Malaya J. Mat. 4(4)(2016) 617–623

Malaya Journal of Matematik

www.malayajournal.org

MJM
an international journal of mathematical sciences with computer applications...

ISSN : 2319-3786

Exact soliton solutions of the generalized combined and the generalized double combined sinh-cosh-Gordon equations

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Abstract

In this paper, the extended tanh method is used to construct exact solutions of the generalized combined sinh-cosh-Gordon equations and the generalized double combined sinh-cosh-Gordon equations which arises in mathematical physics and has a wide range of scientific applications that range from chemical reactions to water surface gravity waves. The extended tanh method is an efficient method for obtaining exact solutions of nonlinear partial differential equations. This method can be applied to nonintegrable equations as well as to integrable ones.

Keywords: Extended tanh method, Combined sinh-cosh-Gordon equations, Double combined sinh-cosh-Gordon equation, soliton.

2010 MSC: 47F05, 35QXX.

1 Introduction

Phenomena in physics and other fields are often described by nonlinear evolution equations. When we want to understand the physical mechanism of phenomena in nature, described by nonlinear evolution equations, exact solutions for the nonlinear evolution equations have to be explored. For example, the wave phenomena observed in fluid dynamics, plasma and elastic media and optical fibers, etc.

Thus, the methods for deriving exact solutions for the governing equations have to be developed. Recently, many powerful methods have been established and improved. Among these methods, we cite the tanh and extended tanh methods [1-9], $(\frac{G'}{G})$-expansion method [10-13], the homogeneous balance method [14], the Jacobi elliptic function method [15, 16], the exp-function method [17], the first-integral method [18-20], the sine-cosine method [21] and so on.

The pioneer work Malfiet in [2, 3] introduced the powerful tanh method for a reliable treatment of the nonlinear wave equations. The useful tanh method is widely used by many work and by the references therein. Later, the extended tanh method, developed by Wazwaz [4, 5], is a direct and effective algebraic method for handling nonlinear equations. Various extensions of the method were developed as well.

The aim of this paper is to find exact soliton solutions of the generalized combined and the generalized double combined sinh-cosh-Gordon equations [22], by using the extended tanh method.

The paper is arranged as follows. In Section 2, we describe briefly the extended tanh method. In Section 3 and 4, we apply this method to find exact soliton solutions of the generalized combined and the generalized double combined sinh-cosh-Gordon equations. In Section 5, some conclusions are given.

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2 The extended tanh method and tanh method

A PDE
$$F(u,u_x,u_t,u_{xx},u_{xt},u_{xxx},....)=0, \tag{2.1}$$
can be converted to an ODE
$$G(u,u',u'',u''',....)=0, \tag{2.2}$$
upon using a wave variable $\xi = x-ct$. Eq. (2.2) is then integrated as long as all terms contain derivatives where integration constants are considered zeros.
Introducing a new independent variable
$$Y=tanh(\mu\xi), \quad \xi=x-ct, \tag{2.3}$$
leads to the change of derivatives:
$$\frac{d}{d\xi}=\mu(1-Y^2)\frac{d}{dY},$$
$$\frac{d^2}{d\xi^2}=-2\mu^2Y(1-Y^2)\frac{d}{dY}+\mu^2(1-Y^2)^2\frac{d^2}{dY^2}. \tag{2.4}$$
The extended tanh method admits the use of the finite expansion
$$u(\mu\xi)=S(Y)=\sum_{k=0}^{M}a_kY^k+\sum_{k=1}^{M}b_kY^{-k}, \tag{2.5}$$
where M is a positive integer, in most cases, that will be determined. Expansion (2.5) reduces to the standard tanh method for $b_k=0,(k=1,...,M)$. Substituting (2.5) into the ODE (2.2) results in an algebraic equation in powers of Y.

To determine the parameter M, we usually balance the linear terms of highest order in the resulting equation with the highest order nonlinear terms. We then collect all coefficients of powers of Y in the resulting equation where these coefficients have to vanish. This will give a system of algebraic equations involving the parameters $a_k(k=0,..,M)$, $b_k(k=1,..,M)$, μ and c. Having determined these parameters we obtain an analytic solution $u(x,t)$ in a closed form.

3 The generalized combined sinh-cosh-Gordon equation

Let us consider the generalized combined sinh-cosh-Gordon equations
$$u_{tt}-ku_{xx}+\alpha\sinh(nu)+\beta\cosh(nu)=0. \tag{3.6}$$
Using the variable $u(x,t)=u(\mu\xi)$, $\xi=x-ct$, carries Eq. (3.6) into the ODE
$$(c^2-k)u''+\alpha\sinh(nu)+\beta\cosh(nu)=0. \tag{3.7}$$
We use the Painleve property
$$v=e^{nu}, \tag{3.8}$$
or equivalently
$$u=\frac{1}{n}lnv, \tag{3.9}$$
from which we find
$$u'=\frac{1}{n}\frac{v'}{v}, \quad u''=\frac{1}{n}\frac{vv''-(v')^2}{v^2}. \tag{3.10}$$
The transformation (3.8) also gives
$$\sinh(nu)=\frac{v-v^{-1}}{2}, \quad \cosh(nu)=\frac{v+v^{-1}}{2}, \tag{3.11}$$

that also gives

$$u = \frac{1}{n} arccosh[\frac{v + v^{-1}}{2}]. \tag{3.12}$$

Substituting the transformations introduced above into Eq. (3.7) gives the ODE

$$(\alpha+\beta)nv^3 - (\alpha-\beta)nv + 2(c^2-k)vv'' - 2(c^2-k)(v')^2 = 0. \tag{3.13}$$

Balancing vv'' with v^3 in Eq. (3.13) gives

$$2M + 2 = 3M,$$

then

$$M = 2.$$

In this case, the extended tanh method the form (2.5) admits the use of the finite expansion

$$v(x,t) = S(Y) = a_0 + a_1 Y + a_2 Y^2 + \frac{b_1}{Y} + \frac{b_2}{Y^2}. \tag{3.14}$$

Substituting the form (3.14) into Eq. (3.13) and using (2.4), collecting the coefficients of Y we obtain:

Coefficient of Y^6: $n(\alpha+\beta)a_2^3 + 4(c^2-k)\mu^2 a_2^2$.

Coefficient of Y^5: $3n(\alpha+\beta)a_1a_2^2 + 8(c^2-k)\mu^2 a_1a_2$.

Coefficient of Y^4: $3n(\alpha+\beta)(a_0a_2^2 + a_1^2a_2) + 2(c^2-k)\mu^2(6a_0a_2 + a_1^2)$.

Coefficient of Y^3: $n(\alpha+\beta)(3b_1a_2^2 + 6a_0a_1a_2 + a_1^3) + 4(c^2-k)\mu^2(a_0a_1 - a_1a_2 + 5a_2b_1)$.

Coefficient of Y^2: $3n(\alpha+\beta)(a_0a_1^2 + a_0^2a_2 + 2a_1a_2b_1 + b_2a_2^2) - n(\alpha-\beta)a_2$
$+ 4(c^2-k)\mu^2(2a_1b_1 - 4a_0a_2 + 8a_2b_2 - a_2^2)$.

Coefficient of Y^1: $3n(\alpha+\beta)(a_0^2a_1 + a_1^2b_1 + 2a_0a_2b_1 + 2a_1a_2b_2) - n(\alpha-\beta)a_1$
$+ 4(c^2-k)\mu^2(-a_0a_1 - a_1a_2 + 4a_1b_2 - 9a_2b_1)$.

Coefficient of Y^0: $3n(\alpha+\beta)(a_1^2b_2 + a_2b_1^2 + 2a_0a_1b_1 + 2a_0a_2b_2) + n(\alpha+\beta)a_0^3 - n(\alpha-\beta)a_0$
$2(c^2-k)\mu^2(2a_0a_2 + 2a_0b_2 - 32a_2b_2 - a_1^2 - 8a_1b_1 - b_1^2)$.

Coefficient of Y^{-1}: $3n(\alpha+\beta)(a_0^2b_1 + a_1b_1^2 + 2a_0a_1b_1 + 2a_2b_1b_2) - n(\alpha-\beta)b_1$
$+ 4(c^2-k)\mu^2(-a_0b_1 - b_1b_2 + 4a_2b_1 - 9a_1b_2)$.

Coefficient of Y^{-2}: $3n(\alpha+\beta)(a_0b_1^2 + a_0^2b_2 + 2a_1b_1b_2 + a_2b_2^2) - n(\alpha-\beta)b_2$
$+ 4(c^2-k)\mu^2(2a_1b_1 - 4a_0b_2 + 8a_2b_2 - b_2^2)$.

Coefficient of Y^{-3}: $n(\alpha+\beta)(3a_1b_2^2 + 6a_0b_1b_2 + b_1^3) + 4(c^2-k)\mu^2(a_0b_1 - b_1b_2 + 5a_1b_2)$.

Coefficient of Y^{-4}: $3n(\alpha+\beta)(a_0b_2^2 + b_1^2b_2) + 2(c^2-k)\mu^2(6a_0b_2 + b_1^2)$.

Coefficient of Y^{-5}: $3n(\alpha+\beta)b_1b_2^2 + 8(c^2-k)\mu^2 b_1b_2$.

Coefficient of Y^{-6}: $n(\alpha+\beta)b_2^3 + 4(c^2-k)\mu^2 b_2^2$.

Setting these coefficients equal to zero, and solving the resulting system, by using Maple, we find the following sets of solutions:

$$a_0 = 0,\ \ a_1 = 0,\ \ a_2 = -\sqrt{\frac{\alpha-\beta}{\alpha+\beta}},\ \ b_1 = 0,\ \ b_2 = 0,\ \ \mu = \frac{\sqrt{n}}{2}\frac{\sqrt[4]{\alpha^2-\beta^2}}{\sqrt{c^2-k}}. \tag{3.15}$$

$$a_0 = 0,\ a_1 = 0,\ a_2 = 0,\ b_1 = 0,\ b_2 = -\sqrt{\frac{\alpha-\beta}{\alpha+\beta}},\ \mu = \frac{\sqrt{n}}{2}\frac{\sqrt[4]{\alpha^2-\beta^2}}{\sqrt{c^2-k}}. \tag{3.16}$$

$$a_0 = \frac{1}{2}\sqrt{\frac{\alpha-\beta}{\alpha+\beta}},\ a_1 = 0,\ a_2 = \frac{1}{4}\sqrt{\frac{\alpha-\beta}{\alpha+\beta}},\ b_1 = 0,\ b_2 = \frac{1}{4}\sqrt{\frac{\alpha-\beta}{\alpha+\beta}}, \tag{3.17}$$

$$\mu = \frac{\sqrt{n}}{4}\frac{\sqrt[4]{\alpha^2-\beta^2}}{\sqrt{c^2-k}}.$$

Recall that

$$u = \frac{1}{n}arccosh[\frac{v+v^{-1}}{2}].$$

The sets (3.15)-(3.17) give the solitons solutions for $\alpha > \beta,\ c^2 > k$

$$u_1(x,t) = \frac{1}{n}arccosh\{\frac{-(\alpha-\beta)\tanh^2[\mu(x-ct)] - (\alpha+\beta)\coth^2[\mu(x-ct)]}{2\sqrt{\alpha^2-\beta^2}}\}, \tag{3.18}$$

$$u_2(x,t) = \frac{1}{n}arccosh\{\frac{-(\alpha-\beta)\coth^2[\mu(x-ct)] - (\alpha+\beta)\tanh^2[\mu(x-ct)]}{2\sqrt{\alpha^2-\beta^2}}\}, \tag{3.19}$$

where $\mu = \frac{\sqrt{n}}{2}\frac{\sqrt[4]{\alpha^2-\beta^2}}{\sqrt{c^2-k}}$,

$$u_3(x,t) = \frac{1}{n}arccosh\{\frac{(\alpha-\beta)(2+\tanh^2[\mu(x-ct)]+\coth^2[\mu(x-ct)])^2 + 16(\alpha+\beta)}{8\sqrt{\alpha^2-\beta^2}(2+\tanh^2[\mu(x-ct)]+\coth^2[\mu(x-ct)])}\}, \tag{3.20}$$

where $\mu = \frac{\sqrt{n}}{4}\frac{\sqrt[4]{\alpha^2-\beta^2}}{\sqrt{c^2-k}}$.
However for $c^2 < k$, we obtain the travelling wave solutions

$$u_4(x,t) = \frac{1}{n}arccosh\{\frac{(\alpha-\beta)\tan^2[\mu(x-ct)] + (\alpha+\beta)\cot^2[\mu(x-ct)]}{2\sqrt{\alpha^2-\beta^2}}\}, \tag{3.21}$$

$$u_5(x,t) = \frac{1}{n}arccosh\{\frac{(\alpha-\beta)\cot^2[\mu(x-ct)] + (\alpha+\beta)\tan^2[\mu(x-ct)]}{2\sqrt{\alpha^2-\beta^2}}\}, \tag{3.22}$$

where $\mu = \frac{\sqrt{n}}{2}\frac{\sqrt[4]{\alpha^2-\beta^2}}{\sqrt{c^2-k}}$,

$$u_6(x,t) = \frac{1}{n}arccosh\{\frac{(\alpha-\beta)(2+\tan^2[\mu(x-ct)]+\cot^2[\mu(x-ct)])^2 + 16(\alpha+\beta)}{8\sqrt{\alpha^2-\beta^2}(2+\tan^2[\mu(x-ct)]+\cot^2[\mu(x-ct)])}\}, \tag{3.23}$$

where $\mu = \frac{\sqrt{n}}{4}\frac{\sqrt[4]{\alpha^2-\beta^2}}{\sqrt{c^2-k}}$.

4 The generalized double combined sinh-cosh-Gordon equation

In this section we study the generalized double combined sinh-cosh-Gordon equation

$$u_{tt} - ku_{xx} + \alpha\sinh(nu) + \alpha\cosh(nu) + \beta\sinh(2nu) + \beta\cosh(2nu) = 0. \tag{4.24}$$

We take the transformation

$$u(x,t) = u(\mu\xi),\ \ \xi = x - ct.$$

The substitution of the transformation into (4.24) yields the ODE

$$(c^2-k)u'' + \alpha\sinh(nu) + \alpha\cosh(nu) + \beta\sinh(2nu) + \beta\cosh(2nu) = 0. \tag{4.25}$$

We use the Painleve property

$$v = e^{nu}, \tag{4.26}$$

or equivalently

$$u = \frac{1}{n} lnv, \tag{4.27}$$

from which we find

$$u' = \frac{1}{n}\frac{v'}{v}, \quad u'' = \frac{1}{n}\frac{vv'' - (v')^2}{v^2}. \tag{4.28}$$

The transformation (4.26) also gives

$$\sinh(nu) = \frac{v - v^{-1}}{2}, \quad \cosh(nu) = \frac{v + v^{-1}}{2}, \sinh(2nu) = \frac{v^2 - v^{-2}}{2}, \tag{4.29}$$

$$\cosh(2nu) = \frac{v^2 + v^{-2}}{2},$$

that also gives

$$u = \frac{1}{n} arccosh[\frac{v + v^{-1}}{2}]. \tag{4.30}$$

Substituting the transformations introduced above into Eq. (4.25) gives the ODE

$$2\beta n v^4 + 2\alpha n v^3 + 2(c^2 - k)vv'' - 2(c^2 - k)(v')^2 = 0. \tag{4.31}$$

Balancing vv'' with v^4 in Eq. (4.31) gives

$$2M + 2 = 4M,$$

then

$$M = 1.$$

In this case, the extended tanh method the form (2.5) admits the use of the finite expansion

$$v(x,t) = S(Y) = a_0 + a_1 Y + \frac{b_1}{Y}. \tag{4.32}$$

Substituting the form (4.32) into Eq. (4.31) and using (2.4), collecting the coefficients of Y we obtain:

Coefficient of Y^4: $2n\beta a_1^4 + 2(c^2 - k)\mu^2 a_1^2$.

Coefficient of Y^3: $2na_1^3(4\beta a_0 + \alpha) + 4(c^2 - k)\mu^2 a_0 a_1$.

Coefficient of Y^2: $8n\beta a_1^3 b_1 + 6na_0 a_1^2(2\beta a_0 + \alpha) + 8(c^2 - k)\mu^2 a_1 b_1$.

Coefficient of Y^1: $6na_1^2 b_1(4\beta a_0 + \alpha) + 2na_0^2 a_1(4\beta a_0 + 3\alpha) - 4(c^2 - k)\mu^2 a_0 a_1$.

Coefficient of Y^0: $2na_0^3(\beta a_0 + \alpha) - 2(c^2 - k)\mu^2(a_1^2 + b_1^2 + 8a_1 b_1) + 12n\beta a_1 b_1(2a_0^2 + a_1 b_1 + \alpha n a_0)$.

Coefficient of Y^{-1}: $6na_1 b_1^2(4\beta a_0 + \alpha) + 2na_0^2 b_1(4\beta a_0 + 3\alpha) - 4(c^2 - k)\mu^2 a_0 b_1$.

Coefficient of Y^{-2}: $8n\beta b_1^3 a_1 + 6na_0 b_1^2(2\beta a_0 + \alpha) + 8(c^2 - k)\mu^2 a_1 b_1$.

Coefficient of Y^{-3}: $2nb_1^3(4\beta a_0 + \alpha) + 4(c^2 - k)\mu^2 a_0 b_1$.

Coefficient of Y^{-4}: $2n\beta b_1^4 + 2(c^2 - k)\mu^2 b_1^2$.

Setting these coefficients equal to zero, and solving the resulting system, by using Maple, we find the following sets of solutions:

$$a_0 = -\frac{\alpha}{2\beta}, \quad a_1 = 0, \quad b_1 = \pm\frac{\alpha}{2\beta}, \quad \mu = \pm\frac{\alpha}{2}\sqrt{\frac{n}{\beta(k - c^2)}}. \tag{4.33}$$

$$a_0 = -\frac{\alpha}{2\beta}, \quad a_1 = \pm\frac{\alpha}{2\beta}, \quad b_1 = 0, \quad \mu = \pm\frac{\alpha}{2}\sqrt{\frac{n}{\beta(k - c^2)}}. \tag{4.34}$$

$$a_0 = -\frac{\alpha}{2\beta}, \quad a_1 = \pm\frac{\alpha}{4\beta}, \quad b_1 = \pm\frac{\alpha}{4\beta}, \quad \mu = \pm\frac{\alpha}{4}\sqrt{\frac{n}{\beta(k-c^2)}}. \tag{4.35}$$

Recall that

$$u = \frac{1}{n}arccosh[\frac{v+v^{-1}}{2}].$$

The sets (4.33)-(4.35) give the soliton solutions

$$u_1(x,t) = \frac{1}{n}arccosh\{\frac{-\alpha^2(1\pm\coth[\mu(x-ct)])^2-4\beta^2}{4\alpha\beta(1\pm\coth[\mu(x-ct)])}\}, \tag{4.36}$$

$$u_2(x,t) = \frac{1}{n}arccosh\{\frac{-\alpha^2(1\pm\tanh[\mu(x-ct)])^2-4\beta^2}{4\alpha\beta(1\pm\tanh[\mu(x-ct)])}\}, \tag{4.37}$$

where $\mu = \pm\frac{\alpha}{2}\sqrt{\frac{n}{\beta(k-c^2)}}$, $k > c^2$.

$$u_3(x,t) = \frac{1}{n}arccosh\{\frac{-\alpha^2(2\pm\tanh[\mu(x-ct)]\pm coth[\mu(x-ct)])^2-16\beta^2}{8\alpha\beta(2\pm\tanh[\mu(x-ct)]\pm coth[\mu(x-ct)])}\}, \tag{4.38}$$

where $\mu = \pm\frac{\alpha}{4}\sqrt{\frac{n}{\beta(k-c^2)}}$.
However, for $k < c^2$, complex solutions can be obtained that are not needed in this work.

5 Conclusion

In this paper, the extended tanh method has been successfully applied to find the exact solutions for the generalized combined and the generalized double combined sinh-cosh-Gordon equations. The results indicate the efficiency and reliability of the method. Thus, we can say that the proposed method can be extended to solve the problems of nonlinear partial differential equations which arising in the theory of solitons and other areas.

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Received: October 14, 2015; *Accepted*: June 9, 2016

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