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Perturbation of Differential Linear System

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Abstract

The main theme studied concerns perturbation of differential linear system with constant coefficients:

$$\frac{dX}{dt} = AX + b. \tag{0.1}$$

The data of the system (0.1) provides the expression of a vector field X of $\mathbb{R}^n$, in the coordinates $X_1, X_2, ..., X_n$. The singularity of the system (0.1) or the field X, expressed by coordinates $X_1, X_2, ..., X_n$ is given by the solutions of the system of equations $AX + b = 0$.

In general, a small perturbation of a regular linear standard real matrix M is a matrix of the form:

$$M' = M + \epsilon.$$

where $\epsilon = (\epsilon_{ij})$is a matrix with elements infinitely small.

We study the regular linear perturbation when the singularity is a point with various situations and practical examples and in the case where the singular place is a line with various practical situations. we hope that our contribution is in fact to use certain technical of non standard Analysis (infinitesimal calculus) which simplify obviously the proves.

Keywords: perturbation singular, regular, critical points, exact solution, differential system, orbits, infinitely-small , infinitely-large.

2010 MSC: 39B55, 39B52, 39B82.

1 Introduction

The study of linear stability informs us about the stability of the system when the non-linear terms are taken into account.When the two eigenvalues have a strictly negative real part, linear stability implies non-linear stability.

In the case of unstable systems and when the two eigenvalues are strictly positive real parts. A system which is unstable by linear stability it remains when the non-linear contributions are taken into consideration. On the other hand, when at least one of the real part of the eigenvalues is zero, i.e. in the case of centers, taking into account the non-linear terms can lead to different results from those obtained by linearization. we hope that our contribution is in fact to use certain Technics of non standard Analysis (infinitesimal calculus) which simplify obviously the proves.

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2 Regular linear perturbations

A translation of the origin, we assume $b = 0$, that gives a homogeneous linear system:

$$\frac{dx}{dt} = Ax.$$

Since $rank(A) = 1$, there exist elements $(\alpha, \beta) \in \mathbb{R}^2$, $(a, b) \in \mathbb{R}^2$, $a^2 + b^2 \succ 0$ such as $a_{11} = \alpha a$, $a_{12} = \alpha b$, $a_{21} = \beta a$, $a_{22} = \beta b$.
The matrix A is written as:

$$A = \begin{pmatrix} \alpha a & \alpha b \\ \beta a & \beta b \end{pmatrix} \quad (2.2)$$

and the system becomes:

$$\begin{cases} x_1' = \alpha a x_1 + \alpha b x_2 \\ x_2' = \beta a x_1 + \beta b x_2 \end{cases}$$

3 Notation

The following abbreviations will be adopted.

NSO: Non singular orbits.
S: Singularity to indicate that a quantity does not take the value 0.
We will write indifferently $a \neq 0$ or $(\bar{a})$.
NSO: $x_1 = constant$ whence $x_1' = 0$ Thus $\alpha = 0$.
S: $x_2 = 0$ whence $x_2' = \beta b x_2$.

$$\begin{cases} x_1' = 0; & \alpha = 0, a = 0, \beta \neq 0, b \neq 0, \\ x_2' = \beta b x_2; & \beta b \prec 0. \end{cases}$$

$$\begin{cases} x_1' = 0; & \alpha = 0, a = 0, \beta \neq 0, b \neq 0, \\ x_2' = \beta b x_2; & \beta b \prec 0. \end{cases}$$

NSO: it is a line of positive slope.
S: $x_2 = 0$.

$$\begin{cases} x_1' = \alpha b x_2 & \alpha \neq 0, a = 0, \beta \neq 0, b \neq 0 \\ x_2' = \beta b x_2 & \alpha b \prec 0, \beta b \prec 0 \end{cases}$$

NSO: $x_2 = constant$ whence $x_2' = 0$ Thus $\beta = 0$
S: it is a line of positive slope.
NSO: They are line of negative slope.
S: it is a line of negative slope parallel with NOS.

$$\begin{cases} x_1' = \alpha (a x_1 + b x_2) & \alpha \neq 0, a \neq 0, \beta \neq 0, b \neq 0 \\ x_2' = \beta (a x_1 + b x_2) & \alpha a \prec 0, \beta a \prec 0 \end{cases}$$

The non singular orbits are perpendicular to the singularity, the system which makes it possible to describe them is

$$\begin{cases} x_1' = 0 \\ x_2' = 0 \end{cases}$$

The non singular orbits are parallel to the singularity described by the following system

$$\begin{cases} x_1' = x_2 \\ x_2' = 0 \end{cases} .$$

Since $rankA = 1$, there exist elements $(\alpha, \beta) \in \mathbb{R}^2$, $(a, b) \in \mathbb{R}^2$, $a^2 + b^2 \succ 0$ such as $a_{11} = \alpha a$, $a_{12} = \alpha b$,$a_{21} = \beta a$, $a_{22} = \beta b$.

The matrix (A, b) is written as:

$$(A, b) = \begin{pmatrix} \alpha a & \alpha b & b_1 \\ \beta a & \beta b & b_1 \end{pmatrix}$$

and the system (0.1) become:

$$\begin{cases} x_1' = \alpha (ax_1 + bx_2) + b_1 \\ x_2' = \beta (ax_1 + bx_2) + b_2 \end{cases}$$

first case $b_1 = b_2 = 0$ we must take α, β, a, b non zero so that the matrix (A, b) remains of $rank2$ second case $b_1 = 0, b_2 \neq 0$ we have:

$$\begin{cases} x_1' = \alpha (ax_1 + bx_2) \\ x_2' = \beta (ax_1 + bx_2) + b_2 \end{cases}$$

If $\alpha = 0, \beta \neq 0$ is impossible because $rank\,(A, b)$ will not be equal any more to 2

Remark 3.1. *For a linear differential connection of* $\mathbb{R}^2$ *with constant coefficients*

$$\frac{dX}{dt} = AX + b$$

with:

$$rank\,(A, b) = 1 + rank\,(A) = 2.$$

There exists two possible models.

Ame exotique or parabola The trajectories of the parabola of equation $x_2 = \frac{1}{2}x_1^2 + k, k \in \mathbb{R}$ corresponding with the system:

$$\begin{cases} x_1' = 1 \\ x_2' = x_1. \end{cases}$$

Ame stable The trajectories are exponential curves of equation $x_2 = k \exp(x_1), k \in \mathbb{R}$ corresponding with the system:

$$\begin{cases} x_1' = 1 \\ x_2' = x_2. \end{cases}$$

The axis of the traces represents the states with a comb type.
The axis of the traces represents the states of the heart type.

4 Regular linear perturbation when the singular place is a point

If B is a real matrix of order p and ϵ a real matrix of p order have infinitely small elements, then it exists a real infinitely small ϵ,such as $\det(B + \epsilon) = \det B + \epsilon$.
The various situations or the singularity is a point.
With a loss less of general information, it can be limited to the homogeneous systems:

$$\frac{dx}{dt} = Ax.$$

with $rankA = 2$, A standard matrix.
First case : A is not diagonal.
We work with the figure X, giving the qualitative states in term of trace.
Defined as $Tr : \mathbb{R}^2 \to \mathbb{R}$ is a continuous standard function.
Thus $Tr\,(B + \epsilon) = TrB + \epsilon$ where ϵ is Infinitesimal. If B is a standard function and ϵ a matrix of Infinitesimals.

The linear differential system $\frac{dx}{dt} = Ax$, A standard matrix.
In SDN the state (respectively IDN) ($\det A \succ 0, TrA \prec 0$ respectively $TrA \succ 0$).
$\det A = \frac{1}{4}(TrA)^2$, undergoing a small regular linear perturbation: $\frac{dx}{dt} = (A+\epsilon)\,x$ and ϵ a matrix of infinitesimals.
SDN the state (respectively IDN) changes into SDN(respectively IDN).
if $\det(A+\epsilon) = \frac{1}{4}(Tr\,(A+\epsilon))^2$ (it is said that states SDN and IDN resist).
The SDN state (respectively IDN) changes into SN (respectively IN).
if $\det(A+\epsilon) \prec \frac{1}{4}(Tr\,(A+\epsilon))^2$.
The linear differential system $\frac{dx}{dt} = Ax$, A standard.
In the state C $(detA \succ 0, TrA = 0)$ undergoing a small regular linear perturbation. $\frac{dx}{dt} = (A+\epsilon)\,x$ and ϵ a matrix of infinitesimals.Then the state C resist if $(Tr\,(A+\epsilon)) = 0$ and the state C transforms into FI if $(Tr(A+\epsilon)) \succ 0$ and the state C transform into FS if $(Tr(A+\epsilon)) \prec 0$

Example 4.1. *Let be the system* $\frac{dx}{dt} = Ax$ *in the state C*

$$A = \begin{pmatrix} -2 & 2 \\ -3 & 2 \end{pmatrix}$$

$\det A = 2, TrA = 0$
The C state resists if the matrix ϵ is chosen null.
The state C transforms it self into FI if we take

$$\epsilon = \begin{pmatrix} \epsilon_{11} & 0 \\ 0 & 0 \end{pmatrix}$$

with ϵ_{11} $i,p \succ 0$

$$A+\epsilon = \begin{pmatrix} -2+\epsilon_{11} & 2 \\ -3 & 2 \end{pmatrix}$$

$\det(A+\epsilon) = 2 + 2\epsilon_{11} \succ 0$, $Tr(A+\epsilon) = \epsilon_{11}$

The C state transforms it self into FS if we take $\epsilon = \begin{pmatrix} \epsilon_{11} & 0 \\ 0 & 0 \end{pmatrix}$ with $\epsilon_{11}i,\ p \succ 0$

We obtain:

$$\det(A+\epsilon) = 2 + 2\epsilon_{11} \succ 0, Tr\,(A+\epsilon) = \epsilon_{11} \succ 0. \tag{4.3}$$

$$\det(A+\epsilon) = 2 + 2\epsilon_{11} \succ \frac{1}{4}(Tr\,(A+\epsilon))^2 = \frac{1}{4}\epsilon_{11}^2 \succ 0. \tag{4.4}$$

qualitative state $colC$ with three answers over looked the small regular linear perturbation, to resist ,change into $colFS$, change into $colFI$

5 Conclusion

we used non-standard matrices infinitely close to standard matrices, then we try to see if the Poincares Classification looses its properties at the singular points. Our goal is to find, for non-linear systems when the linearized is a matrix close to a standard matrix, a possible link between what we do and to generalize our results.

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