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θ-local function and ψ_θ-operator

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Abstract

In this paper, we introduce the notion of a θ-local function and investigate some of their properties. Also, we define two operators $()^{*\theta}$ and ψ_θ in an ideal topological space.

Keywords: θ-local function, $()^{*\theta}$-operator, θ-compatible and ψ_θ-operator.

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1 Introduction

In 1968, Velicko[22] introduced the notions of θ-open subsets, θ-closed subsets and θ-closure, for the sake of studying the important class of H-closed spaces in terms of arbitrary filterbases. In 1990, Jankovic and Hamlett[7,8] defined the concept of $\mathcal{I}$-open set via local function which was given by Vaidyanathaswamy. O.Njastad[16,17] introduced the concept of compatible ideals in 1966. This ideal was also called as supercompact by Vaidyanathaswamy[20,21]. In an ideal topological space, the local function was introduced by Kuratowski[11]. After that so many mathematicians like Hayashi [7], Natkaniec[15] and Modak and Bandyopadhyay[14] have studied this field and proved some new results in an ideal topological spaces. In 2009, Jeong Gi Kang and Chang Su Kim [10] defined pre-local function, semi-local function and α -local function. In 2011, Shyamapada Modak [16] introduced δ-local function and an operator ψ_δ in the ideal topological spaces. In 2013, Arokia Rani and Nithya[2] introduced precompatible ideals, Al-Omari and Noiri[1] defined the local closure function and an operator ψ_Γ and K. Bhavani[3,4] introduced g-local function and ψ_g -operator in the ideal topological spaces.

In this paper, we introduce the notion of a θ-local function and investigate some of their properties. We also introduce two operators $()^{*\theta}$ and ψ_θ a $*\theta$-closure operator in lines with kuratowski. Also, we discuss θ-compatibility of topological spaces.

2 Preliminaries

Let (X, τ) be a topological space with no separation properties assumed. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure of A and the interior of A respectively. (X, τ) and (Y, σ) will be replaced by X and Y if there is no chance of confusion. A subset A of X is said to be semi open[9] (resp. pre open[10] and α -open[13] if A$\subset$ cl(int(A)) (resp. A $\subset$ int(cl(A)) and A $\subset$ int(cl(int(A)))). The complement of semi open (resp. pre open and α -open) is called semi closed (resp. pre closed and α-closed).

A set A is said to be θ-open[1] if every point of A has an open neighborhood whose closure is contained in A. It is very well known that the family of all θ-open subsets of (X, τ) are topologies on X which we shall denote by τ^θ. From the definitions it follows immediately that $\tau^\theta \subset \tau$. A space (X, τ) is regular if and only if $\tau^\theta = \tau$. A point x $\in$ X is said to be in the θ-closure of a subset A$\subseteq X$[6] if for each open neighbourhood U of x

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we have cl(U)$\cap$ A $\neq \phi$. We shall denote θ-closure by $cl_\theta(A)$. A subset $A \subseteq X$ is called θ-closed if A = $cl_\theta(A)$. In general, the θ- closure of a given set need not be a θ-closed set. But it is always closed. A point x $\in$ A is said to be a θ-limit point of A[5] in X if for each θ-open set U containing x, such that U $\cap$ (A - $\{x\}$)$\neq \phi$ .The set all θ-limit points of A is called a θ-derived set of A and is denoted by $D_\theta(A)$.

An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies (i) A $\in \mathcal{I}$ and B$\subseteq$ A implies B$\in \mathcal{I}$ and (ii) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $(A \cup B) \in \mathcal{I}$. A topological space (X, τ) with an ideal $\mathcal{I}$ on X is called an ideal topological space and is denoted by $(X, \tau, \mathcal{I})$. For a subset A $\subseteq X$, $A^*(\mathcal{I})$ = $\{x \in X : U \cap A \notin \mathcal{I}$ for every $U \in \tau(x)\}$ is called the local function of A with respect to $\mathcal{I}$ and τ[4]. We simply write A* in case there is no chance for confusion. A Kuratowski[11] closure operator cl*(.) for a topology $\tau^*(\mathcal{I})$ called the τ^*-topology finer than τ is defined $cl^*(A)$ = $A \cup A^*$. A subset A of an ideal space $(X, \tau, \mathcal{I})$ is τ^*-closed [18] (resp. $*$ -dense in itself [18], $*$-perfect [18]) if $A^* \subset A$ (resp. $A \subset A^*$, $A = A^*$). Clearly, A is $*$-perfect if and only if A is τ^*-closed and $*$-dense in itself. An ideal $\mathcal{I}$ in a space (X, τ) is said to be compatible with respect to τ[9], denoted by $\mathcal{I} \sim \tau$, if for every subset A of X and for each $x \in A$, there exists a neighborhood U of x such that $U \cap A \in \mathcal{I}$, then $A \in \mathcal{I}$. Let (X, τ) be a topological space with $\mathcal{I}$ an ideal on X, then τ is pre-compatible[2] with $\mathcal{I}$, if for every $A \subseteq X$, and for every $x \in A$, there exists a $U \in PO(x)$ such that $U \cap A \in \mathcal{I}$, then $A \in \mathcal{I}$ and is denoted by $\mathcal{I} \sim_P \tau$. An operator[8] $\psi : \wp(X) \to \tau$ is defined as: $\psi(A)$ = $\{x \in X$: there exists an open set O_x such that $O_x - A \in \mathcal{I}\}$, for every $A \in \wp(X)$. Its equivalent definition is $\psi(A)$ = $X - (X - A)^*$. Let A be a subset of an ideal topological space $(X, \tau, \mathcal{I})$. Then the set (1) ${A^*}_p(\mathcal{I}, \tau)$ = $\{x \in X : U \cap A \notin \mathcal{I}$ for each $U \in \tau^p(x)\}$ is called the pre-local function with respect to $\mathcal{I}$ and τ . (2) ${A^*}_s(\mathcal{I}, \tau)$ = $\{x \in X : U \cap A \notin \mathcal{I}$ for each $U \in \tau^s(x)\}$ is called the semi-local function with respect to $\mathcal{I}$ and τ . (3) ${A^*}_\alpha(\mathcal{I}, \tau)$ = $\{x \in X : U \cup A \notin \mathcal{I}$ for each $U \in \tau^\alpha(x)\}$ is called the α-local function with respect to $\mathcal{I}$ and τ . Al-Omari and Noiri[1] defined the local closure function and an operator ψ_Γ in an ideal topological spaces as follows:$\Gamma(A)(\mathcal{I}, \tau)$ = $\{x \in X : A \cap cl(U) \notin \mathcal{I}$ for every $U \in \tau(x)\}$ and $\psi_\Gamma(A)$ = X - $\Gamma(X - A)$ where $\psi : \wp(X) \to \tau$. K. Bhavani[3,4] introduced g-local function and ψ_g-operator in the ideal topological spaces as: : $A^*(\mathcal{I}, \tau_g) = \{x \in X : U \cap A \notin \mathcal{I}$ for every g-open set U containing $x\}$ and $\psi_g(A) = \{x \in X$: there exists a g-open set U_x containing x such that $U_x - A \in \mathcal{I}\}$ for every $A \in \wp(X)$ where $\psi_g : \wp(X) \to \wp(X)$.

Result 2.1 Let A be a subset of a topological space (X, τ). If $A \in \tau^\theta$, then $cl_\theta(A)$ = A

Lemma 2.1. *[1]. Let A be a subset of a topological space (X, τ). Then*

1. *if A is open, then cl(A) = $cl_\theta(A)$*
2. *if A is closed, then int(A) = $int_\theta(A)$*

Lemma 2.2. *If $(X, \tau, \mathcal{I})$ is an ideal topological space, then $\mathcal{I}$ is codense[18] if and only in $A \subset A^*$ for every open set A of X.*

Lemma 2.3. *[18]. If $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subset X$. If $A \subset A^*$, then A^* = $cl(A^*)$ = $cl(A)$ = $cl^*(A)$.*

3 The Operator() $^{*\theta}$

In this section we shall introduce an operator $()^{*\theta}$ and discuss various properties of this operator.

Definition 3.1. *Let A be a subset of an ideal topological space $(X, \tau, \mathcal{I})$. Then, the θ-local function of $\mathcal{I}$ on X is defined as $A^{*\theta}(\mathcal{I}, \tau)$ = $\{x \in X : U_x \cap A \notin \mathcal{I}$ for every $U_x \in \theta O(X, x)\}$ with respect to $\mathcal{I}$ and τ and is denoted as $A^{*\theta}$ for $A^{*\theta}(\mathcal{I}, \tau)$.*

Lemma 3.1. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. Then every subset A of X,*

(1) ${A^*}_p(\mathcal{I}, \tau) \subseteq A^{*\theta}(\mathcal{I}, \tau)$.

(2) ${A^*}_s(\mathcal{I}, \tau) \subseteq A^{*\theta}(\mathcal{I}, \tau)$.

(3) ${A^*}_\alpha(\mathcal{I}, \tau) \subseteq A^{*\theta}(\mathcal{I}, \tau)$.

(4) $\Gamma(A)(\mathcal{I}, \tau) \subseteq A^{*\theta}(\mathcal{I}, \tau)$.

(5) ${A^*}_g(\mathcal{I}, \tau) \subseteq A^{*\theta}(\mathcal{I}, \tau)$.

Proof. Straight forward. □

Remark 3.1. *The converse of the Lemma 3.1 need not be true as seen in the following examples.*

Example 3.1. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space with $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, \{b, c\}, X\}$ and $I = \{\phi, \{c\}\}$. If $A = \{a, b\}$, then $A^{*\theta} = \{a, b, c\} \not\subset \{a, b\} = A^*{}_p$.*

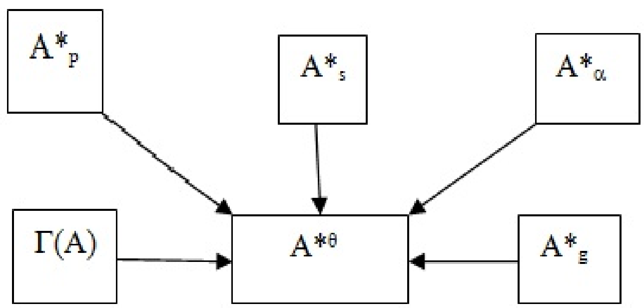
Example 3.2. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space with $X = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, X\}$ and $\mathcal{I} = \{\phi, \{b\}, \{c\}, \{b, c\}\}$. If $A = \{a, b, c\}$, then $A^{*\theta} = \{a, c, d\} \not\subset \{a, d\} = A^*{}_s$.*

Example 3.3. *In example 3.2, if $A = \{b, c, d\}$ then, $A^{*\theta} = \{a, c, d\} \not\subset \{d\} = A^*{}_\alpha$.*

Example 3.4. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space with $X = \{a, b, c, d\}$, $\tau = \{\phi, \{d\}, \{a, c\}, \{a, c, d\}, X\}$ and $I = \{\phi, \{c\}\}$. If $A = \{a\}$, then $A^{*\theta} = \{a, b, c, d\} \not\subset \{a, b, c\} = \Gamma(A)$.*

Example 3.5. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space with $X = \{a, b, c, d, e\}$, $\tau = \{\phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}, X\}$ and $\mathcal{I} = \{\phi, \{c\}, \{d\}, \{c, d\}\}$. If $A = \{a, b, c, d\}\}$, then $A^{*\theta} = X \not\subset \{a, b\} = A^*{}_g$.*

Remark 3.2. *The above discussions are summarized in the following diagram.*



Remark 3.3. *$A \subset A^{*\theta}$ and $A^{*\theta} \subset A$ are not true in general as shown in the following example.*

Example 3.6. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space with $X = \{a, b, c, d, e\}$, $\tau = \{\phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}, X\}$ and $\mathcal{I} = \{\phi, \{b\}, \{c\}, \{b, c\}\}$. (i) If $A = \{a, b\}$, then $A^{*\theta} = \{a\}$. Therefore, $A \not\subset A^{*\theta}$. (ii) If $A = \{a, b, d\}$, then $A^{*\theta} = X$. Therefore, $A^{*\theta} \not\subset A$.*

Remark 3.4. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subset X$. Then, $cl^{*\theta}(A) = A \cup A^{*\theta}$ is a $*\theta$-closure operator.*

Remark 3.5. *Open sets of $\tau^{*\theta}$. Let (X, τ) be a topological space and $\mathcal{I}$ an ideal on X and observe that A is $\tau^{*\theta}$-closed iff $\tau^{*\theta} \subset A$. Now we have $U \in \tau^{*\theta}$ iff $X - U$ is $\tau^{*\theta}$-closed iff $(X - U)^{*\theta} \subseteq X - U$ iff $U \subseteq X - (X - U)^{*\theta}$. Therefore, $x \in U \to x \notin (X - U)^{*\theta} \to$ there exists a θ-neighbourhood V such that $V \cap (X - U) \in \mathcal{I}$. Now let $I = V \cap (X - U)$ and we have $x \in V - I \subseteq U$, where $I \in \mathcal{I}$. We shall denote $\beta(\mathcal{I}, \tau^\theta) = \{V - I : V \in \tau^\theta, I \in \mathcal{I}\}$.*

Theorem 3.1. *Let (X, τ) be a topological space and $\mathcal{I}$ an ideal on X. Then β is a basis for $\tau^{*\theta}$.*

Lemma 3.2. *If $(X, \tau, \mathcal{I})$ is an ideal topological space and $A \subset X$. If $A \subset A^{*\theta}$, then $A^{*\theta} = cl_\theta(A) = cl^{*\theta}(A)$.*

Proof. Always $cl^{*\theta}(A) \subset cl_\theta(A)$. Let $x \notin cl^{*\theta}(A)$. Then, there exists a $\tau^{*\theta}$-open set G containing x such that $G \cap A = \phi$. By Remark 3.5, there exists $V \in \tau^\theta$ and $I \in \mathcal{I}$ such that $x \in V - I \subset G$. Since $G \cap A = \phi \Rightarrow (V - I) \cap A = \phi \Rightarrow (V \cap A) - I = \phi \Rightarrow ((V \cap A) - I)^{*\theta} = \phi^{*\theta} \Rightarrow (V \cap A)^{*\theta} - I^{*\theta} = \phi \Rightarrow (V \cap A)^{*\theta} = \phi \Rightarrow V \cap A^{*\theta} = \phi \Rightarrow x \notin cl_\theta(A)$. Therefore, $cl_\theta(A) \subset cl^{*\theta}(A)$. Hence $cl^{*\theta}(A) = cl_\theta(A) - - - (1)$. We know that $cl^{*\theta}(A) = A \cup A^{*\theta} = A^{*\theta}$—(2), since $A \subset A^{*\theta}$. From (1) and (2), $A^{*\theta} = cl_\theta(A) = cl^{*\theta}(A)$. □

Definition 3.2. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subset X$. If $A \subset A^{*\theta}$, then A is said to be $*\theta$-dense in itself.*

Definition 3.3. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subset X$. If $A^{*\theta} \subset A$, then A is said to be $*\theta$-closed.*

Remark 3.6. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subset X$. Then, $\tau^{*\theta} = \{X - A : cl^{*\theta}(A) = A\}$.*

Proposition 3.1. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subset X$. Then A is $\tau^{*\theta}$-closed if and only if $A^{*\theta} \subset A$.*

Proof. Let A be $\tau^{*\theta}$-closed. Then, A = $cl^{*\theta}(A) \Rightarrow A = A \cup A^{*\theta} \Rightarrow A^{*\theta} \subset A$. Conversely, let $A^{*\theta} \subset A$. By assumption, $A \cup A^{*\theta} = A$. i.e. $cl^{*\theta}(A) = A$. Hence, A is $\tau^{*\theta}$ -closed. □

Proposition 3.2. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. Then the following hold for every subset A of X, $cl^{*\theta}(A) \subset cl_\theta(A)$;*

Proof. Let $x \in cl^{*\theta}(A)$. Then, $x \in A$ or $x \in A^{*\theta}$. If $x \in A^{*\theta}$, then there exists a θ-open set U_x containing x such that $U_x \cap A \notin \mathcal{I}$. That is $U_x \cap A \neq \phi$. This implies that $x \in cl_\theta(A)$. Thus, $cl^{*\theta}(A) \subset cl_\theta(A)$. □

Proposition 3.3. *Let $x \in cl^{*\theta}(A)$ if and only if $V \cap A \neq \phi$ for every $*\theta$-open set $V \subseteq X$.*

Properties of $()^{*\theta}$ operator

Theorem 3.2. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and let A,B be subsets of X. Then for θ-local functions the following properties hold:*

(i) $\phi^{*\theta} = \phi$.

(ii) $A \subset B$ *implies* $A^{*\theta} \subset B^{*\theta}$.

(iii) *For an another ideal* $\mathcal{J} \supset \mathcal{I}$ *on X,* $A^{*\theta}(\mathcal{J}) \subset A^{*\theta}(\mathcal{I})$.

(iv) $A^{*} \subset A^{*\theta}$.

(v) $A^{*\theta} \subset cl_\theta(A)$.

(vi) $(A^{*\theta})^{*\theta} \subset A^{*\theta}$, *if A is θ-closed.*

(vii) $A^{*\theta} \cup B^{*\theta} = (A \cup B)^{*\theta}$.

(viii) $(A \cap B)^{*\theta} \subset A^{*\theta} \cap B^{*\theta}$.

(ix) *for a θ-open set U,* $U \cap A^{*\theta} = U \cap (U \cap A)^{*\theta} \subset (U \cap A)^{*\theta}$.

(x) *For* $I \in \mathcal{I}$, $(A \cup I)^{*\theta} = A^{*\theta} = (A - I)^{*\theta}$.

(xi) $(A - B)^{*\theta} - B^{*\theta} = (A^{*\theta} - B^{*\theta}) \subset (A - B)^{*\theta}$.

(xii) $(A - A^{*\theta}) \cap (A - A^{*\theta})^{*\theta} = \phi$.

(xiii) *If* $A \in \mathcal{I}$, *then* $A^{*\theta} = \phi$.

(xiv) $A^{*\theta}(\mathcal{I} \cap \mathcal{J}) \supset A^{*\theta}(\mathcal{I}) \cup A^{*\theta}(\mathcal{J})$.

Proof. (i) From the definition of θ-local function, $\phi^{*\theta} = \phi$ is obvious.

(ii) Let $x \in A^{*\theta}$. Then for every θ-open set U_x containing x, $U_x \cap A \notin \mathcal{I}$. Since $A \subset B$ implies that $U_x \cap A \subset U_x \cap B \notin \mathcal{I}$. Therefore, $U_x \cap B \notin \mathcal{I}$. This implies that $x \in B^{*\theta}$. Hence, $A^{*\theta} \subset B^{*\theta}$.

(iii) Let $x \in A^{*\theta}(\mathcal{J})$. Then for every θ-open set U_x containing x, such that $U_x \cap A \notin \mathcal{J}$. This implies that $U_x \cap A \notin \mathcal{I}$, since $\mathcal{I} \subset \mathcal{J}$. So, $x \in A^{*\theta}(\mathcal{I})$. Hence, $A^{*\theta}(\mathcal{J}) \subset A^{*\theta}(\mathcal{I})$.

(iv) Let $x \in A^{*}$. We assert that $x \in A^{*\theta}$. If not, then there is a θ-open set U_x containing x such that $U_x \cap A \in \mathcal{I}$. Since every θ-open is open, U_x is open and since, $U_x \cap A \in \mathcal{I}$ contradicts the assumption $x \in A^{*}$. Therefore, $x \in A^{*\theta}$. This implies that $A^{*} \subset A^{*\theta}$.

(v) Let $x \in A^{*\theta}$. Then for every θ-open set U_x containing x, $U_x \cap A \notin \mathcal{I}$. Since every θ-open is open, U_x is open. This implies that $U_x \cap A \neq \phi$ for every θ-open set containing x. Hence, $x \in cl_\theta(A)$.

(vi) From (v) $A^{*\theta} \subset cl_\theta(A)$. $(A^{*\theta})^{*\theta} \subset (cl_\theta(A))^{*\theta}$. But $A = cl_\theta(A)$, since A is θ-closed. This implies that $(A^{*\theta})^{*\theta} \subset A^{*\theta}$.

(vii) Since $A \subset A \cup B$ and $B \subset A \cup B$. Then from (ii) $A^{*\theta} \subset (A \cup B)^{*\theta}$ and $B^{*\theta} \subset (A \cup B)^{*\theta}$. Hence, $A^{*\theta} \cup B^{*\theta} \subset (A \cup B)^{*\theta}$. Conversely suppose that $x \notin A^{*\theta} \cup B^{*\theta}$. Then, $x \notin A^{*\theta}$ and $x \notin B^{*\theta}$. If $x \notin A^{*\theta}$, then there exists θ-open set U_x containing x such that $U_x \cap A \in \mathcal{I}$. Similarly since $x \notin B^{*\theta}$, there exists θ-open set V_x containing x such that $V_x \cap A \in \mathcal{I}$. Then by the hereditary property of ideal, $A \cap (U_x \cap V_x) \in \mathcal{I}$ and $B \cap (U_x \cap V_x) \in \mathcal{I}$. Again, by the finite additivity of the ideal, $(A \cup B) \cap (U_x \cap V_x) \in \mathcal{I}$. Hence, $x \notin (A \cup B)^{*\theta}$. So, $(A \cup B)^{*\theta} \subset A^{*\theta} \cup B^{*\theta}$. Hence $A^{*\theta} \cup B^{*\theta} = (A \cup B)^{*\theta}$.

(viii) Since $A \cap B \subset A$ and $A \cap B \subset B$, from (2), $(A \cap B)^{*\theta} \subset A^{*\theta}$ and $(A \cap B)^{*\theta} \subset B^{*\theta}$. Hence, $(A \cap B)^{*\theta} \subset A^{*\theta} \cap B^{*\theta}$.

(ix) Let $x \in U \cap A^{*\theta}$. Let V_x be a θ-open set containing x, then $A \cap (U \cap V_x) \notin \mathcal{I}$, since $x \in A^{*\theta}$ and $U \cap V_x$ is a θ-open set containing x . Hence, $x \in (U \cap A)^{*\theta}$. Therefore, $U \cap A^{*\theta} \subset (U \cap A)^{*\theta}$.
Therefore, $U \cap A^{*\theta} \subset U \cap (U \cap A)^{*\theta} - - - -(1)$. Again for $U \cap A \subset A$, $(U \cap A)^{*\theta} \subset A^{*\theta}$. So, $U \cap (U \cap A)^{*\theta} \subset U \cap A^{*\theta} - - - -(2)$. From (1) and (2) we have $U \cap A^{*\theta} = U \cap (U \cap A)^{*\theta}$. Hence, $U \in \tau^{\theta}$, $U \cap A^{*\theta} = U \cap (U \cap A)^{*\theta} \subset (U \cap A)^{*\theta}$.

(x) Since $A \subset A \cup I$, $A^{*\theta} \subset (A \cup I)^{*\theta}[by(i)] - - - - - (1)$. Let $x \in (A \cup I)^{*\theta}$. Then for every θ-open set U_x containing x, $U_x \cap (A \cup I) \notin \mathcal{I}$. Since $U_x \cap I \in \mathcal{I}$, it follows that $U_x \cap A \notin \mathcal{I}$. Hence $x \in A^{*\theta}$ which implies that $(A \cup I)^{*\theta} \subset A^{*\theta} - - - - - (2)$. From (1) and (2), we have $(A \cup I)^{*\theta} = A^{*\theta} - - - - - (3)$. Since $(A - I) \subset A$, then $(A - I)^{*\theta} \subset A^{*\theta} - - - - - (4)$. Now, for reverse inclusion, let $x \in A^{*\theta}$. We claim that $x \in (A - I)^{*\theta}$. If not, then there is some θ-open set U_x containing x such that $U_x \cap (A - I) \in \mathcal{I}$. Since, $I \in \mathcal{I}$, $I \cup (U_x \cap (A - I)) \in \mathcal{I}$. This implies $I \cup (U_x \cap A) \in \mathcal{I}$. So, $U_x \cap A \in \mathcal{I}$, a contradiction to the fact that $x \in A^{*\theta}$. Hence, $A^{*\theta} \subset (A - I)^{*\theta} - - - - - - -(5)$. From (4) and (5), we have, $A^{*\theta} = (A - I)^{*\theta}$. Again from (3) and (6) we have $(A \cup I)^{*\theta} = A^{*\theta} = (A - I)^{*\theta}$.

(xi) Let $x \in A^{*\theta} - B^{*\theta}$. Then, $x \in A^{*\theta}$ and $x \notin B^{*\theta}$. This implies that $U_x \cap A \notin \mathcal{I}$, for every θ-open set U_x containing x and $V_x \cap B \in \mathcal{I}$, for some θ-open set V_x containing x. Hence $V_x \cap A \notin \mathcal{I}$ and $V_x \cap B \in \mathcal{I}$. Suppose that $(A - B) \cap V_x \in \mathcal{I}$. Since $((A - B) \cap V_x) \cup (B \cap V_x) = (A \cup B) \cap V_x$, by finite additivity property of ideal, $(A \cup B) \cap V_x \in \mathcal{I}$. Since $A \cap V_x \subset (A \cup B) \cap V_x$, $A \cap V_x \in \mathcal{I}$, which is a contradiction to the fact that $V_x \cap A \notin \mathcal{I}$. Therefore, $(A - B) \cap V_x \notin \mathcal{I}$ and so, $x \in (A - B)^{*\theta} - - - (1)$. Therefore, $A^{*\theta} - B^{*\theta} \subset (A - B)^{*\theta} - - - -(2)$.
Also, $x \notin B^{*\theta}$ implies that $x \in (A - B)^{*\theta} - B^{*\theta}$. Therefore, $A^{*\theta} - B^{*\theta} \subset (A - B)^{*\theta} - B^{*\theta} - - - -(3)$. Let $x \in (A - B)^{*\theta} - B^{*\theta}$. Then $x \in (A - B)^{*\theta}$ and $x \notin B^{*\theta}$. If $x \in (A - B)^{*\theta}$, then for every θ-open set U_x containing x such that $(A - B) \cap U_x \notin \mathcal{I}$. Suppose that $x \notin A^{*\theta}$, then there is some θ-open set V_x containing x, $A \cap V_x \in \mathcal{I}$. Since, $x \notin B^{*\theta}$, then there is some θ-open set W_x containing x, such that $B \cap W_x \in \mathcal{I}$. Since $((A - B) \cap V_x) \cup (B \cap V_x) = (A \cup B) \cap V_x = (A \cap V_x) \cup (B \cap V_x)$ by finite additive property of the ideal, $(A \cup B) \cap V_x \in \mathcal{I}$. Since $(A - B) \cap V_x \subset (A \cup B) \cap V_x$, $(A - B) \cap V_x \in \mathcal{I}$ which is a contradiction. Therefore, $A \cap V_x \notin \mathcal{I}$, $x \in A^{*\theta}$ and $x \notin B^{*\theta}$. Therefore, $x \in A^{*\theta} - B^{*\theta}$. Thus $(A - B)^{*\theta} - B^{*\theta} \subset A^{*\theta} - B^{*\theta} - - - (4)$. From (3) and (4), we have $(A^{*\theta} - B^{*\theta}) = (A - B)^{*\theta} - B^{*\theta}$. Using (2), we have $(A - B)^{*\theta} - B^{*\theta} = (A^{*\theta} - B^{*\theta}) \subset (A - B)^{*\theta}$.

(xii) Since $A - A^{*\theta} \subset X - A^{*\theta}$. So, $(A - A^{*\theta}) \cap A^{*\theta} = \phi$. Since $(A - A^{*\theta}) \subset A$, $(A - A^{*\theta})^{*\theta} \subset A^{*\theta}$. It follows that $(A - A^{*\theta}) \cap (A - A^{*\theta})^{*\theta} = \phi$.

(xiii) Suppose that $x \in A^{*\theta}$. Then, there exists some θ-open set containing x such that $U_x \cap A \notin \mathcal{I}$. But, since $A \in \mathcal{I}$, $U_x \cap A \in \mathcal{I}$ for every $U_x \in \tau^{\theta}$. This is a contradiction. Hence, $A^{*\theta} = \phi$.

□

Remark 3.7. *In Theorem 3.2, the reverse inclusions of (iii), (viii) are not valid as in the following example.*

Example 3.7. *Let* $X = \{a, b, c, d\}$ *with* $\tau = \{\phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}, X\}$, $\mathcal{J} = \{\phi, \{b\}, \{c\}, \{b, c\}\}$ *and* $\mathcal{I} = \{\phi, \{c\}\}$.

(1) Let $A = \{a, b\}$. *Then,* $A^{*\theta}(\mathcal{I}) = \{a, b, c, d\} \not\subset \{a\} = A^{*\theta}(\mathcal{J})$.

(2) Let $A = \{a, b, c, d\}$, $A^{*\theta} = X$, $B = \{a, b, c, e\}$, $B^{*\theta} = X$, $A \cap B = \{a, b, c\}$, $(A \cap B)^{*\theta} = \{a\}$. *Therefore* $A^{*\theta} \cap B^{*\theta} = X \not\subset \{a\} = (A \cap B)^{*\theta}$.

Proposition 3.4. *Let* $(X, \tau, \mathcal{I})$ *be an ideal topological space and* $A \subset X$ *where* $\mathcal{I} = \{\phi\}$. *Then* $A^{*\theta} = cl_{\theta}(A)$.

Proof. Let $\mathcal{I} = \{\phi\}$. We know that $cl_\theta(A) = A \cup D_\theta(A)$ where $D_\theta(A)$ is the θ-derived set of A. Let $x \in A \cup D_\theta(A)$ and let U_x be a θ-open set containing x. Then $x \in A$ or $x \in D_\theta(A)$. If $x \in A$ then $x \in U_x \cap A$ and so $U_x \cap A \neq \phi$. If $x \in D_\theta(A)$, then $\phi \neq [U_x - \{x\}] \cap A \subset U_x \cap A$ and thus $U_x \cap A \neq \phi$. Hence, $cl_\theta(A) = A \cup D_\theta(A) \subset A^{*\theta}$. By Theorem 3.2(v), $A^{*\theta} \subset cl_\theta(A)$. Therefore, $A^{*\theta} = cl_\theta(A)$. □

Proposition 3.5. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subset X$ where $\mathcal{I} = \wp(X)$. Then $A^{*\theta} = \phi$ for every $A \subset X$.*

Proof. Since $A^{*\theta} = \{x \in X : U_x \cap A \notin \wp(X)$ for every θ-open set U_x containing $x\} = \phi$. Therefore, $A^{*\theta} = \phi$ for every $A \subset X$. □

Theorem 3.3. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and let A,B be subsets of X. Then for θ-local functions the following properties hold:*

1. *$A^{*\theta} = cl_\theta(A^{*\theta}) \subseteq cl_\theta(A)$ and $A^{*\theta}$ is θ-closed.*
2. *If $A \subseteq A^{*\theta}$ and $A^{*\theta}$ is open, then, $A^{*\theta} = cl_\theta(A)$.*

Proof.
1. Always $A^{*\theta} \subseteq cl_\theta(A^{*\theta})$. Let $x \in cl_\theta(A^{*\theta})$. Then , there exists some open set U_x containing x such that $A^{*\theta} \cap U_x \neq \phi$. Therefore, there exists some $y \in A^{*\theta} \cap U_x$ and $U_x \in \tau^\theta(x)$. Since $y \in A^{*\theta}$, there exists some θ-open set V_x such that $A \cap V_x \cap U_x = A \cap V_x \notin \mathcal{I}$. Therefore, $x \in A^{*\theta}$. Hence, $A^{*\theta} = cl_\theta(A^{*\theta})$ and $A^{*\theta} = cl_\theta(A^{*\theta}) \subseteq cl_\theta(A)$ by Theorem 3.2 (v).
2. For any subset A of X, by(1) we have $A^{*\theta} = cl_\theta(A^{*\theta}) \subseteq cl_\theta(A)$. Since $A \subseteq A^{*\theta}$ and $A^{*\theta}$ is open, by Lemma 1.2, $cl_\theta(A) \subseteq cl_\theta(A^{*\theta}) = cl(A^{*\theta}) = A^{*\theta} \subseteq cl_\theta(A)$ and hence, $A^{*\theta} = cl_\theta(A)$.

□

Theorem 3.4. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. Then, $A^{*\theta} \supset A - \cup\{U \subset X : U \in \mathcal{I}\}$ for all $A \subset X$.*

Proof. Let $B = \cup\{U \subset X : U \in \mathcal{I}\}$ and let $x \in (A - B)$. Then $x \in A$ and $x \notin B$. This implies that $x \notin U$ for all $U \in \mathcal{I}$ so that $\{x\} = \{x\} \cap A \notin \mathcal{I}$ because $x \in A$. For every $G \in \tau^\theta(x)$, we have $\{x\} \cap A \subset G \cap A \notin \mathcal{I}$ by the heredity of ideal. Hence, $x \in A^{*\theta}$. □

Remark 3.8. *The converse of the theorem 3.4 need not be true as seen in the following example.*

Example 3.8. *Let $X = \{a, b, c, d\}$ with $\tau = \{\phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}, X\}$ and $I = \{\phi, \{b\}, \{c\}, \{b, c\}\}$. Let $A = \{a, b, c, d\}$. $B = \cup\{U \subset X : U \in I\} = \{b, c\}$. A - B = $\{a, d\}$. $A^{*\theta}$ =X $\not\subset \{a, d\} = A - B$.*

Theorem 3.5. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $B = \cup\{U \subset X : U \in \mathcal{I}\}$. If $B \in \mathcal{I}$ then $(A^{*\theta})^{*\theta} = A^{*\theta}$ for all $A \subset X$.*

Proof. Let A be a subset of X. Then, $(A^{*\theta})^{*\theta} \subset A^{*\theta}$ by Theorem 3.2(vi) . Furthermore, $A^{*\theta} \supset A - B$ by Theorem 3.4. It follows from Theorem 3.2(ii) that $(A^{*\theta})^{*\theta} \supset (A - B)^{*\theta}$. Since $B \in \mathcal{I}$, by Theorem 3.2 (x) implies that $(A^{*\theta})^{*\theta} \supset (A - B)^{*\theta} = A^{*\theta}$. Therefore, $(A^{*\theta})^{*\theta} = A^{*\theta}$. □

Theorem 3.6. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space in which $\tau^\theta = \wp(X)$. Then $A^{*\theta} = A - \cup\{U \subset X : U \in \mathcal{I}\}$ for all $A \subset X$.*

Proof. Let B = $A - \cup\{U \subset X : U \in \mathcal{I}\}$ and let $x \in A^{*\theta}$. Then $\{x\} \cap A \notin \mathcal{I}$ because $\{x\} \in \tau^\theta = \wp(X)$. Since ideal $\mathcal{I}$ always contains ϕ, $\{x\} \cap A \neq \phi$ and so $x \in A$. It follows that $\{x\} = \{x\} \cap A \notin \mathcal{I}$ so that $x \notin U$ for all $U \in \mathcal{I}$. Hence, $x \notin B$ and therefore, $x \in A - B$. Hence, $A^{*\theta} \subset A - B$. The reverse inclusion is obvious by Theorem 3.4. □

Remark 3.9. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space in which every member of τ is clopen. Then $A^{*\theta} = A - \cup\{U \subset X : U \in \mathcal{I}\}$ for all $A \subset X$.*

Proof. Let B = $A - \cup\{U \subset X : U \in \mathcal{I}\}$ and let $A \in \wp(X)$. Then every clopen set is θ-open. Hence $A \in \tau^\theta$, which means that $\wp(X) \subset \tau^\theta$ so that $\wp(X) = \tau^\theta$. By Theorem 3.6 $A^{*\theta} = A - B$. □

Theorem 3.7. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. Then, the following properties holds.*

1. *If $\mathcal{I} = \{\phi\}$, then $cl^{*\theta}(A) = cl_\theta(A)$.*

2. *If $\mathcal{I} = \wp(X)$, then $cl^{*\theta}(A) = A$.*
3. *If $A \in \mathcal{I}$, then $cl^{*\theta}(A) = A$.*

Proof. Obvious. □

Theorem 3.8. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and let A,B be subsets of X. Then for $*\theta$-local functions the following properties hold:*

(i) $cl^{*\theta}(\phi) = \phi$.

(ii) *If $A \subset B$, then $cl^{*\theta}(A) \subset cl^{*\theta}(B)$.*

(iii) *For an another ideal $\mathcal{J} \supseteq \mathcal{I}$ on X, $cl^{*\theta}(A, \tau, \mathcal{J}) \subset cl^{*\theta}(A, \tau, \mathcal{I})$.*

(iv) $cl^{*}(A) \subset cl^{*\theta}(A)$.

(v) $cl^{*\theta}(A) \subset cl_{\theta}(A)$.

(vi) *$cl^{*\theta}(cl^{*\theta}(A)) \subset cl^{*\theta}(A)$ if A is θ-closed.*

(vii) $cl^{*\theta}(A) \cup cl^{*\theta}(B) = cl^{*\theta}(A \cup B)$.

(viii) $cl^{*\theta}(A \cap B) \subset cl^{*\theta}(A) \cap cl^{*\theta}(B)$.

Proof. It is obvious by using Remark 3.5 and Theorem 3.7. □

Remark 3.10. *In Theorem 3.8, The reverse inclusions of (ii), (iv), (v) and the converse of (iii) and (viii) are not valid as seen in the following examples.*

Example 3.9. *(iii) Let $X = \{a, b, c, d\}$ with $\tau = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, X\}$ and $\mathcal{I} = \{\phi, \{a\}\}$, $\mathcal{J} = \{\phi, \{b\}, \{c\}, \{b, c\}\}$. Let $A = \{a, d\}$, $cl^{*\theta}(A, \tau, \mathcal{J}) = \{a, c, d\} \supset \{a, d\} = cl^{*\theta}(A, \tau, \mathcal{I})$ but $\mathcal{J} \not\subset \mathcal{I}$.*

Example 3.10. *Let $X = \{a, b, c, d\}$ with $\tau = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, X\}$ and $\mathcal{I} = \{\phi, \{b\}, \{c\}, \{b, c\}\}$.*

(ii) *Let $A = \{c\}$,$B = \{a, b\}$. Then $cl^{*\theta}(A) = \{c\} \subset X = cl^{*\theta}(B)$, but$A \not\subset B$.*

(iv) *Let $A = \{a\}$. Then $cl^{*\theta}(A) = \{a, c, d\} \not\subset \{a, d\} = cl^{*}(A)$.*

(viii) *Let $A = \{b, c\}$, $B = \{b, d\}$. Then $cl^{*\theta}(A) = \{b, c\}$, $cl^{*\theta}(B) = X$, $A \cap B = \{b\}$. $cl^{*\theta}(A \cap B) = \{b\}$. So, $cl^{*\theta}(A) \cap cl^{*\theta}(B) = \{b, c\} \not\subset \{b\} = cl^{*\theta}(A \cap B)$.*

(v) *Let $A = \{b, c\}$. Then, $cl_{\theta}(A) = X \not\subset \{b, c\} = cl^{*\theta}(A)$.*

Remark 3.11. *$D_{\theta}(A) \subset cl^{*\theta}(A)$ and $cl^{*\theta}(A) \subset D_{\theta}(A)$ are not true in general as shown in the following example.*

Example 3.11. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space with $X = \{a, b, c, d, e\}$, $\tau = \{\phi$,$\{a\}$,$\{c, d\}$,$\{a, c, d\}$, $\{b, c, d, e\}$,$X\}$ and $\mathcal{I} = \{\phi, \{c\}, \{d\}, \{c, d\}\}$.*

(i) *If $A = \{c, d\}$, then $A^{*\theta} = \phi$. Therefore, $D_{\theta}(A) = \{b, c, d, e\} \not\subset \{c, d\} = cl^{*\theta}(A)$.*

(ii) *If $A = \{a, b, d\}$, then $A^{*\theta} = X$. Therefore, $cl^{*\theta}(A) = X \not\subset \{b, c, d, e\} = D_{\theta}(A)$.*

Proposition 3.6. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. For any subset A of X, the following properties are hold.*

(i) $A^{*\theta} - A \subset cl_{\theta}(A) - A \subset D_{\theta}(A)$.

(ii) *If $\mathcal{I} = \{\phi\}$, then $A^{*\theta} - A = cl_{\theta}(A) - A \subset D_{\theta}(A)$.*

(iii) *If $\mathcal{I} = \wp(X)$, then $A^{*\theta} = D_{\theta}(A)$.*

Proof. (i)From Theorem 3.2(v), we have $A^{*\theta} \subset cl_{\theta}(A)$. Then, $A^{*\theta} - A \subset cl_{\theta}(A) - A$. Since $cl_{\theta}(A) = A \cup D_{\theta}(A)$, $cl_{\theta}(A) - A \subset D_{\theta}(A)$. It follows that $A^{*\theta} - A \subset cl_{\theta}(A) - A \subset D_{\theta}(A)$.
(ii) and (iii) are straight forward by Proposition 3.4 and Proposition 3.5. □

4 θ - Compatibility

Definition 4.1. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space, then τ is θ-compatible with the ideal $\mathcal{I}$, if for every $A \subseteq X$ and if for every $x \in A$, there exists $U \in \tau^\theta(x)$ such that $U \cap A \in \mathcal{I}$, then $A \in \mathcal{I}$ and it is denoted by $\tau \sim^\theta \mathcal{I}$.*

Theorem 4.1. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space, then the following properties are equivalent:*

1. *$\tau \sim^\theta \mathcal{I}$;*
2. *If a subset A of X has a cover of θ-open sets each of whose intersection with A is in $\mathcal{I}$, then $A \in \mathcal{I}$;*
3. *For every $A \subseteq X$, $A \cap A^{*\theta} = \phi$ implies that $A \in \mathcal{I}$;*
4. *For every $A \subseteq X$, $A - A^{*\theta} \in \mathcal{I}$.*
5. *For every $A \subseteq X$, if A contains no nonempty subset B with $B \subseteq B^{*\theta}$, then $A \in \mathcal{I}$.*

Proof. $(1) \Rightarrow (2)$. The proof is obvious.

$(2) \Rightarrow (3)$. Let$A \subseteq X$ and $x \in A$. Since $A \cap A^{*\theta} = \phi$, $x \notin A^{*\theta}$ and there exists some θ-open set $V_x \in \tau^\theta$ such that $V_x \cap A \in \mathcal{I}$. Therefore, we have $A \subseteq \bigcup\{V_x : x \in A\}$ and $V_x \in \tau^\theta$ and by (2) $A \in \mathcal{I}$.

$(3) \Rightarrow (4)$. For any $A \subseteq X$, $A - A^{*\theta} \subseteq A$ and $(A - A^{*\theta}) \cap (A - A^{*\theta})^{*\theta} \subseteq (A - A^{*\theta}) \cap A^{*\theta} = \phi$. By (3), $A - A^{*\theta} \in \mathcal{I}$.

$(4) \Rightarrow (5)$. By (4), for every $A \subseteq X$, $A - A^{*\theta} \in \mathcal{I}$. Let $A - A^{*\theta} = J \in \mathcal{I}$, $A = J \cup (A \cap A^{*\theta})$ and by Theorem 3.17 (vii) and (xiii), $A^{*\theta} = J^{*\theta} \cup (A \cap A^{*\theta})^{*\theta} = (A \cap A^{*\theta})^{*\theta}$. Therefore, we have $(A \cap A^{*\theta}) = A \cap (A \cap A^{*\theta})^{*\theta} \subseteq (A \cap A^{*\theta})^{*\theta}$ and $(A \cap A^{*\theta}) \subseteq A$. By the assumption $A \cap A^{*\theta} = \phi$ and hence $A =(A - A^{*\theta}) \in \mathcal{I}$.

$(5) \Rightarrow (1)$. Let $A \subseteq X$ and assume that for every $x \in A$, there exists some θ-open set U_x containing x, $U_x \cap A \in \mathcal{I}$. Then $A \cap A^{*\theta} = \phi$. Suppose that A contains B such that $B \subseteq B^{*\theta}$. Then $B = B \cap B^{*\theta} \subseteq A \cap A^{*\theta} = \phi$. Therefore, A contains no nonempty subset B with $B \subseteq B^{*\theta}$. Hence $A \in \mathcal{I}$. □

Lemma 4.1. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. If $\tau \sim^\theta \mathcal{I}$, then for every $A \subseteq X$, $A \cap A^{*\theta} = \phi$ implies that $A^{*\theta} = \phi$.*

Proof. Let A be any subset of X and $A \cap A^{*\theta} = \phi$. By Theorem 4.1, $A \in \mathcal{I}$ and by Theorem 3.2 (xiii), $A^{*\theta} = \phi$. □

Theorem 4.2. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. If $\tau \sim^\theta \mathcal{I}$ then the following properties are equivalent:*

1. *For every $A \subseteq X$, $A \cap A^{*\theta} = \phi$ implies that $A^{*\theta} = \phi$.*
2. *For every $A \subseteq X$, $(A - A^{*\theta})^{*\theta} = \phi$.*
3. *For every $A \subseteq X$, $(A \cap A^{*\theta})^{*\theta} = A^{*\theta}$.*

Proof. $(1) \Rightarrow (2)$. Assume that every $A \subseteq X$, $A \cap A^{*\theta} = \phi$ implies that $A^{*\theta} = \phi$. Let $B = A - A^{*\theta}$, then $B \cap B^{*\theta} = (A - A^{*\theta}) \cap (A - A^{*\theta})^{*\theta} = (A \cap (X - A^{*\theta})) \cap (A \cap (X - A^{*\theta}))^{*\theta} \subseteq (A \cap (X - A^{*\theta})) \cap (A^{*\theta} \cap (X - A^{*\theta})^{*\theta}) = \phi$. By (1), we have $B^{*\theta} = \phi$. Hence $(A - A^{*\theta})^{*\theta} = \phi$.

$(2) \Rightarrow (3)$ Assume for every $A \subseteq X$, $(A - A^{*\theta})^{*\theta} = \phi$. $A = (A - A^{*\theta}) \cup (A \cap A^{*\theta})$. $A^{*\theta} = [(A - A^{*\theta}) \cup (A \cap A^{*\theta})]^{*\theta} = (A - A^{*\theta})^{*\theta} \cup (A \cap A^{*\theta})^{*\theta} = (A \cap A^{*\theta})^{*\theta}$.

$(3) \Rightarrow (1)$ Assume for every $A \subseteq X$, $A \cap A^{*\theta} = \phi$ and $(A \cap A^{*\theta})^{*\theta} = A^{*\theta}$. This implies that $\phi = \phi^{*\theta} = A^{*\theta}$. □

Definition 4.2. *If $(X, \tau, \mathcal{I})$ is an ideal topological space, then $\mathcal{I}$ is $*\theta$-codense if and only if $A \subset A^{*\theta}$ for every θ-open set A of X.*

Characterization of θ -local function in $*\theta$-codense ideal topological space.

Theorem 4.3. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. Then the following are equivalent:*

1. $X = X^{*\theta}$.
2. $\tau^\theta \cap \mathcal{I} = \{\phi\}$.

3. *If $I \in \mathcal{I}$, then $int_\theta(I) = \phi$.*

4. *For every $U \in \tau^\theta$, $U \subset U^{*\theta}$.*

Proof. (1) $\Rightarrow$ (2): Let $U \in \tau^\theta \cap \mathcal{I}$. Then $U \in \tau^\theta$ and $U \in \mathcal{I}$. Suppose that $x \in U$. Since $x \in X$, this implies $x \in X^{*\theta}$. Since U is a θ-open set containing x, $U \cap X \notin \mathcal{I}$ implies that $U \notin \mathcal{I}$ which is a contradiction. Therefore, $x \notin U$ for every $x \in X$. This implies that $U = \phi$ and so $\tau^\theta \cap \mathcal{I} = \{\phi\}$.

(2) $\Rightarrow$ (3): Suppose that (2) holds. Let $I \in \mathcal{I}$ be such that $I \neq \phi$. Then $int_\theta(I) \in \tau^\theta$ and $int_\theta(I) \subset I$ implies that $int_\theta(I) \in \mathcal{I}$. Therefore, by (2), $int_\theta(I) = \phi$.

(3) $\Rightarrow$ (4): $U \in \tau^\theta$ and $x \in U$. Suppose that $x \notin U^{*\theta}$. Then there exists a θ-open set V_x containing x such that $V_x \cap U \in \mathcal{I}$. Since $U \cap V_x$ is a θ-open set containing x, $U \cap V_x = int_\theta(U \cap V_x) = \phi$ by (3). Since $x \in V_x$,$x \notin U$. Thus $U \subset U^{*\theta}$ for every $U \in \tau^\theta$.

(4) $\Rightarrow$ (1): Since X is θ-open, by (4), $X \subset X^{*\theta}$, $X = X^{*\theta}$.

□

Theorem 4.4. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $I \in \mathcal{I}$. Then, I is $\tau^{*\theta}$-closed.*

Proof. Let $I \in \mathcal{I}$. By Theorem 3.22 (x) $I^{*\theta} = (I - I)^{*\theta} = \phi^{*\theta} = \phi$. Hence $cl^{*\theta}(I) = I \cup I^{*\theta}$ = I which implies that I is $\tau^{*\theta}$ -closed. □

Theorem 4.5. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $A \subset X$. Then $A^{*\theta}(\tau^{*\theta}, \mathcal{I}) \subset A^{*\theta}(\tau^\theta, \mathcal{I})$.*

Proof. Let $x \in A^{*\theta}(\tau^{*\theta}, \mathcal{I})$. Suppose that $x \notin A^{*\theta}(\tau^{*\theta}, \mathcal{I})$. Then there exists a θ-open set U_x containing x, such that $A \cap U_x \in \mathcal{I}$. Since $U_x \in \tau^\theta \subset \tau^{*\theta}$, $A \cap U_x \in \mathcal{I}$ for a $\tau^{*\theta}$-open set U_x containing x. Therefore, $x \notin A^{*\theta}(\tau^{*\theta}, \mathcal{I})$ which implies that $A^{*\theta}(\tau^{*\theta}, \mathcal{I}) \subset A^{*\theta}(\tau^\theta, \mathcal{I})$. □

Theorem 4.6. *Let (X, τ) be an ideal topological space where $\mathcal{I}$ and $\mathcal{J}$ are ideals on X and $A \subset X$. Then the following hold:*

(i) $A^{*\theta}(\mathcal{I} \cap \mathcal{J}) = A^{*\theta}(\mathcal{I}) \cup A^{*\theta}(\mathcal{J})$.

(ii) If $\mathcal{I} \subset \mathcal{J}$, *then* $\tau^{*\theta}(\mathcal{I}) \subset \tau^{*\theta}(\mathcal{J})$.

(iii) $\tau^{*\theta}(\mathcal{I} \cap \mathcal{J}) = \tau^{*\theta}(\mathcal{I}) \cap \tau^{*\theta}(\mathcal{J})$.

Proof. (i) Let $x \notin A^{*\theta}(\mathcal{I} \cap \mathcal{J})$ if and only if there exists a θ-open set U_x containing x, such that $A \cap U_x \in \mathcal{I} \cap \mathcal{J}$ if and only if $A \cap U_x \in \mathcal{I}$ and $A \cap U_x \in \mathcal{J}$ if and only if $x \notin A^{*\theta}(\mathcal{I})$ and $x \notin A^{*\theta}(\mathcal{J})$ if and only if $x \notin A^{*\theta}(\mathcal{I}) \cup A^{*\theta}(\mathcal{J})$. Hence, $A^{*\theta}(\mathcal{I} \cap \mathcal{J}) = A^{*\theta}(\mathcal{I}) \cup A^{*\theta}(\mathcal{J})$ for every subset $A \subset X$.

(ii) Let $\mathcal{I} \subset \mathcal{J}$. Now if $X - A \in \tau^{*\theta}(\mathcal{I})$, then $A \cup A^{*\theta}(\mathcal{I}) = A$ which implies that $A^{*\theta}(\mathcal{I}) \subset A$. Since $\mathcal{I} \subset \mathcal{J}$, $A^{*\theta}(\mathcal{J}) \subset A^{*\theta}(\mathcal{I}) \subset A$ by Theorem 3.17 (iii). Therefore, $X - A \in \tau^{*\theta}(\mathcal{J})$ which implies that $\tau^{*\theta}(\mathcal{I}) \subset \tau^{*\theta}(\mathcal{J})$.

(iii) Let $A \subset X$ and $X - A \in \tau^{*\theta}(\mathcal{I} \cap \mathcal{J})$. Since $\mathcal{I} \cap \mathcal{J}$ is a subset of $\mathcal{I}$ and $\mathcal{J}$, $X - A \in \tau^{*\theta}(\mathcal{I})$ and $X - A \in \tau^{*\theta}(\mathcal{J})$ if and only if A is $\tau^{*\theta}(\mathcal{I})$- closed and $\tau^{*\theta}(\mathcal{J})$- closed if and only if $A^{*\theta}(\mathcal{I}) \subset A$ and $A^{*\theta}(\mathcal{J}) \subset A$. Hence, $A^{*\theta}(\mathcal{I}) \cup A^{*\theta}(\mathcal{J}) \subset A$ if and only if $A^{*\theta}(\mathcal{I} \cap \mathcal{J}) \subset A$ by (i). This implies that A is $\tau^{*\theta}(\mathcal{I} \cap \mathcal{J})$-closed. Therefore, $\tau^{*\theta}(\mathcal{I} \cap \mathcal{J}) = \tau^{*\theta}(\mathcal{I}) \cap \tau^{*\theta}(\mathcal{J})$.

□

5 The operator ψ_θ

Definition 5.1. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. An operator $\psi_\theta : \wp(X) \to \tau$ is defined as $\psi_\theta(A) = \{x \in X:$ there exists a θ-open set U_x containing x such that $U_x - A \in \mathcal{I}\}$, for every $A \in \wp(X)$. We observe that $\psi_\theta(A) = X - (X - A)^{*\theta}$.*

Theorem 5.1. *Let $(X, \tau, \mathcal{I})$ be a ideal topological space. Then, for $A \in \wp(X)$, $\psi_\theta(A) = X - (X - A)^{*\theta}$.*

Proof. Let $x \in \psi_\theta(A)$. Then there exists a θ-open set U_x containing x such that $U_x - A \in \mathcal{I}$. Then $X \cap (U_x - A) \in \mathcal{I}$, implies that $U_x \cap (X - A) \in \mathcal{I}$. So $x \notin (X - A)^{*\theta}$ and hence, $x \in X - (X - A)^{*\theta}$. Therefore, $\psi_\theta(A) \subset X - (X - A)^{*\theta}$. For reverse inclusion, if $x \in X - (X - A)^{*\theta}$, then $x \notin (X - A)^{*\theta}$ and so there exists a θ-open set U_x containing x such that $U_x \cap (X - A) \in \mathcal{I}$ which implies that $U_x - A \in \mathcal{I}$. Hence $x \in \psi_\theta(A)$. Thus $X - (X - A)^{*\theta} \subset \psi_\theta(A)$ and so $\psi_\theta(A) = X - (X - A)^{*\theta}$. □

Theorem 5.2. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and let A , B be subsets of X, then the following hold:*

(i) If $A \subseteq B$, then $\psi_\theta(A) \subseteq \psi_\theta(B)$.

(ii) If $A, B \in \wp(X)$, then $\psi_\theta(A) \cup \psi_\theta(B) \subset \psi_\theta(A \cup B)$

(iii) If $A, B \in \wp(X)$, then $\psi_\theta(A) \cap \psi_\theta(B)$=$\psi_\theta(A \cap B)$.

(iv) If $A \subseteq X$, $\psi_\theta(A) \subset \psi(A)$.

(v) If $U \in \tau^\theta$, then $U \subseteq \psi_\theta(U)$. Also, if $U \in \tau^{*\theta}$, then $U \subseteq \psi_\theta(U)$.

(vi) If $A \subseteq X$, then $\psi_\theta(A) \subseteq \psi_\theta(\psi_\theta(A))$.

(vii) If $A \subseteq X$, then $\psi_\theta(A) = \psi_\theta(\psi_\theta(A))$ if and only if $((X - A)^{*\theta})^{*\theta} = (X - A)^{*\theta}$.

(viii) If $A \subseteq X$ and $I \in \mathcal{I}$, then $\psi_\theta(A - I) = \psi_\theta(A) = \psi_\theta(A \cup I)$.

(ix) If $(A - B) \cup (B - A) \in \mathcal{I}$, then $\psi_\theta(A) = \psi_\theta(B)$.

Proof. (i) Since $A \subseteq B$, then $(X - A) \supseteq (X - B)$. Then by Theorem 3.22 (ii), $(X - A)^{*\theta} \supseteq (X - B)^{*\theta}$ and hence $\psi_\theta(A) \subseteq \psi_\theta(B)$.

(ii) Since $A \subset A \cup B$ and $B \subset A \cup B$, by (i) $\psi_\theta(A) \cup \psi_\theta(B) \subset \psi_\theta(A \cup B)$.

(iii) $\psi_\theta(A \cap B) = X - (X - (A \cap B))^{*\theta} = X - ((X - A) \cup (X - B))^{*\theta}$. This implies that $\psi_\theta(A \cap B) = X - ((X - A)^{*\theta} \cup (X - B)^{*\theta})$, from Theorem 3.22(xi). Therefore, $\psi_\theta(A \cap B) = (X - (X - A)^{*\theta}) \cup (X - (X - B)^{*\theta})$ and hence, $\psi_\theta(A \cap B) = \psi_\theta(A) \cap \psi_\theta(B)$.

(iv) From Theorem 3.17 (iv), we have that $(X - A)^* \subset (X - A)^{*\theta}$. This implies that $X - (X - A)^* \supset X - (X - A)^{*\theta}$ and $\psi_\theta(A) \subset \psi(A)$.

(v) Since $U \in \tau^\theta$, then $X - U$ is a θ-closed set. So, $cl_\theta(X - U) = X - U$. By theorem 3.22 (vi), $(X - U)^{*\theta} \subseteq cl_\theta(X - U) = (X - U)$. Then, $U \subseteq X - (X - U)^{*\theta} = \psi_\theta(U)$ for every $U \in \tau^\theta$. If $U \in \tau^{*\theta}$, then $X - U$ is a $\tau^{*\theta}$-closed which implies that $(X - U)^{*\theta} \subseteq (X - U)$ and so, $U \subseteq X - (X - U)^{*\theta} = \psi_\theta(U)$.

(vi) This follows from (i) and (v).

(vii) Since $\psi_\theta(\psi_\theta(A)) = X - (X - \psi_\theta(A))^{*\theta} = X - (X - (X - (X - A)^{*\theta}))^{*\theta} = X - ((X - A)^{*\theta})^{*\theta} = X - (X - A)^{*\theta} = \psi_\theta(A)$ if and only if $((X - A)^{*\theta})^{*\theta} = (X - A)^{*\theta}$.

(viii) We know that $X - (X - (A - \mathcal{I}))^{*\theta} = X - ((X - A) \cup \mathcal{I})^{*\theta} = X - (X - A)^{*\theta}$, (Theorem3.22(xvi)). So, $\psi_\theta(A - \mathcal{I}) = \psi_\theta(A)$. Also, we know that $X - (X - (A \cup \mathcal{I}))^{*\theta} = X - ((X - A) - \mathcal{I})^{*\theta} = X - (X - A)^{*\theta}$, (from Theorem 3.22(xvi)). So, $\psi_\theta(A - \mathcal{I}) = \psi_\theta(A)$. Also, $\psi_\theta(A \cup \mathcal{I}) = \psi_\theta(A)$.

(ix) Given that $(A - B) \cup (B - A) \in \mathcal{I}$, and let $A - B = I_1$, $B - A = I_2$. We observe that I_1 and $I_2 \in \mathcal{I}$ by heredity. Also, observe that, $B = ((A - I_1) \cup I_2)$. Thus, $\psi_\theta(A) = \psi_\theta((A - I_1) \cup I_2) = \psi_\theta(B)$.

□

Corollary 5.1. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. Then $U \subseteq \psi_\theta(U)$ for every θ-open set $U \subseteq X$.*

Proof. We know that $\psi_\theta(U) = X - (X - U)^{*\theta}$. Now $(X - U)^{*\theta} \subseteq cl_\theta(X - U) = X - U$, since $X - U$ is θ-closed. Therefore, $U = X - (X - U) \subseteq X - (X - U)^{*\theta} = \psi_\theta(U)$. □

Remark 5.1. *The following example shows that a set A is not θ-open but satisfies $A \subseteq \psi_\theta(A)$.*

Example 5.1. *Let* $X = \{a, b, c, d\}$, $\tau = \{\phi, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, X\}$ *and* $\mathcal{I} = \{\phi, \{c\}, \{c, d\}\}$. *Let* $A = \{b\}$. *Then* $\psi_\theta(\{b\}) = X - (X - \{b\})^{*\theta} = X - (\{a, c, d\})^{*\theta} = X - \{a\} = \{b, c, d\}$. *Therefore,* $A \subseteq \psi_\theta(A)$*, But* A *is not* θ*-open.*

Theorem 5.3. *Let* $(X, \tau, \mathcal{I})$ *be an ideal topological space. If* $A \subseteq X$*, then,* $A \cap \psi_\theta(A) = int_\theta(A)$.

Proof. If $x \in A \cap \psi_\theta(A)$, then $x \in A$ and there exists a θ-open set U_x containing x, such that $U_x - A \in \mathcal{I}$. Then, by Remark 3.5, $U_x - (U_x - A) \in \tau^\theta$-open neighborhood of x and $x \in int_\theta(A)$. On the other hand, if $x \in int_\theta(A)$ there exists a basic τ^θ-open neighborhood $V_x - A$ of x, where $V_x - A \in \tau$ and $I \in \mathcal{I}$, such that $x \in V_x - I \subseteq A$ which implies $V_x - A \subseteq I$ and hence $V_x - A \in \mathcal{I}$. Hence, $x \in A \cap \psi_\theta(A)$. □

Theorem 5.4. *Let* $(X, \tau, \mathcal{I})$ *be an ideal topological space and* $A \subseteq X$*. Then the following properties hold:*

1. $\psi_\theta(A) = \bigcup\{U \in \tau^\theta\text{: } U - A \in \mathcal{I}\}$.
2. $\psi_\theta(A) \supseteq \bigcup\{U \in \tau^\theta \text{ :}(U - A) \cup (A - U) \in \mathcal{I}\}$.

Proof. (1) This follows immediately from the definition of ψ_θ-operator.

(2) Since $\mathcal{I}$ is heredity, it is obvious that $\bigcup\{U \in \tau^\theta\text{:}(U - A) \cup (A - U) \in \mathcal{I}\} \subseteq \bigcup\{U \in \tau^\theta\text{: } U - A \in \mathcal{I}\} = \psi_\theta(A)$ for every $A \subseteq X$.

□

Theorem 5.5. *Let* $(X, \tau, \mathcal{I})$ *be an ideal topological space. Then* $\tau \sim^\theta \mathcal{I}$ *if and only if* $\psi_\theta(A) - A \in \mathcal{I}$ *for every* $A \subseteq X$.

Proof. **Necessity:**
Assume $\tau \sim^\theta \mathcal{I}$ and let $A \subseteq X$. Observe that $x \in \psi_\theta(A) - A \in \mathcal{I}$ if and only if $x \notin A$ and $x \notin (X - A)^{*\theta}$ if and only if $x \notin A$ and there exists some θ-open set $U_x \in \tau^\theta(x)$ such that $U_x - A \in \mathcal{I}$ if and only if there exists some θ-open set $U_x \in \tau^\theta(x)$ such that $x \in U_x - A \in \mathcal{I}$. Now, for each $x \in \psi_\theta(A) - A$ and $U_x \in \tau^\theta(x)$, $U_x \cap (\psi_\theta(A) - A) \in \mathcal{I}$ by heredity and hence, $\psi_\theta(A) - A \in \mathcal{I}$ by assumption that $\tau \sim^\theta \mathcal{I}$.
Sufficiency:
Let $A \subseteq X$ and assume that for each $x \in A$ there exists some θ-open set $U_x \in \tau^\theta(x)$ such that $U_x \cap A \in \mathcal{I}$. Observe that $\psi_\theta(X - A) - (X - A) = A - A^{*\theta} = \{x : \text{there exists some } \theta\text{-open set } U_x \in \tau^\theta(x) \text{ such that } U_x \cap A \in \mathcal{I}\}$. Thus, we have $A \subseteq \psi_\theta(X - A) - (X - A) \in \mathcal{I}$ and hence, $A \in \mathcal{I}$ by heredity of $\mathcal{I}$. □

Theorem 5.6. *Let* $(X, \tau, \mathcal{I})$ *be an ideal topological space with* $\tau \sim^\theta \mathcal{I}$, $A \subseteq X$*. If N is a nonempty* θ*-open subset of* $A^{*\theta} \cap \psi_\theta(A)$*, then* $N - A \in \mathcal{I}$ *and* $N \cap A \notin \mathcal{I}$.

Proof. If $N \subseteq A^{*\theta} \cap \psi_\theta(A)$, then $N - A \subseteq \psi_\theta(A) - A \in \mathcal{I}$ by Theorem 5.5 and hence $N - A \in \mathcal{I}$ by heredity. Since $N \in \tau^\theta - \{\phi\}$ and $N \subseteq A^{*\theta}$, we have $N \cap A \notin \mathcal{I}$ by the definition of $A^{*\theta}$. □

Remark 5.2. *Let* $(X, \tau, \mathcal{I})$ *be an ideal topological space with* $\tau \sim^\theta \mathcal{I}$*. Then* $\psi_\theta(A) = \psi_\theta(\psi_\theta(A))$ *for every* $A \subseteq X$.

Proof. $\psi_\theta(A) \subseteq \psi_\theta(\psi_\theta(A))$ follows from Theorem 5.2(vi). Since $\tau \sim^\theta \mathcal{I}$, it follows from Theorem 5.5 that $\psi_\theta(A) \subseteq A \cup \mathcal{I}$ for some $I \in \mathcal{I}$, and hence $\psi_\theta(A) = \psi_\theta(\psi_\theta(A))$ by Theorem 5.2 (viii). □

Theorem 5.7. *Let* $(X, \tau, \mathcal{I})$ *be an ideal topological space with* $\tau \sim^\theta \mathcal{I}$*. Then* $\psi_\theta(A) = \bigcup\{\psi_\theta(U)\text{: } U \in \tau^\theta \text{ , } \psi_\theta(U) - A \in \mathcal{I}\}$.

Proof. Let $\Phi(A) = \bigcup\{\psi_\theta(U)\text{: } U \in \tau^\theta \text{ , } \psi_\theta(U) - A \in \mathcal{I}\}$. Clearly $\Phi(A) \subseteq \psi_\theta(A)$. Now let $x \in \psi_\theta(A)$. Then, there exists a θ-open set U, such that $U - A \in \mathcal{I}$. By Corollary 5.1, $U \subseteq \psi_\theta(U)$and $\psi_\theta(U) - A \subseteq [\psi_\theta(U) - U] \cup [U - A]$. By Theorem 5.5 $\psi_\theta(U) - U \in \mathcal{I}$. Hence, $x \in \Phi(A)$ and $\Phi(A) \supseteq \psi_\theta(A)$. Consequently, we obtain $\Phi(A) = \psi_\theta(A)$. □

Theorem 5.8. *Let* $(X, \tau, \mathcal{I})$ *be an ideal topological space with* $\tau \sim^\theta \mathcal{I}$*, where* $\tau^\theta \cap \mathcal{I} = \phi$*. Then for* $A \subseteq X$, $\psi_\theta(A) \subseteq A^{*\theta}$.

Proof. Suppose $x \in \psi_\theta(A)$ and $x \notin A^{*\theta}$. Then, there exists a θ-open set $U_x \in \tau(x)$ such that $U_x \cap A \in \mathcal{I}$. Since $x \in \psi_\theta(A)$, by Theorem 5.4 $x \in \bigcup\{U \in \tau^\theta : U - A \in \mathcal{I}\}$ and there exists a θ-open set $V_x \in \tau^\theta(x)$ such that $V_x - A \in \mathcal{I}$. Now, we have $U_x \cap V_x \in \tau^\theta(x)$, $U_x \cap V_x \cap A \in \mathcal{I}$ and $U_x \cap V_x - A \in \mathcal{I}$ by heredity. Hence, by finite additivity, we have $(U_x \cap V_x \cap A) \cup (U_x \cap V_x - A) = U_x \cap V_x \in \mathcal{I}$. Since $(U_x \cap V_x) \in \tau^\theta$, this is contrary to $\tau^\theta \cap \mathcal{I} = \phi$. Therefore, $x \in A^{*\theta}$. This implies that $\psi_\theta(A) \subseteq A^{*\theta}$. □

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