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Oscillation theorems for higher order neutral nonlinear dynamic equations on time scales

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Abstract

In this paper, we will establish some oscillation criteria for the even-order nonlinear dynamic equation

$$\left(a\left(x^{\Delta^{n-2}}\right)^{\gamma}\right)^{\Delta^2}(t)+f\left(t,x^{\alpha}(t)\right)=0,\qquad t\in[t_0,\infty)_{\mathbb{T}}$$

on a time scales $\mathbb{T}$ with n is an even integer ≥ 3, where γ and α are the ratios of positive odd integer and a is areal valued rd-continuous function defined on $\mathbb{T}$.

Keywords: Time scale, Oscillation, Neutral delay differential equation.

2010 MSC: 34K11, 39A10, 39A99.

1 Introduction

The theory of time scales was introduced by Hilger [1] in order to unify, extend and generalize ideas from discrete calculus, quantum calculus and continuous calculus to arbitrary time scale calculus. The books on the subjects of time scale, that is, measure chain, by Bohner and Peterson [2], [3], summarize and organize much of time scale calculus.

The theory of oscillations is an important branch of the applied theory of dynamic equations related to the study of oscillatory phenomena in technology and natural and social sciences. In recent years, there has been much research activity concerning the oscillation of solutions of various dynamic equations on time scales.

In this paper, we deal with the oscillation of all solutions of the even-order nonlinear delay dynamic equation

$$\left(a\left(x^{\Delta^{n-2}}\right)^{\gamma}\right)^{\Delta^2}(t)+f\left(t,x^{\alpha}(t)\right)=0,\qquad t\in[t_0,+\infty)_{\mathbb{T}}\tag{1.1}$$

on a time scale $\mathbb{T}$ with $\sup\mathbb{T}=\infty$, n is an even integer ≥ 3. Where α, γ are a quotient of odd positive integer, $a\in\mathcal{C}^1(\mathbb{T},\mathbb{R}^+)$ such that $a^{\Delta}(t)>0$ for $t\in[t_0,\infty)_{\mathbb{T}}$ and f satisfies the following conditions:

$(\mathcal{H}_1)$ $f:\mathbb{T}\times\mathbb{R}\longrightarrow\mathbb{R}$ is continuous,

$(\mathcal{H}_2)$ $f(t,-x)=-f(t,x)$ for all $t\in[t_0,\infty)_{\mathbb{T}}$, $x\in\mathbb{R}$,

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$(\mathcal{H}_3)$ There exist a function $r : \mathbb{T} \longrightarrow \mathbb{R}$ positive and rd-continuous, such that

$$\frac{f(t,x)}{x} \geq r(t), \qquad \text{for all } t \in [t_0, \infty)_{\mathbb{T}}, x \in \mathbb{R} - \{0\}. \tag{1.2}$$

In order to prove our theorems we shall need the following two lemmas.

Lemma 1.1. *[4] If $n \in \mathbb{N}$, $\sup \mathbb{T} = \infty$ and $f \in \mathcal{C}_{rd}^{n}([t_0,\infty)_{\mathbb{T}}, \mathbb{R})$ then the following statements are true.*

1. $\liminf_{t\to\infty} f^{\Delta^n}(t) > 0$ *implies* $\lim_{t\to\infty} f^{\Delta^k}(t) = \infty$ *for all* $k \in [0,n)_{\mathbb{Z}}$.
2. $\limsup_{t\to\infty} f^{\Delta^n}(t) < 0$ *implies* $\lim_{t\to\infty} f^{\Delta^k}(t) = -\infty$ *for all* $k \in [0,n)_{\mathbb{Z}}$.

Lemma 1.2. *[7] Assume that* $\sup \mathbb{T} = \infty$, $f \in \mathcal{C}_{rd}^{1}([t_0,\infty)_{\mathbb{T}}, \mathbb{R}^{+})$ *and* $\lambda > 0$. *Then*

$$f^{\Delta}(f^{\sigma})^{-\lambda} \leq \frac{\left(f^{1-\lambda}\right)^{\Delta}}{1-\lambda} \leq f^{\Delta} f^{-\lambda}, \quad on \ [t_0,\infty)_{\mathbb{T}}.$$

2 Main results

In this section, we establish some sufficient conditions which guarantee that every solution x of (1.1) oscillates on $[t_0,\infty)_{\mathbb{T}}$.

Before stating the main results, we begin with the following lemma.

Lemma 2.3. *Suppose that x is an eventually positive solution of* (1.1) *and*

$$\lim_{t\to\infty} \frac{1}{a(t)} \in \mathbb{R}_{+}^{*}, \qquad \lim_{t\to\infty} \frac{t}{a(t)} \int_{t}^{\infty} r(s)\,\Delta s = \infty. \tag{2.3}$$

Then there exists $t_1 \in [t_0,\infty)_{\mathbb{T}}$ such that

$$\left(a\left(x^{\Delta^{n-2}}\right)^{\gamma}\right)^{\Delta}(t) > 0, \qquad x^{\Delta^{n-2}}(t) > 0, \text{ for all } t \in [t_1,\infty)_{\mathbb{T}}. \tag{2.4}$$

Lemma 2.4. *Assume that x is an eventually positive solution of* (1.1) *and* (2.3) *hold. Suppose there exists a sequence functions $\phi_1, \phi_2, \cdots, \phi_{n-2} \in \mathcal{C}_{rd}^{1}([t_0,\infty)_{\mathbb{T}}, \mathbb{R}^{+})$. Let $A_1, A_2, \cdots A_{n-2}$ are functions defined by*

$$A_1(t,t_1) := \left\{\frac{a(t)}{\phi_1(t)}\right\}^{\frac{1}{\gamma}} \int_{t_1}^{t} \left\{\frac{\phi_1(s)}{a(s)}\right\}^{\frac{1}{\gamma}} \Delta s, \qquad \text{for } t \in [t_1,\infty)_{\mathbb{T}},$$

and

$$A_k(t,t_1) := \frac{1}{\phi_k(t)} \int_{t_1}^{t} \phi_k(s)\,\Delta s, \quad \text{for all } t \in [t_1,\infty)_{\mathbb{T}} \text{ and all } k \in [2,n-1)_{\mathbb{Z}}.$$

where $t_1 \in [t_0,\infty)_{\mathbb{T}}$. Moreover, suppose that

$$\phi_1(t) - \phi_1^{\Delta}(t)(t-t_1) \leq 0, \qquad \text{for } t \in [t_1,\infty)_{\mathbb{T}}, \tag{2.5}$$

and

$$\phi_k(t) - \phi_k^{\Delta}(t) A_{k-1}(t,t_1) \leq 0, \quad \text{for all } t \in [t_1,\infty)_{\mathbb{T}} \text{ and all } k \in [2,n-1)_{\mathbb{Z}}. \tag{2.6}$$

Then

$$x^{\Delta^k}(t) \geq E_k(t,t_1)\, x^{\Delta^{n-2}}(t), \quad \text{for all } t \in [t_1,\infty)_{\mathbb{T}} \text{ and all } k \in [0,n-2)_{\mathbb{Z}},$$

where

$$E_k(t,t_1) := \prod_{m=1}^{m=n-k-2} A_m(t,t_1), \quad \text{for all } t \in [t_1,\infty)_{\mathbb{T}}.$$

Theorem 2.1. *Let* (2.3) *hold and* $\alpha > \gamma$. *Assume that there exist sufficiently large* $t_1 \in [t_0, \infty)_{\mathbb{T}}$, *such that*

$$\int_{t_1}^{\infty} E_1(t, t_1) \left(\frac{t - t_1}{a(t)} \int_{\sigma(t)}^{\infty} r(u) \Delta u \right)^{\frac{1}{\gamma}} \Delta t = \infty, \tag{2.7}$$

where E_1 *is defined as in Lemma 2.4.*
Then equation (1.1) *is oscillatory.*

Proof. Suppose the contrary, that $x(t)$ is a nonoscillatory solution of (1.1). Without loss of generality, we may assume that $x(t)$ is an eventually positive solution of (1.1), since the substitution $y(t) = -x(t)$ transforms equation (1.1) into an equation of the same form. Say $x(t) > 0$ for $t \geq t_1 \geq t_0$.

By (1.2) , we get

$$\left(a \left(x^{\Delta^{n-2}} \right)^{\gamma} \right)^{\Delta^2} (t) \leq -r(t) x^{a}(t), \qquad \text{for } t \in [t_1, \infty)_{\mathbb{T}}. \tag{2.8}$$

Integrating (2.8) form t to ∞, we have

$$\left(a \left(x^{\Delta^{n-2}} \right)^{\gamma} \right)^{\Delta} (t) \geq \int_{t}^{\infty} r(s) x^{\alpha}(s) \Delta s, \qquad \text{for } t \in [t_1, \infty)_{\mathbb{T}}. \tag{2.9}$$

By (2.8), we have that $\left(a \left(x^{\Delta^{n-2}} \right)^{\gamma} \right)^{\Delta}$ is nonincreasing in $[t_1, \infty)_{\mathbb{T}}$. Then, for all $t \in [t_1, \infty)_{\mathbb{T}}$, we obtain

$$a(t) \left(x^{\Delta^{n-2}}(t) \right)^{\gamma} \geq \int_{t_1}^{t} \left(a \left(x^{\Delta^{n-2}} \right)^{\gamma} \right)^{\Delta} (s) \Delta s \geq (t - t_1) \left(a \left(x^{\Delta^{n-2}} \right)^{\gamma} \right)^{\Delta} (t).$$

As above we see that

$$x^{\Delta^{n-2}}(t) \geq \left(\frac{t - t_1}{a(t)} \int_{t}^{\infty} r(s) x^{\alpha}(s) \Delta s \right)^{\frac{1}{\gamma}}, \qquad \text{for } t \in [t_1, \infty)_{\mathbb{T}}.$$

By lemma 2.4, we have

$$x^{\Delta}(t) \geq \left(\frac{t - t_1}{a(t)} \int_{t}^{\infty} r(s) x^{\alpha}(s) \Delta s \right)^{\frac{1}{\gamma}} E_1(t, t_1), \qquad \text{for } t \in [t_1, \infty)_{\mathbb{T}}.$$

Clearly $x^{\Delta}(t) > 0$, for $t \in [t_1, \infty)_{\mathbb{T}}$, then

$$x^{\Delta}(t) x^{\frac{-\alpha}{\gamma}}(\sigma(t)) \geq \left(\frac{t - t_1}{a(t)} \int_{\sigma(t)}^{\infty} r(s) \Delta s \right)^{\frac{1}{\gamma}} E_1(t, t_1), \qquad \text{for } t \in [t_1, \infty)_{\mathbb{T}}.$$

By lemma 1.2, we get

$$\frac{\gamma}{\gamma - \alpha} \left(x^{1 - \frac{\alpha}{\gamma}} \right)^{\Delta} (t) \geq \left(\frac{t - t_1}{a(t)} \int_{\sigma(t)}^{\infty} r(s) \Delta s \right)^{\frac{1}{\gamma}} E_1(t, t_1), \quad \text{for } t \in [t_1, \infty)_{\mathbb{T}}. \tag{2.10}$$

Integrating (2.10) from t_1 to t and letting $t \to \infty$, we have

$$\int_{t_1}^{\infty} E_1(t, t_1) \left(\frac{t - t_1}{a(t)} \int_{\sigma(t)}^{\infty} r(s) \Delta s \right)^{\frac{1}{\gamma}} \Delta t \leq -\frac{\gamma}{\gamma - \alpha} x^{1 - \frac{\alpha}{\gamma}}(t_1).$$

This result is in contradiction with (2.7) . □

Theorem 2.2. *Let* (2.3) *holds and* $\alpha = \gamma \geq 1$. *Assume that there exist positive function* $\delta \in C^1_{rd}([t_0,\infty)_{\mathbb{T}},\mathbb{R})$ *such that for all sufficiently large* $t_1 \in [t_0,\infty)_{\mathbb{T}}$, *for some* $t_2 \in [t_1,\infty)_{\mathbb{T}}$ *such that*

$$\int_{t_2}^{\infty} \delta(t)\, r(t) - \frac{\gamma^\gamma}{(\gamma+1)^{\gamma+1}} \frac{\left(\delta_+^\Delta(t)\right)^{\gamma+1} a(t)}{\delta^\gamma(t)\, E_1^\gamma(t,t_1)\,(t-t_1)} \Delta t = \infty, \tag{2.11}$$

where $\delta_+^\Delta(t) = \max\left(0, \delta^\Delta(t)\right)$ *and* E_1 *is defined as in Lemma 2.4.*
Then equation (1.1) *is oscillatory.*

Proof. Suppose that (1.1) has a nonoscillatory solution x on $[t_0,\infty)_{\mathbb{T}}$. We may assume without loss of generality that there exists $t_1 \in [t_0,\infty)_{\mathbb{T}}$ such that $x(t) > 0$ for $t \in [t_1,\infty)_{\mathbb{T}}$.
We define the function $w(t)$ by

$$w(t) = \delta(t) \frac{\left(a\left(x^{\Delta^{n-2}}\right)^\gamma\right)^\Delta(t)}{x^\gamma(t)}, \qquad t \in [t_1,\infty)_{\mathbb{T}}.$$

Then $w(t) > 0$ for $t \in [t_1,\infty)_{\mathbb{T}}$ and by (2.8) which implies that

$$\begin{aligned} w^\Delta(t) &\leq -\delta(t)\, r(t) + \frac{w^\sigma(t)}{\delta^\sigma(t)} x^\gamma(\sigma(t)) \left\{ \frac{\delta^\Delta(t)\, x^\gamma(t) - \delta(t)\,(x^\gamma)^\Delta(t)}{x^\gamma(t)\, x^\gamma(\sigma(t))} \right\} \\ &\leq -\delta(t)\, r(t) + \frac{\delta^\Delta(t)}{\delta^\sigma(t)} w^\sigma(t) - w^\sigma(t) \frac{\delta(t)\,(x^\gamma)^\Delta(t)}{\delta^\sigma(t)\, x^\gamma(t)}. \end{aligned} \tag{2.12}$$

By Pötzsche's chain rule [2], we get

$$\begin{aligned} (x^\gamma(t))^\Delta &= \gamma x^\Delta(t) \int_0^1 (hx(t) + (1-h)x^\sigma(t))^{\gamma-1}\, dh \\ &\geq x^\Delta(t) x^{\gamma-1}(t). \end{aligned} \tag{2.13}$$

Substituting (2.13) in (2.12), we find

$$w^\Delta(t) \leq -\delta(t)\, r(t) + \frac{\delta^\Delta(t)}{\delta^\sigma(t)} w^\sigma(t) - w^\sigma(t) \frac{\delta(t)\, x^\Delta(t)}{\delta^\sigma(t)\, x(t)}. \tag{2.14}$$

By lemma 2.4, we find

$$\begin{aligned} x^\Delta(t) &\geq \frac{E_1(t,t_1)}{(a(t))^{\frac{1}{\gamma}}} \left[a(t)\left(x^{\Delta^{n-2}}(t)\right)^\gamma\right]^{\frac{1}{\gamma}} \\ &\geq E_1(t,t_1) \left[\frac{t-t_1}{a(t)}\right]^{\frac{1}{\gamma}} \left[\left(a\left(x^{\Delta^{n-2}}\right)^\gamma\right)^\Delta(t)\right]^{\frac{1}{\gamma}} \\ &\geq E_1(t,t_1)\, x(t) \left(\frac{t-t_1}{a(t)\,\delta^\sigma(t)}\right)^{\frac{1}{\gamma}} (w^\sigma(t))^{\frac{1}{\gamma}}. \end{aligned} \tag{2.15}$$

Substituting (2.15) in (2.14), we get

$$w^\Delta(t) \leq -\delta(t)\, r(t) + \frac{\delta^\Delta(t)}{\delta^\sigma(t)} w^\sigma(t) - \frac{\delta(t)\, E_1(t,t_1)}{\delta^\sigma(t)} \left(\frac{t-t_1}{a(t)\,\delta^\sigma(t)}\right)^{\frac{1}{\gamma}} (w^\sigma(t))^{1+\frac{1}{\gamma}}.$$

Using the inequality [10]

$$By - Ay^{1+\frac{1}{\beta}} \leq \frac{\beta^\beta B^{\beta+1}}{(\beta+1)^{\beta+1} A^\beta}, \qquad A > 0,\ B > 0 \text{ and } \beta > 0.$$

which yields

$$w^\Delta(t) \leq -\delta(t)\, r(t) + \frac{\gamma^\gamma}{(\gamma+1)^{\gamma+1}} \frac{\left(\delta_+^\Delta(t)\right)^{\gamma+1} a(t)}{\delta^\gamma(t)\, E_1^\gamma(t,t_1)\,(t-t_1)}.$$

Integrating the last inequality from t_2 to t, we have

$$\int_{t_2}^{t} \delta(s) r(s) - \frac{\gamma^{\gamma} \left(\delta_{+}^{\Delta}(s)\right)^{\gamma+1} a(s)}{(\gamma+1)^{\gamma+1} \delta^{\gamma}(s) E_1^{\gamma}(s,t_1)(s-t_1)} \Delta s \leq w(t_2) - w(t) \leq w(t_2).$$

which contradicts (2.11). This completes the proof. □

Theorem 2.3. *Let* (2.3) *holds and* $\gamma > \alpha$. *Assume that there exist positive function* $\delta \in C_{rd}^{1}([t_0,\infty)_{\mathbb{T}}, \mathbb{R})$ *such that for all sufficiently large* $t_1 \in [t_0,\infty)_{\mathbb{T}}$, *such that*

$$\int_{t_1}^{\infty} \delta^{\sigma}(t) r(t) E_0^{\alpha}(t,t_1) \left(\frac{t-t_1}{a(t)\delta(t)}\right)^{\frac{\alpha}{\gamma}} \Delta t = \infty, \tag{2.16}$$

where $\delta^{\Delta}(t) \leq 0$, *for all* $t \in [t_1,\infty)_{\mathbb{T}}$ *and* E_0 *is defined as in Lemma 2.4.*
Then every solution of (1.1) *is either oscillatory.*

Proof. Suppose that (1.1) has a nonoscillatory solution x on $[t_0,\infty)_{\mathbb{T}}$. We may assume without loss of generality that there exists $t_1 \in [t_0,\infty)_{\mathbb{T}}$ such that $x(t) > 0$ for $t \in [t_1,\infty)_{\mathbb{T}}$.
Let

$$w(t) = \delta(t) \left(a\left(x^{\Delta^{n-2}}\right)^{\gamma}\right)^{\Delta}(t), \qquad t \in [t_1,\infty)_{\mathbb{T}}.$$

Then $w(t) > 0$ for $t \in [t_1,\infty)_{\mathbb{T}}$ and by (1.2), we obtain

$$w^{\Delta}(t) \leq -\delta^{\sigma}(t) r(t) x^{\alpha}(t). \tag{2.17}$$

By lemma 2.4, we get

$$\begin{aligned} x(t) &\geq E_0(t,t_1) x^{\Delta^{n-2}}(t) \\ &\geq E_0(t,t_1) \left(\frac{t-t_1}{a(t)\delta(t)}\right)^{\frac{1}{\gamma}} w^{\frac{1}{\gamma}}(t). \end{aligned} \tag{2.18}$$

Substituting (2.18) in (2.17), we find

$$-w^{\Delta}(t) w^{\frac{-\alpha}{\gamma}}(t) \geq \delta^{\sigma}(t) r(t) E_0^{\alpha}(t,t_1) \left(\frac{t-t_1}{a(t)\delta(t)}\right)^{\frac{\alpha}{\gamma}}.$$

By Lemma 1.2 we have

$$-\frac{\gamma}{\gamma-\alpha} \left(w^{1-\frac{\alpha}{\gamma}}\right)^{\Delta}(t) \geq \delta^{\sigma}(t) r(t) E_0^{\alpha}(t,t_1) \left(\frac{t-t_1}{a(t)\delta(t)}\right)^{\frac{\alpha}{\gamma}}.$$

Integrating this inequality from t_1 to t we obtain

$$\int_{t_1}^{t} \delta^{\sigma}(s) r(s) E_0^{\alpha}(s,t_1) \left(\frac{s-t_1}{a(s)\delta(s)}\right)^{\frac{\alpha}{\gamma}} \Delta s \leq \frac{\gamma}{\gamma-\alpha} w^{1-\frac{\alpha}{\gamma}}(t_1),$$

for all large t. This result is in contradiction with (2.16). This completes the proof. □

3 Example

As some application of the main results, we present the following example.

Example 3.1. *On the quantum set* $\mathbb{T} = \overline{2^{\mathbb{Z}}}$. *Consider the following n-order neutral differential equation*

$$x^{\Delta^n}(t) + t^{-\frac{3}{2}} x^{\alpha}(t) = 0, \quad t \in [1,\infty)_{\overline{2^{\mathbb{Z}}}}. \tag{3.19}$$

where $n \geq 3$ is even integer. Here $a(t) = 1$, $r(t) = t^{-\frac{3}{2}}$, $\gamma = 1$ and α is a quotient of odd positive integer.
It is easy to see that (2.3) *hold.*
Set

$$\phi_1(t) := h_k(t, t_1), \qquad \text{for all } k \in [1, n-1)_{\mathbb{Z}} \text{ and for } t \in [t_1, \infty)_{\overline{2^{\mathbb{Z}}}}.$$

Then (2.6) *and* (2.5) *holds.*
Moreover, for all $k \in [1, n-1)_{\mathbb{Z}}$, we have

$$A_k(t, t_1) = \frac{h_{k+1}(t, t_1)}{h_k(t, t_1)}, \qquad \text{for all } t \in [t_1, \infty)_{\overline{2^{\mathbb{Z}}}}.$$

Then

$$E_1(t, t_1) \left(\frac{(t - t_1)}{a(t)} \int_{\sigma(t)}^{\infty} r(u) \Delta u \right)^{\frac{1}{\gamma}} \geq \frac{h_{n-2}(t, t_1)}{\sqrt{t}}, \quad \text{for all } t \in [t_1, \infty)_{\overline{2^{\mathbb{Z}}}}.$$

By Theorem 2.1, every solution x of (3.19) *is either oscillatory.*

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