V(G) = \{a,b,c,d\}$ and $E(G) = \{ab, ac, bc, bd\}$. Then, $M_2(G) = 19$, $M_1(G) = 18$ where $q = 4, r = 2$ and $\ell = 1$. Then

$$19 = M_2(G) = 3 + 18 - 4 + 2 = 3\ell + M_1(G) - q + r\,.$$

3 Conclusion

In this paper, we defined the Zagreb indices of congraphs and investigate the degree–related properties of the congraphs and the Zagreb indices of congraphs. Moreover, we obtained relations between Zagreb indices of parent graphs and graph invariants such as number of edges of parent graph, number of edges of congraph, the number of all paths of order 3, number of triangles and the number of cycles of size 4 by using adjacency matrix of the parent graph.

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