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On The Cohen p-Nuclear Positive Sublinear Operators

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Abstract

In the present paper, we will introduce the concept of Cohen p-nuclear positive sublinear operators. We give an analogue to "Pietsch's domination theorem" and we study some properties concerning this notion.

Keywords: Cohen p-nuclear operators, Pietsch's domination theorem, Strongly p-summing operators, Positive operator, Sublinear operators.

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1 Introduction

For a Banach space X, X^* will denote its topological dual and B_X will denote its closed unit ball. For a Banach lattice E, E^+ will denote its positive cone. Throughout the paper, X, Y will be Banach spaces and E, F will be Banach lattices. Let $\mathcal{L}(X;Y)$ denote the Banach space of all continuous linear operators from X to Y. For $1 \leq p < \infty$, let p^* be its conjugate, that is, $1/p + 1/p^* = 1$.

The notion of Cohen p-nuclear operators $(1 \leq p \leq \infty)$ was initiated by Cohen in [9]. A linear operator u between two Banach spaces X, Y is Cohen p-nuclear for $(1 < p < \infty)$ if there is a positive constant C such that for all $n \in \mathbb{N}; x_1, ..., x_n \in X$ and $y_1, ..., y_n \in Y$ we have

$$\left|\sum_{i=1}^{n} \langle u(x_i), y_i^* \rangle\right| \leq C \sup_{x^* \in B_{X^*}} \left(\sum_{i=1}^{n} |x_i(x^*)|^p \, d\mu_1(x^*)\right)^{\frac{1}{p}} \times$$
$$\times \sup_{y \in B_Y} \left(\sum_{i=1}^{n} |y_i^*(y)|^{p^*} \, d\mu_2(y)\right)^{\frac{1}{p^*}}.$$

The smallest constant C which is noted by $n_p(u)$, such that the above inequality holds, is called the Cohen p-nuclear norm on the space $\mathcal{N}_p(X, Y)$ of all Cohen p-nuclear operators from X into Y which is a Banach space. We have $\mathcal{N}_1(X, Y) = \Pi_1(X, Y)$ (the Banach space of all 1-summing operators) and $\mathcal{N}_\infty(X, Y) = \mathcal{D}_\infty(X, Y)$ (the Banach space of all strongly ∞-summing operators).

In [9, Theorem 2.3.2], Cohen proves that, if u verifies a domination theorem then u is p-nuclear and he asked if the statement of this theorem characterizes p-nuclear operators. In [6], Achour et al. generalized this notion to the sublinear operators and they gave an analogue to "Pietsch's domination theorem" for this category of operators. Motivated by that, we study this notion with the positive sublinear maps and we propose, among others, an analogue to "Pietsch's domination theorem" for this category of operators which is one of the main results of this paper and we also discuss some properties concerning this class. It remains to prove the Pietsch's factorization theorem.

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This paper is organized as follows: In the first section, we give some basic definitions and terminology concerning Banach lattices. We also recall some standard notations. In the second section, we present some definitions and properties concerning positive sublinear operators. We give the definition of positive p-summing operators introduced by Blasco [7, 8] and we present the notion of strongly p-summing sublinear operators initiated in [6]. In Section 3, we generalize the class of Cohen p-nuclear operators to the positive sublinear operators. This category verifies a domination theorem, which is the principal result. We used another Technics than the Ky Fan's lemma. We end in Section 4, by studying a relation between some classes of positive sublinear operators (p-nuclear and p-summing).

2 Preliminary

We start by recalling the abstract definition of Banach lattices. Let E be a Banach space. If E is a vector lattice and $\|x\| \leq \|y\|$ whenever $|x| \leq |y|$ we say that E is a Banach lattice. If the lattice is complete, we say that E is a complete Banach lattice and for all x in $E, \|x\| = \||x|\|$. The dual E^* of a Banach lattice E is a complete endowed with the natural order $x_1, x_2 \in E$

$$x_1^* \leq x_2^* \Longleftrightarrow \langle x_1^*, x \rangle \leq \langle x_2^*, x \rangle, \forall x \in E^+.$$

where $\langle ., . \rangle$ denotes the bracket of duality. If we consider E as a Sublattice of E^{**} we have for

$$x_1 \leq x_2 \Longleftrightarrow \langle x_1, x^* \rangle \leq \langle x_2, x^* \rangle, \forall x^* \in E^{*+}.$$

for more details on this, the interested reader can consult the references [11].

Given $1 \leq p < \infty$ we will write $\ell_p^n(X)$ for the space of all sequences $(x_i)_{i=1}^n$ in X with the norm

$$\left\|(x_i)_{i=1}^n\right\|_p = \left(\sum_{i=1}^n \|x_i\|^p\right)^{\frac{1}{p}},$$

and $\ell_p^{n,w}(X)$ for the space of all sequences $(x_i)_{i=1}^n$ in X with the norm

$$\left\|(x_i)_{i=1}^n\right\|_{p,w} = \sup_{\|\phi\|_{X^*} \leq 1} \left(\sum_{i=1}^n |\phi(x_i)|^p\right)^{\frac{1}{p}},$$

where X^* denotes the topological dual of X. The closed unit ball of X will be denoted by B_X. Let $\ell_p(X)$ be the Banach space of all absolutely p-summable sequences $(x_i)_{i=1}^\infty$ in X with the norm

$$\left\|(x_i)_{i=1}^\infty\right\|_p = \left(\sum_{i=1}^\infty \|x_i\|^p\right)^{\frac{1}{p}}.$$

We denote by $\ell_p^w(X)$ the Banach space of all weakly p-summable sequences $(x_i)_{i=1}^\infty$ in X with the norm

$$\left\|(x_i)_{i=1}^\infty\right\|_{p,w} = \sup_{\|\phi\|_{X^*} \leq 1} \left(\sum_{i=1}^\infty |\phi(x_i)|^p\right)^{\frac{1}{p}},$$

Note that $\ell_p^w(X) = \ell_p(X)$ for some $1 \leq p < \infty$ if, and only if, X is finite dimensional. We continue in specifying definitions of the convexity and the concavity.

Definition 2.1. *Let* $1 \leq p \leq \infty$.

(i) *A sublinear operator* $T : F \longrightarrow E$ *is called a* p*-convex if there exists a constant* C *such that for every* n *in* $\mathbb{N}$ *the operators*

$$\begin{array}{rccc} T_n : & \ell_p^n(F) & \longrightarrow & E\left(\ell_p^n\right) \\ & (x_1, ..., x_n) & \longmapsto & (T(x_1), ..., T(x_n)) \end{array}$$

are uniformly bounded by C.

(ii) *A sublinear operator $T : E \longrightarrow F$ is called a p-convex if there exists a constant C such that for every n in $\mathbb{N}$ the operators*

$$\begin{array}{llll} T_n : & E\left(\ell_p^n\right) & \longrightarrow & \ell_p^n(F) \\ & (x_1, ..., x_n) & \longmapsto & (T(x_1), ..., T(x_n)) \end{array}$$

are uniformly bounded by C.

The space E is p-convex (p-concave) if id_E is p-convex (p-concave).

3 Positive sublinear operators

We give in this section some elementary definitions and fundamental properties relative to positive sublinear operators, for example see [6].

Definition 3.1. *An operator T from X into F is said to be positive sublinear if we have for all x, y in X and λ in $\mathbb{R}_+$.*

$$\begin{array}{ll} i) & T(\lambda x) = \lambda T(x), \\ ii) & T(x+y) \leq T(x) + T(y), \\ iii) & T(x) \geq 0. \end{array}$$

Let us denote by

$$\mathcal{SL}^+(X, F) = \{\text{positive sublinear operators, } T : X \longrightarrow F\}.$$

A positive sublinear operator is continuous if, and only if, there is $C > 0$ such that for all $x \in X$, $\|T(x)\| \leq C\|x\|$. In this case, we said that T is bounded and we write

$$\|T\| = \sup_{x \in B_X} \|T(x)\|$$

and we put

$$\mathcal{SB}^+(X, F) = \{\text{bounded positive sublinear operators, } T : X \longrightarrow F\}.$$

Remark 3.2. If $u : X \longrightarrow F$ is a linear operator, then $|u|$ is a positive sublinear operator.

Proposition 3.3. *Let T be a symmetric sublinear operator between X and F. Then, T is positive.*

Proof. For every x in X

$$\begin{array}{rcl} 0 & = & T(x-x) \\ & \leq & T(x) + T(-x) \\ & \leq & 2T(x). \quad \blacksquare \end{array}$$

Lemma 3.4. *Let $T : E \longrightarrow F$ be an increasing sublinear operator, if $|T|$ exist, then*

$$|T(x)| \leq |T|(|x|)$$

for all $x \in E$.

Proof. As $x \leq |x|$ and $-x \leq |x|$. Then by the monotonicity of T, we have

$$\forall x \in E, T(x) \leq T(|x|),$$

and

$$\forall x \in E, -T(x) \leq T(-x) \leq T(|x|),$$

and also

$$|T(x)| \leq T(|x|) \leq |T|(|x|)$$

for all $x \in E$. ■

Now, we study the continuity of an increasing positive sublinear operator. We adapt the same demonstration as in the linear case see [1, 12].

Theorem 3.5. *Let $T : E \longrightarrow F$ be an increasing positive sublinear operator. Then, T is continuous.*

Proof. We assume that T is not continuous. Then there exists a sequence $(x_n)_n$ in E with $\|x_n\| = 1$ such that $\|T(x_n)\| \geq n^3$ for all $n \in \mathbb{N}$. We have $|T(x_n)| \leq T(|x_n|)$, one can take $x_n \geq 0$ for all n. As $\sum\limits_{n \geq 1} \frac{\|x_n\|}{n^2} < \infty$ and E is complete, then the serie $\sum\limits_{n \geq 1} \frac{x_n}{n^2}$ converges in norm in E. Let $x = \sum\limits_{n \geq 1} \frac{x_n}{n^2}$. Then, it is clear that $0 \leq \frac{x_n}{n^2} \leq x$ for all n, and $T\left(\frac{x_n}{n^2}\right) \leq T(x)$ for all n, since T is increasing, we write $n \leq \left\|T\left(\frac{x_n}{n^2}\right)\right\| \leq \|T(x)\| < \infty$, for all n by the monotonicity of the norm of F, contradiction. Then T is continuous. ■

Remark 3.6. Without increase, we not know the answer. But we conjucture it's true.

Definition 3.7. *We said that a positive sublinear operator T between X, F is p-regular, $1 \leq p < \infty$, if there exist a constant $C > 0$ such that for all $(x_i)_1^n \subset X$, we have*

$$\left\| \left(\sum_{i=1}^{n} |T(x_i)|^p \right)^{\frac{1}{p}} \right\|_F \leq C \left\| \left(\sum_{i=1}^{n} |x_i|^p \right)^{\frac{1}{p}} \right\|_X \tag{3.1}$$

if $p < +\infty$, and if $p = +\infty$, we take the sup.

We note by

$$\rho_p(X, F) = \{p\text{-regular positive sublinear operators } T : X \longrightarrow F\}$$

and

$$\rho_p(T) = \inf\{C, \text{verifying the inequality } (3.1)\}.$$

The above proposition is not true for positive sublinear operators.

Proposition 3.8 [11, Proposition $1.d.9$**].** *Let $T : E \longrightarrow F$ be a positive operator. Then, for every $1 \leq p \leq \infty$, T is p-regular.*

The following counterexample (communicated by Gilles Godefroy, 2002), shows that the positive sublinear operator T isn't 2-regular..

We define a function S_r by

$$\begin{array}{llll} S_r: & L_2(T) & \longrightarrow & L_1(\Omega, \mu); \qquad\qquad T = \mathbb{R}/2\pi\mathbb{Z} \\ & f & \longrightarrow & S_r(f) = \frac{1}{2r}\int_{x-r}^{x+r} |f(y)|^2\, dy, \quad \forall x \in \mathbb{R} \text{ et } 0 < r \leq \pi. \end{array}$$

We put $T_r f = \sqrt{S_r f}$, hence the operator T_r is sublinear, and the operator T defined by

$$Tf = \sup\{T_r f : 0 < r < \pi\}.$$

For more details, see **[4].**

Proposition 3.9. *Let $1 \leq p < \infty$. Then $i) \Longleftrightarrow ii)$. Such that:*
i) F is p-concave.
ii) Every p-regular positive sublinear operators $T : X \longrightarrow F$, is p-concave.

Proof.
$ii) \Longrightarrow i)$ We put $X = F$ and $T = Id_X$.
$i) \Longrightarrow ii)$ We suppose that F is p-concave, i.e.,

$$\forall f_1, ..., f_n \in F, \left(\sum_{i=1}^{n} \|f_i\|^p\right)^{\frac{1}{p}} \le K \left\| \left(\sum_{i=1}^{n} |f_i|^p\right)^{\frac{1}{p}} \right\|.$$

For all $x_1, ..., x_n$ in X,

$$\begin{aligned}\left(\sum_{i=1}^{n} \|T(x_i)\|^p\right)^{\frac{1}{p}} &\le K \left\| \left(\sum_{i=1}^{n} |T(x_i)|^p\right)^{\frac{1}{p}} \right\| \\ &\le K' \left\| \left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}} \right\|, \ K' = K\|T\|.\end{aligned}$$

Then T is concave. ■

Corollary 3.10. *Every p-regular positive sublinear operators $T : X \longrightarrow L_p$, $1 \le p < \infty$, is bounded.*

Proof. It is easy.

Proposition 3.11. *Let $1 < p < \infty$. Then $i) \Longleftrightarrow ii)$. Such that:*
i) E is p-convex.
ii) Every p-regular positive sublinear operators $T : E \longrightarrow Y$, is p-convex.

Proof.
$i) \Longrightarrow ii)$ We have, for all $x_1, ..., x_n$ in E

$$\begin{aligned}\left\| \left(\sum_{i=1}^{n} |T(x_i)|^p\right)^{\frac{1}{p}} \right\|_Y &\le \|T\| \left\| \left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}} \right\|_E, \ p\text{-regular} \\ &\le C\|T\| \left(\sum_{i=1}^{n} \|x_i\|_E^p\right)^{\frac{1}{p}}.\end{aligned}$$

Then T is p-convex. The converse is obvious. ■

4 Cohen p-nuclear positive sublinear operators

To conclude this section, we recall the definition of positive p-summing sublinear operators, which was first stated in the linear case by Blasco in [7].

Definition 4.1. *Let $T : X \longrightarrow F$ be a positive sublinear operator. We will say that T is "p-summing" ($1 \le p < +\infty$) (we write $T \in S\Pi_p^+(X, F)$), if there exists a positive constant C such that for all $n \in \mathbb{N}$ and all $\{x_1, ..., x_n\} \subset X$, we have*

$$\|(T(x_i))\|_{\ell_p^n(F)} \le C \|(x_i)\|_{\ell_p^{nw}(X)}. \tag{4.2}$$

We put $\pi_p^+(T) = \inf\{C$ verifying the inequality (4.2)$\}$.

We introduce the following extension of the class of Cohen p-nuclear operators. We give the domination theorem for such a category.

Definition 4.2. *Let $1 < p < \infty$. A positive sublinear operator T between X and F is p-nuclear if there is $C > 0$ such that for all $n \in \mathbb{N}$ and $x_1, ..., x_n$ in X, $y_1^*, ..., y_n^*$ in F^{*+} we have:*

$$\left|\sum_{i=1}^{n}\left\langle T\left(x_i\right),y_i^*\right\rangle\right| \leq C \sup_{x^*\in B_{X^*}^+}\left(\sum_{i=1}^{n}\left(\left|x_i\right|\left(x^*\right)\right)^p d\mu_1\left(x^*\right)\right)^{\frac{1}{p}} \times$$
$$\times \sup_{y^{**}\in B_{F^{**}}^+}\left(\sum_{i=1}^{n}\left(y_i^*\left(y^{**}\right)\right)^{p^*} d\mu_2\left(y^{**}\right)\right)^{\frac{1}{p^*}} \qquad (I)$$

We denote by $n_p^+(T)$ the smallest constant C which verified the inequality (I), called p-nuclear norm on $\mathcal{SN}_p^+(X,F)$, the Banach space of all p-nuclear positive sublinear operators. If $p=1$, we obtain the Banach space of all 1-summing positive sublinear operators.

Theorem 4.3. (**Composition theorem**). *Let X be a Banach space, E and F two Banach lattices. Let T be in $\mathcal{SB}^+(X,E)$, u a positive operator in $\mathcal{L}(E,F)$ and v in $\mathcal{L}(Y,X)$.*

i) If T is Cohen p-nuclear, then $u\circ T$ is p-nuclear positive sublinear operator and $n_p^+(u\circ T)\leq \|u\|\, n_p^+(T)$.

ii) If T is Cohen p-nuclear, then $T\circ v$ is p-nuclear positive sublinear operator and $n_p^+(T\circ v)\leq \|v\|\, n_p^+(T)$.

Theorem 4.4. *A positive sublinear operator between X,F is p-summing $(1\leq p<+\infty)$, if, and only if, there exists a positive constant $C>0$ and a Borel probability μ on $B_{X^*}^+$ such that*

$$\|T(x)\| \leq \pi_p^+(T)\left(\int_{B_{E^*}^+}\left(|x|\left(x^*\right)\right)^p d\mu\left(x^*\right)\right)^{\frac{1}{p}} \tag{4.3}$$

for every $x\in X$. Moreover, in this case $\pi_p^+(T)=\inf\{C>0$: for all C verifying the inequality $(4.3)\}$.

Proof. It is similar to the linear case (see [7]).

The main result of this section is the next theorem.

Theorem 4.5. *Let T be a bounded positive sublinear operator from X into F. Then the two following properties are equivalent.*

1) *The operator T is in $\mathcal{SN}_p^+(X,F)$.*

2) *There are some Banach space Z, a positive p-summing sublinear operator $u: X\longrightarrow Z$ and a positive strongly p-summing operator $v: Z\longrightarrow F$ such that $T=vu$.*

Proof. 1) $\Longrightarrow$ 2) We consider the operator $u_0: x\in X\longrightarrow \langle |x|,.\rangle\in L_p\left(B_{X^*}^+,\mu\right)$, we notice that $\|Tx\|\leq C\left\|u_0(x)\right\|$, for all $x\in X$, let Z be a closed subspace of $L_p(\mu)$ such that $Z=\overline{u_0(X)}$, and let $u: X\longrightarrow Z$ the induite operator. Notice that u is a positive p-summing sublinear operator from X into Z with $\pi_p^+(u)\leq 1$. We write $T=vu$, for some $v\in\mathcal{L}(Z,F)$. If $y^*\in F^{*+}$, then

$$\begin{aligned}
\left\|v^*\left(y^*\right)\right\| &= \sup\left\{\left|\left\langle u(x),v^*\left(y^*\right)\right\rangle\right|\right\}: \|u(x)\|_p\leq 1\\
&= \sup\left|\left\langle T(x),y^*\right\rangle\right|: \int_{B_{X^*}^+}\left|\left\langle x^*,|x|\right\rangle\right|^p d\mu\left(x^*\right)\leq 1\\
&\leq C\left(\int_{B_{F^{**}}^+}\left|\left\langle y^{**},y^*\right\rangle\right|^{p^*} d\lambda\left(y^{**}\right)\right)^{\frac{1}{p^*}}.
\end{aligned}$$

by Pietsch's domination theorem for positive p-summing operators, $v^*\in\Pi_{p^*}^+(F^*,Z^*)$ and $\pi_{p^*}^+(v^*)\leq C$. This implies that v is a positive strongly p-summing operator, see [2, Theorem 4.6].

2) $\Longrightarrow$ 1) It's clear. ∎

5 Applications

The main result of this section is the next extension of the Pietsch's domination theorem to this class of operators. For proof, we will use Theorem 4.5. In [6], Achour et al. used Ky Fan's lemma to prove the domination theorem.

Theorem 5.1. *The following two conditions are equivalent.*

1) $T : X \longrightarrow F$ *is Cohen p-nuclear positive sublinear operator and* $n_p^+(T) \leq C$.

2) *There exists a constant* $C \geq 0$ *and two positives Radon measures* μ_1 *on* $B_{X^*}^+$ *and* μ_2 *on* $B_{F^{**}}^+$, *such that for all* $x \in E$ *and* $y^* \in F^{*+}$, *we have*

$$|\langle T(x), y^*\rangle| \leq C\left(\int_{B_{X^*}^+} (|x|(x^*))^p \, d\mu_1(x^*)\right)^{\frac{1}{p}} \left(\int_{B_{F^{**}}^+} (y^*(y^{**}))^{p^*} \, d\mu_2(y^{**})\right)^{\frac{1}{p^*}} \qquad (J)$$

in this case

$$n_p(T) = \inf\{C > 0, \text{ for all } C, \text{verifying the inequality } (J)\}.$$

Proof. 2) $\Longrightarrow$ 1) Letting $x_1, ..., x_n \in X$ and $y_1^*, ..., y_n^* \in F^{*+}$ according to (J), we have

$$|\langle T(x_i), y_i^*\rangle| \leq C\left(\int_{B_{X^*}^+} (|x_i|(x^*))^p \, d\mu_1(x^*)\right)^{\frac{1}{p}} \left(\int_{B_{F^{**}}^+} (y_i^*(y^{**}))^{p^*} \, d\mu_2(y^{**})\right)^{\frac{1}{p^*}}.$$

We deduce,

$$\begin{aligned}
\left|\sum_{i=1}^n \langle T(x_i), y_i^*\rangle\right| &\leq \\
&\leq C\sum_{i=1}^n \left(\int_{B_{X^*}^+} (|x_i|(x^*))^p \, d\mu_1(x^*)\right)^{\frac{1}{p}} \left(\int_{B_{F^{**}}^+} (y_i^*(y^{**}))^{p^*} \, d\mu_2(y^{**})\right)^{\frac{1}{p^*}} \\
&\leq C\left(\sum_{i=1}^n \int_{B_{X^*}^+} (|x_i|(x^*))^p \, d\mu_1(x^*)\right)^{\frac{1}{p}} \left(\sum_{i=1}^n \int_{B_{F^{**}}^+} (y_i^*(y^{**}))^{p^*} \, d\mu_2(y^{**})\right)^{\frac{1}{p^*}} \\
&\leq C \sup_{x^* \in B_{X^*}^+} \left(\sum_{i=1}^n (|x_i|(x^*))^p \, d\mu_1(x^*)\right)^{\frac{1}{p}} \sup_{y^{**} \in B_{F^{**}}^+} \left(\sum_{i=1}^n (y_i^*(y^{**}))^{p^*} \, d\mu_2(y^{**})\right)^{\frac{1}{p^*}}
\end{aligned}$$

This implies that T is a p-nuclear positive sublinear operator.

1) $\Longrightarrow$ 2) If $T \in S\mathcal{N}_p^+(E, F)$, thus, according to the above $T = vu$ where $u \in S\Pi_p^+(E, Z)$ and $v \in \mathcal{D}_p^+(Z, F)\left[v^* \in \pi_{p^*}^+(F^*, Z^*)\right]$. by [6, Thm 2.4] and [2, Theorem 4.13] there exist a constant $C > 0$, two positive Radon measures μ_1 on $B_{E^*}^+$ and μ_2 on $B_{F^{**}}^+$, endowed with their weak$*$ topologies, such that for all $x \in E$ and $y^* \in F^{*+}$,

$$\begin{aligned}
|\langle T(x), y^*\rangle| &= |\langle vu(x), y^*\rangle| \\
&= |\langle u(x), v^*(y^*)\rangle| \\
&\leq \|u(x)\| \, \|v^*(y^*)\| \\
&\leq C\left(\int_{B_{E^*}^+} (\langle |x|, x^*\rangle)^p \, d\mu_1(x^*)\right)^{\frac{1}{p}} \left(\int_{B_{F^{**}}^+} \langle y^*, y^{**}\rangle^{p^*} \, d\mu_2(y^{**})\right)^{\frac{1}{p^*}}.
\end{aligned}$$

This was proven. ■

Now we are ready to use the Grothendieck–Maurey theorem in the positive sublinear case.

Theorem 5.2. *Let E, F and G be three Banach lattices where G is 2-concave space. Let $T : C(K) \longrightarrow E$ be 2-regular positive sublinear operator, $w : E \longrightarrow F$ a positive 2-concave operator and a positive strongly p-summing operator $v : F \longrightarrow G$. Then vwT is Cohen 2-nuclear positive sublinear operator and $n_2^+(vwT) \leq d_2^+(v)\, C_2^+(w)\, \rho_2(T)$.*

Proof. The operator wT is positive 2-summing sublinear [5, Theorem 3.6] and by Theorem 4.5, the operator vwT is Cohen 2-nuclear positive sublinear. ■

Proposition 5.3. *We have*

$$S\mathcal{N}_p^+(E,F) \subseteq S\Pi_p^+(E,F) \text{ and } \pi_p^+(T) \leq n_p^+(T).$$

Proof. Let T be an operator in $S\mathcal{N}_p^+(E,F)$. For all $x \in E$, we have

$$\begin{aligned}
\|T(x)\| &= \sup_{y^* \in B_{F^*}^+} |\langle T(x), y^* \rangle| \\
&\leq \sup_{y^* \in B_{F^*}^+} n_p^+(T) \left(\int_{B_{E^*}^+} (|x|(x^*))^p \, d\mu_1(x^*) \right)^{\frac{1}{p}} \left(\int_{B_{F^{**}}^+} (y^*(y^{**}))^{p^*} \, d\mu_2(y^{**}) \right)^{\frac{1}{p^*}} \\
&\leq n_p^+(T) \left(\int_{B_{E^*}^+} (|x|(x^*))^p \, d\mu_1(x^*) \right)^{\frac{1}{p}} \sup_{y^* \in B_{F^*}^+} \|y^*\| \\
&\leq n_p^+(T) \left(\int_{B_{E^*}^+} (|x|(x^*))^p \, d\mu_1(x^*) \right)^{\frac{1}{p}}.
\end{aligned}$$

Then, T is a positive p-summing sublinear operator and $\pi_p^+(T) \leq n_p^+(T)$. ■

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