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Study on combinatorial dual graph in intuitionistic fuzzy environment

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Abstract

In this paper, intuitionistic fuzzy planar graphs are defined and various properties are studied. The intuitionistic fuzzy graphs are more efficient than fuzzy graphs, since it was found that one component is not sufficient to illustrate some special types of information. The notion of intuitionistic fuzzy dual graph and one of its close association namely intuitionistic fuzzy combinatorial dual graph is presented here. Some properties on intuitionistic fuzzy combinatorial dual graphs are investigated here.

Keywords: Intuitionistic fuzzy graphs, intuitionistic fuzzy planar graphs, intuitionistic fuzzy dual graphs, intuitionistic fuzzy combinatorial dual graphs.

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1 Introduction

Graph theory has a numerous applications in different research areas to structuring and designing of several models, its structures are used to represent various networking problems namely traffic network, telephone network, railway network, communication problems etc. The notion of fuzzy set (FS) was first introduced by Zadeh [11] (1965) to handle uncertainty in real life problems. After that it was found that one component is not sufficient to represent some special types of information. In this situations, a component namely non-membership value is needed to illustrate the information completely. To overcome this limitation of FS Atanassov [2] (1986) introduced the notion of intuitionistic fuzzy set (IFS) in addition to a new component known as degree of non-membership. Fuzzy graph (FG) theory was introduced by Rosenfeld [5] in 1975. Samanta et al. [6–8] defined fuzzy planar graph (FPG) in a different way where crossing between edges are allowed. Some related works are also found in [3, 4]. The idea of intuitionistic fuzzy graph (IFG) discussed by Shannon et al. [10]. Alshehri et al. [1] introduced the notion of intuitionistic fuzzy planar graphs (IFPG). Shriram et al. [9] defined fuzzy combinatorial dual graph.

In this work, we present IFPG, intuitionistic fuzzy faces, intuitionistic fuzzy dual graphs (IFDG), intuitionistic fuzzy combinatorial dual graphs (IFCDG) which is one of the classification of IFDGs. Also, introduced the terms strong (weak) IFPGs, strength of an edge, intersecting value between the edges. The IFMGs, IFPGs, IFDGs and IFCDGs are illustrated by an examples and lot of are presented of these graphs.

2 Preliminaries

This section, we give some related terminologies and results.

Definition 2.1. [5] *A FG is of the form $\zeta = (\tilde{V}, \sigma, \mu)$ where $\tilde{V}$ is the vertex set, $\sigma : \tilde{V} \to [0,1]$ and $\mu : \tilde{V} \times \tilde{V} \to [0,1]$ denote the degree of membership of $r \in \tilde{V}$ and edge $(r,s) \in \zeta$, respectively such that $\mu(r,s) \leq min(\sigma(r), \sigma(s))$ $\forall$ $r, s \in \tilde{V}$.*

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Definition 2.2. [2] *Let χ be the universe. Then a IFS $\tilde{A}$ is defined on X as $\tilde{A} = \{r, (\mu_{\tilde{A}}(r), \nu_{\tilde{A}}(r)) : r \in X\}$, where $\mu_{\tilde{A}}(r)$ and $\nu_{\tilde{A}}(r)$ are independent denote the degree of membership (DMS) and degree of non-membership (DNS) of $r \in \tilde{A}$, respectively with $0 \leq \mu_{\tilde{A}}(r) + \nu_{\tilde{A}}(r) \leq 1 \; \forall \; r \in X$. Also $\forall \; r \in X$, $D_{\tilde{A}}(r) = 1 - (\mu_{\tilde{A}}(r) + \nu_{\tilde{A}}(r))$ represent denial degree of r in $\tilde{A}$.*

Definition 2.3. [1] *A intuitionistic fuzzy relation (IFR) R is a intuitionistic fuzzy (IF) subset of $X \times Y$ is given by $\mathrm{R} = \{(r,s), \mu_{\mathrm{R}}(r,s), \nu_{\mathrm{R}}(r,s) | (r,s) \in X \times Y\}$, where $\mu_{\mathrm{R}}, \nu_{\mathrm{R}} : X \times Y \to [0,1]$ denote DMS and DNS of an edge (r,s) in R, respectively with $0 \leq \mu_{\mathrm{R}}(r,s) + \nu_{\mathrm{R}}(r,s) \leq 1$ for every $(r,s) \in X \times Y$.*

Definition 2.4. [1] *A IFG is of the form $\tilde{G} = (\tilde{V}, \tilde{A}, \tilde{B})$ where $\tilde{A} = (\mu_{\tilde{A}}, \nu_{\tilde{A}})$, $\tilde{B} = (\mu_{\tilde{B}}, \nu_{\tilde{B}})$ and*
(i) $\tilde{V} = \{r_1, r_2, .., r_n\}$ such that that $\mu_{\tilde{A}}, \nu_{\tilde{A}} : \tilde{V} \to [0,1]$ denote the DMS and DNS of $r_i \in \tilde{V}$, respectively with $0 \leq \mu_{\tilde{A}}(r_i) + \nu_{\tilde{A}}(r_i) \leq 1 \; \forall r_i \in \tilde{V}$, $(i = 1, 2, .., n)$.
(ii) $\mu_{\tilde{B}}, \nu_{\tilde{B}} : \tilde{V} \times \tilde{V} \to [0,1]$ denote the DMS and DNS of an edge (r_i, r_j), respectively such that $\mu_{\tilde{B}}(r_i, r_j) \leq min\{\mu_{\tilde{A}}(r_i), \mu_{\tilde{A}}(r_j)\}$ and $\nu_{\tilde{B}}(r_i, r_j) \leq max\{\nu_{\tilde{A}}(r_i), \nu_{\tilde{A}}(r_j)\}$ with $\mu_{\tilde{B}}(r_i, r_j) + \nu_{\tilde{B}}(r_i, r_j) \leq 1$ for every (r_i, r_j), $(i, j = 1, 2, .., n)$.

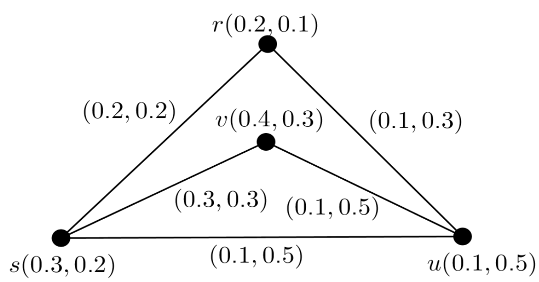

Figure 1: Example of a IFG

Definition 2.5. [1] *A intuitionistic fuzzy multiset (IFMS) M is given by $\mathrm{M} = \{(r, \mu^i_{\mathrm{M}}(r), \nu^i_{\mathrm{M}}(r)) : i = 1, 2, .., n | r \in \tilde{V}\}$, where $n = max\{i : \mu^i_{\mathrm{M}}(r) \neq 0$ or $\nu^i_{\mathrm{M}}(r) \neq 0\}$ and $\mu^i_{\mathrm{M}}(r), \nu^i_{\mathrm{M}}(r) \in [0,1]$ are the DMS and DNS of $r \in \tilde{V}$, respectively with $0 \leq \mu^i_{\mathrm{M}}(r) + \nu^i_{\mathrm{M}}(r) \leq 1 \; \forall \; r \in \tilde{V}$.*

Now, we introduce the notion of IFPG, for that it needs to define Intuitionistic fuzzy multigraph (IFMG) using the concept of IFMS.

Definition 2.6. [1] *Let $\tilde{A} = (\mu_{\tilde{A}}, \nu_{\tilde{A}})$ be a IFS on a non-empty set $\tilde{V}$ and $\tilde{B} = \{(rs, \mu^i_{\tilde{B}}(rs), \nu^i_{\tilde{B}}(rs)) : i = 1, 2, \ldots, n_{rs} | rs \in \tilde{V} \times \tilde{V}\}$ be a IFMS on $\tilde{V} \times \tilde{V}$ such that $\mu^i_{\tilde{B}}(rs) \leq min\{\mu_{\tilde{A}}(r), \mu_{\tilde{A}}(s)\}$, $\nu^i_{\tilde{B}}(rs) \leq max\{\nu_{\tilde{A}}(r), \nu_{\tilde{A}}(s)\}$ for all $i = 1, 2, \ldots, n_{rs}$, where $n_{rs} = max\{i : \mu^i_{\tilde{B}}(rs) \neq 0$ or $\nu^i_{\tilde{B}}(rs) \neq 0\}$ is the number of edges between r and s. Then $\tilde{G} = (\tilde{V}, \tilde{A}, \tilde{B})$ is called IFMG where $\mu_{\tilde{A}}(r), \nu_{\tilde{A}}(r)$ and $\mu^i_{\tilde{B}}(rs), \nu^i_{\tilde{B}}(rs)$ represent the DMS and DNS of vertex r and the i^{th} edge between r and s in $\tilde{G}$, respectively.*

Definition 2.7. [1] *Let $\tilde{G} = (\tilde{V}, \tilde{A}, \tilde{B})$ be IFMG, where $\tilde{B} = \{(rs, \mu^i_{\tilde{B}}(rs), \nu^i_{\tilde{B}}(rs)) : i = 1, 2, \ldots, n_{rs} | rs \in \tilde{V} \times \tilde{V}\}$ and $n_{rs} = max\{i : \mu^i_{\tilde{B}}(rs) \neq 0$ or $\nu^i_{\tilde{B}}(rs) \neq 0\}$. A multiedge rs is strong in $\tilde{G}$ if $\frac{1}{2}min\{\mu_{\tilde{A}}(r), \mu_{\tilde{A}}(s)\} \leq \mu^i_{\tilde{B}}(rs)$, $\frac{1}{2}max\{\nu_{\tilde{A}}(r), \nu_{\tilde{A}}(s)\} \leq \nu^i_{\tilde{B}}(rs)$ for all $i = 1, 2, \ldots, n_{rs}$.*

Example 2.1. *Consider a MG $\tilde{G}^* = (\tilde{V}, E)$, where $\tilde{V} = \{r, s, u, v\}$ and $E = \{rs, su, sv, sv, uv\}$. Let $\tilde{A} = (\mu_{\tilde{A}}, \nu_{\tilde{A}})$ be a IFS on $\tilde{V}$ and $\tilde{B} = (\mu_{\tilde{B}}, \nu_{\tilde{B}})$ be a IFMS on $\tilde{V} \times \tilde{V}$ given in Table 1 and 2. Fig.2 is a IFMG.*

Table 1: IFS $\tilde{A}$

$\tilde{A}$	r	s	u	v
$\mu_{\tilde{A}}$	0.4	0.45	0.3	0.3
$\nu_{\tilde{A}}$	0.4	0.1	0.25	0.4

Table 2: IFMS $\tilde{B}$

$\tilde{B}$	rs	su	sv	sv	uv
$\mu_{\tilde{B}}$	0.35	0.3	0.3	0.2	0.3
$\nu_{\tilde{B}}$	0.2	0.25	0.3	0.4	0.25

Here, rs and uv be two strong edges as $\frac{1}{2}min\{0.4, 0.45\} \leq 0.35$, $\frac{1}{2}max\{0.4, 0.1\} = 0.2$ and $\frac{1}{2}min\{0.3, 0.3\} \leq 0.3$, $\frac{1}{2}max\{0.25, 0.4\} \leq 0.25$.

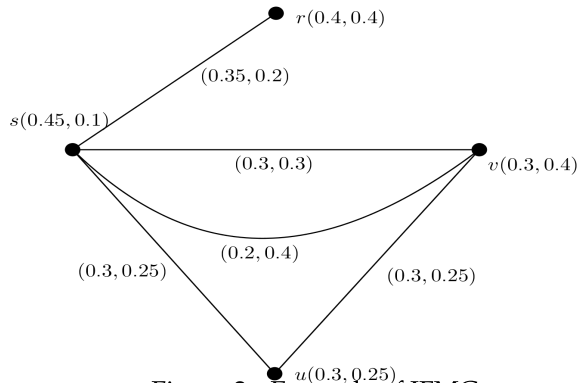

Figure 2: Example of IFMG

Definition 2.8. [1] *Let in $\tilde{G}$, P is the intersecting point between the edges $(rs, \mu^i_{\tilde{B}}(rs), \nu^i_{\tilde{B}}(rs))$ and $(uv, \mu^j_{\tilde{B}}(uv), \nu^j_{\tilde{B}}(uv))$, where i, j are fixed integers. The strength of the edge rs is defined as $I_{rs} = (t_{rs}, f_{rs}) = \left(\frac{\mu^i_{\tilde{B}}(rs)}{min(\mu_{\tilde{A}}(r), \mu_{\tilde{A}}(s))}, \frac{\nu^i_{\tilde{B}}(rs)}{max(\nu_{\tilde{A}}(r), \nu_{\tilde{A}}(s))}\right)$. The edge rs is strong if $t_{rs} \geq 0.5$ and $f_{rs} \geq 0.5$ otherwise weak. At P the intersecting value is $\widetilde{I}_P = (t_p, f_p) = \left(\frac{t_{rs}+t_{uv}}{2}, \frac{f_{rs}+f_{uv}}{2}\right)$.*

Example 2.2. *In Fig.3, strength of the edges (r, u) and (s, v) are $I_{ru} = (0.8, 0.8)$ and $I_{sv} = (0.66, 0.88)$, respectively. Thus at P intersecting value is $\widetilde{I}_P = (0.73, 0.84)$.*

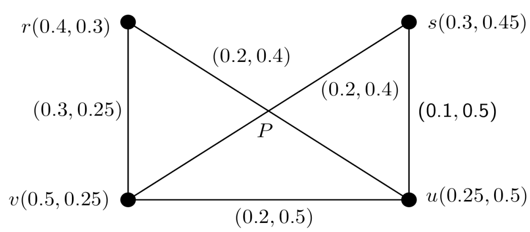

Figure 3: Intersecting value between two edges

Definition 2.9. [1] *Let $P_1, P_2, \ldots, P_k$ be k (integer) intersecting points between the edges of IFMG $\tilde{G}$. Then $\tilde{G}$ is IFPG with DP $f = (f_t, f_f)$, where $f_t = \frac{1}{1+\{t_{P_1}+t_{P_2}+\ldots+t_{P_k}\}}$ and $f_f = \frac{1}{1+\{f_{P_1}+f_{P_2}+\ldots+f_{P_k}\}}$. Clearly, $f = (f_t, f_f)$ is bounded as $0 < f_t \leq 1$ and $0 < f_f \leq 1$. DP increases if intersecting points decreases.*

Example 2.3. *Consider a IFMG $\tilde{G}^* = (\tilde{V}, E)$, where $\tilde{V} = (r, s, u, v)$ and $E = \{rs, ru, ru, su, sv, sv, rv, uv\}$. Let $\tilde{A} = (\mu_{\tilde{A}}, \nu_{\tilde{A}})$ be a IFS of $\tilde{V}$ and $\tilde{B} = (\mu_{\tilde{B}}, \nu_{\tilde{B}})$ be a IFMS of $\tilde{V} \times \tilde{V}$ given in Table 3 and 4.*

Table 3: IFS $\tilde{A}$

$\tilde{A}$	r	s	u	v
$\mu_{\tilde{A}}$	0.5	0.4	0.6	0.3
$\nu_{\tilde{A}}$	0.2	0.3	0.1	0.4

Table 4: IFMS $\tilde{B}$

$\tilde{B}$	rs	ru	ru	su	sv	sv	rv	uv
$\mu_{\tilde{B}}$	0.4	0.3	0.3	0.4	0.2	0.2	0.2	0.2
$\nu_{\tilde{B}}$	0.2	0.1	0.2	0.1	0.3	0.2	0.2	0.3

In Fig.4, a IFPG is considered with two intersecting points P_1 and P_2, between the edges $(ru, (0.3, 0.1))$, $(sv, (0.2, 0.3))$ and $(ru, (0.3, 0.2))$, $(sv, (0.2, 0.2))$, respectively. The strength of $(ru, (0.3, 0.1))$, $(sv, (0.2, 0.3))$, $(ru, (0.3, 0.2))$ and $(sv, (0.2, 0.2))$ are respectively $I_{ru} = (0.6, 0.5)$, $I_{sv} = (0.66, 0.75)$, $I_{ru} = (0.6, 1)$ and $I_{sv} = (0.66, 0.5)$. At P_1, intersecting value is $\widetilde{I}_{P_1} = (0.63, 0.62)$ and at P_2, $\widetilde{I}_{P_2} = (0.63, 0.75)$. Thus, the DP of $\tilde{G}^*$ is $f = (0.44, 0.42)$.

Definition 2.10. [1] *A IFPG $\tilde{G}$ is strong if its DP $f = (f_t, f_f)$ is such that $f_t > 0.5$ and $f_f > 0.5$. Otherwise weak.*

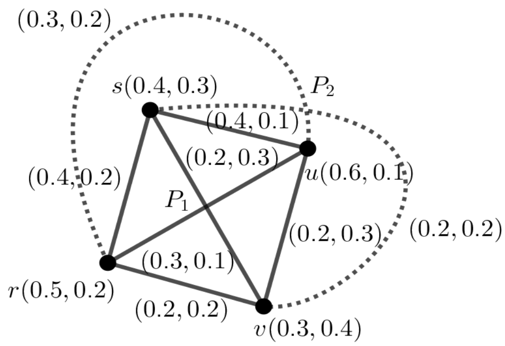

Figure 4: Example of IFPG

Now we present a special type of IFPG called 0.67-IFPG with DP $f = (f_t, f_f)$, where $f_t \geq 0.67$ and $f_f \geq 0.67$. When DP is $(1, 1)$, its geometrical representation is like as crisp planar graph. The above theorem state that, if DP is $f = (f_t, f_f)$, where $f_t \geq 0.67$ and $f_f \geq 0.67$, then two strong edges not intersect in $\tilde{G}$ and if there is any crossing, this is the crossing between the edges except both are strong. Thus any IFPG having no intersecting point between the edges is a IFPG with DP $(1, 1)$. Therefore, it is a 0.67-IFPG.

3 Intuitionistic fuzzy dual graph (IFDG)

At first we present intuitionistic fuzzy face (IFF) of a IFPG. Face is a region bounded by IF edges in a IFG. The presence of a IFF depending on minimum strength of its boundary edges. Because if all boundary edges of a IFF have DMS and DNS 1 and 0, respectively, it turn out crisp face but if we removed one of such edge or has membership degrees 0 and 1, respectively, the IFF does not exit. A IFF with its membership degrees are defined below.

Definition 3.11. *Let $\tilde{G}$ be a IFPG and $\tilde{B} = \{(rs, \mu^i_{\tilde{B}}(rs), \nu^i_{\tilde{B}}(rs)), i = 1, 2, \ldots, n_{rs} | rs \in \tilde{V} \times \tilde{V}\}$, where $n_{rs} = max\{i : \mu^i_{\tilde{B}}(rs) \neq 0$ or $\nu^i_{\tilde{B}}(rs) \neq 0\}$. A IFF of $\tilde{G}$ is a region, enclosed by the set of IF edges $\mathrm{E}' \subset E$. The DMS and DNS of IFF are, respectively $min\{\frac{\mu^i_{\tilde{B}}(rs)}{min(\mu_{\tilde{A}}(r), \mu_{\tilde{A}}(s))}, i = 1, 2, \ldots, n_{rs} | rs \in \mathrm{E}'\}$ and $max\{\frac{\nu^i_{\tilde{B}}(rs)}{max(\nu_{\tilde{A}}(r), \nu_{\tilde{A}}(s))}, i = 1, 2, \ldots, n_{rs} | rs \in \mathrm{E}'\}$.*

Definition 3.12. *A IFF strong if its DMS > 0.5 and DNS < 0.5 and weak otherwise. Each IFPG has an outer face with an infinite region and inner faces with finite region.*

Example 3.4. *In Fig.5, the IFPG has the faces: $\tilde{F}_1$ (inner face) is enclosed by the edges $(r_1r_2, 0.4, 0.1)$, $(r_2r_3, 0.6, 0.1)$ and $(r_1r_3, 0.4, 0.1)$. $\tilde{F}_2$ (outer face) is enclosed by the edges $(r_1r_4, 0.4, 0.1)$, $(r_1r_3, 0.4, 0.1)$, $(r_2r_3, 0.6, 0.1)$ and $(r_2r_4, 0.5, 0.1)$. $\tilde{F}_3$ (inner face) is enclosed by the edges $(r_1r_2, 0.4, 0.1)$, $(r_1r_4, 0.4, 0.1)$ and $(r_2r_4, 0.5, 0.1)$. The IFFs $\tilde{F}_1, \tilde{F}_2$ and $\tilde{F}_3$ are strong as all have same DMS and DNS 0.8 and 0.33, respectively.*

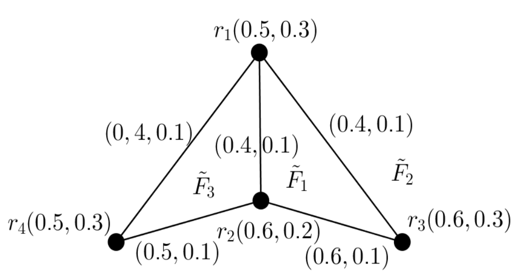

Figure 5: Example of faces in IFPG

Now we introduce dual of IFPG with DP $(1, 1)$. The vertices of IFDG are imposed corresponding to strong IFFs and edges are imposed corresponding to common border edges of IFFs.

Definition 3.13. *Let $\tilde{G} = (\tilde{V}, \tilde{A}, \tilde{B})$ be a 0.67-IFPG and $\tilde{F}_1, \tilde{F}_2, \ldots, \tilde{F}_n$ be its strong IFFs. The IFDG of $\tilde{G}$ is a IFPG $\tilde{G}_1 = (\tilde{V}_1, \tilde{A}_1, \tilde{B}_1)$, where $\tilde{V}_1 = \{t_i, i = 1, 2, \ldots, n\}$ and each t_i in $\tilde{G}_1$ is considered corresponding to the face $\tilde{F}_i$ of $\tilde{G}$. The DMS and DNS of vertices are given by the mapping $\tilde{A}_1 = (\mu_{\tilde{A}_1}, \nu_{\tilde{A}_1}) : \tilde{V}_1 \to [0,1] \times [0,1]$ such that*
$\mu_{\tilde{A}_1}(t_i) = max\{\mu^i_{\tilde{B}}(rs), i = 1, 2, \ldots, n_{rs} | rs \text{ is the border edge of } \tilde{F}_i\}$,
$\nu_{\tilde{A}_1}(t_i) = min\{\nu^i_{\tilde{B}}(rs), i = 1, 2, \ldots, n_{rs} | rs \text{ is the border edge of } \tilde{F}_i\}$.
In IFDG $\tilde{G}_1$, may exists more than one edge between t_i and t_j as two faces $\tilde{F}_i$ and $\tilde{F}_j$ of $\tilde{G}$ may exists more than one common edge. Let $\mu^l_{\tilde{B}}(t_it_j)$ and $\nu^l_{\tilde{B}}(t_it_j)$ denotes the DMS and DNS of the l-th edge between t_i and t_j, respectively. The DMS and DNS of IF edges in IFDG are given by $\mu^l_{\tilde{B}_1}(t_it_j) = \mu^i_{\tilde{B}}(rs)^l$ and $\nu^l_{\tilde{B}_1}(t_it_j) = \nu^i_{\tilde{B}}(rs)^l$, where $(rs)^l$ is border edge between $\tilde{F}_i$ and $\tilde{F}_j$ and $l = 1, 2, \ldots, p$, where p is the number of common border edges between $\tilde{F}_i$ and $\tilde{F}_j$ or the edges between t_i and t_j. If in a IFDG present any strong pendent edge, then for that there is a self-loop in $\tilde{G}_1$. The DMS and DNS of the self-loop of $\tilde{G}_1$ and pendent edge of $\tilde{G}$ are same.

Example 3.5. *In Fig.6, consider a IFPG $\tilde{G} = (\tilde{V}, \tilde{A}, \tilde{B})$, where $\tilde{V} = \{r, s, u, v\}$, $\tilde{A} = \{(r, 0.4, 0.3), (s, 0.6, 0.2), (u, 0.7, 0.3), (v, 0.3, 0.3)\}$ and $\tilde{B} = \{(rs, 0.4, 0.1), (ru, 0.3, 0.1), (rv, 0.3, 0.1), (su, 0.6, 0.1), (uv, 0.3, 0.1), (rv, 0.2, 0.1)\}$.*

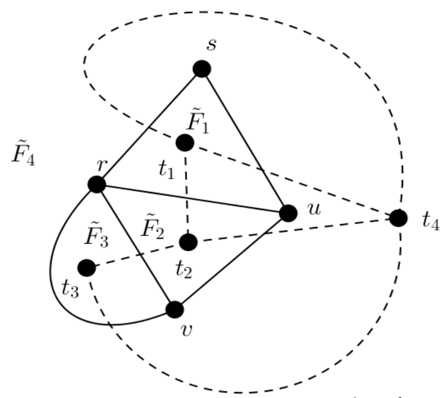

Figure 6: Example of IFDG

This graph has four faces $\tilde{F}_1, \tilde{F}_2, \tilde{F}_3$ and $\tilde{F}_4$, where $\tilde{F}_1$ is enclosed by the edges $(rs, 0.4, 0.1)$, $(ru, 0.3, 0.1)$ and $(su, 0.6, 0.1)$, $\tilde{F}_2$ is enclosed by $(ru, 0.3, 0.1), (rv, 0.3, 0.1)$ and $(uv, 0.3, 0.1)$, $\tilde{F}_3$ is enclosed by $(rv, 0.3, 0.1), (rv, 0.2, 0.1)$ and outer face $\tilde{F}_4$ is enclosed by $(rs, 0.4, 0.1), (su, 0.6, 0.1)$, $(uv, 0.3, 0.1)$ and $(rv, 0.2, 0.1)$. Since IFFs are strong, the vertex set of IFDG is $\tilde{V}_1 = \{t_1, t_2, t_3, t_4\}$, where each t_i is assigned corresponding to each $\tilde{F}_i, i = 1, 2, 3, 4$. Thus $\mu_{\tilde{A}_1}(t_1) = 0.6$, $\nu_{\tilde{A}_1}(t_1) = 0.1$, $\mu_{\tilde{A}_1}(t_2) = 0.3$, $\nu_{\tilde{A}_1}(t_2) = 0.1$, $\mu_{\tilde{A}_1}(t_3) = 0.3$, $\nu_{\tilde{A}_1}(t_3) = 0.1$, $\mu_{\tilde{A}_1}(t_4) = 0.6$, $\nu_{\tilde{A}_1}(t_4) = 0.1$.
It is seen that rs and su are the common edges between $\tilde{F}_1$ and $\tilde{F}_4$. So in IFDG $\tilde{G}_1$ there exists two edges between t_1 and t_4. The DMS and DNS of these edges are given by
$\mu_{\tilde{B}_1}(t_1t_4) = \mu_{\tilde{B}}(rs) = 0.4$, $\nu_{\tilde{B}_1}(t_1t_4) = \nu_{\tilde{B}}(rs) = 0.1$,
$\mu_{\tilde{B}_1}(t_1t_4) = \mu_{\tilde{B}}(su) = 0.6$, $\nu_{\tilde{B}_1}(t_1t_4) = \nu_{\tilde{B}}(su) = 0.1$.
Also,
$\mu_{\tilde{B}_1}(t_1t_2) = \mu_{\tilde{B}}(ru) = 0.3$, $\nu_{\tilde{B}_1}(t_1t_2) = \nu_{\tilde{B}}(ru) = 0.1$,
$\mu_{\tilde{B}_1}(t_2t_3) = \mu_{\tilde{B}}(rv) = 0.3$, $\nu_{\tilde{B}_1}(t_2t_3) = \nu_{\tilde{B}}(rv) = 0.1$,
$\mu_{\tilde{B}_1}(t_2t_4) = \mu_{\tilde{B}}(uv) = 0.3$, $\nu_{\tilde{B}_1}(t_2t_4) = \nu_{\tilde{B}}(uv) = 0.1$,
$\mu_{\tilde{B}_1}(t_3t_4) = \mu_{\tilde{B}}(rv) = 0.2$, $\nu_{\tilde{B}_1}(t_3t_4) = \nu_{\tilde{B}}(rv) = 0.1$.
Therefore, the edge set of IFDG is $\tilde{B}_1 = \{(t_1t_4, 0.4, 0.1), (t_1t_4, 0.6, 0.1), (t_1t_2, 0.3, 0.1), (t_2t_3, 0.3, 0.1), (t_2t_4, 0.3, 0.1), (t_3t_4, 0.2, 0.1)\}$. The IFDG $\tilde{G}_1$ of $\tilde{G}$ is drawn by dotted line in Fig.6.

4 Intuitionistic fuzzy combinatorial dual graph (IFCDG)

In this section, we define one of the classification of IFDG known as IFCDG and give some theorems of it.

Definition 4.14. *Let $\tilde{G} = (\tilde{V}, \tilde{A}, \tilde{B})$ be a 0.67-IFPG. The IFCDG of $\tilde{G}$ is $\tilde{G}'_1 = (\tilde{V}'_1, \tilde{A}'_1, \tilde{B}'_1)$, where $\tilde{V}'_1 = \{t'_i, i = 1, 2, ..., n\}$ is the vertex set of $\tilde{G}'_1$. The DMS and DNS of the vertices of $\tilde{G}'_1$ are given by the mapping $\tilde{A}'_1 = (\mu_{\tilde{A}'_1}, \nu_{\tilde{A}'_1}) : \tilde{V}'_1 \rightarrow [0,1] \times [0,1]$ such that*
$\mu_{\tilde{A}'_1}(t'_i) = max\{\mu^r(t'_it'_j), r = 1, 2, ..., n_{t'_it'_j} | t'_it'_j \text{ is an edge adjacent to } t'_i\}$,
$\nu_{\tilde{A}'_1}(t'_i) = min\{\nu^r(t'_it'_j), r = 1, 2, ..., n_{t'_it'_j} | t'_it'_j \text{ is an edge adjacent to } t'_i\}$.
Between the edges of $\tilde{G}$ and $\tilde{G}'_1$ there is a one-to-one correspondence such that the DMS and DNS of the edges of $\tilde{G}'_1$ are the DMS and DNS of the edges in $\tilde{G}$ with the condition each cycle of $\tilde{G}$ is cut set of $\tilde{G}'_1$.

Example 4.6. *Consider a 0.67-IFPG $\tilde{G} = (\tilde{V}, \tilde{A}, \tilde{B})$, where $\tilde{V} = \{r, s, u, v, w\}$, $\tilde{A} = \{(r, 0.5, 0.3), (s, 0.4, 0.2), (u, 0.6, 0.3), (v, 0.3, 0.2), (w, 0.5, 0.3)\}$ and $\tilde{B} = \{(e_1, 0.4, 0.3), (e_2, 0.4, 0.3), (e_3, 0.3, 0.3), (e_4, 0.3, 0.3), (e_5, 0.4, 0.3), (e_6, 0.3, 0.2)\}$ (see Fig.7). The cycles of $\tilde{G}$ are $\{e_1, e_2, e_3, e_4, e_5\}$, $\{e_2, e_3, e_6\}$ and $\{e_1, e_6, e_4, e_5\}$ form cut sets in IFCDG $\tilde{G}'_1 = (\tilde{V}'_1, \tilde{A}'_1, \tilde{B}'_1)$, where $\tilde{V}'_1 = \{t'_1, t'_2, t'_3\}$, $\tilde{A}'_1 = \{(t'_1, 0.4, 0.2), (t'_3, 0.4, 0.3)\}$ and $\tilde{B}'_1 = \{(e'_1, 0.4, 0.3), (e'_2, 0.4, 0.3), (e'_3, 0.3, 0.3), (e'_4, 0.3, 0.3), (e'_5, 0.4, 0.3), (e'_6, 0.3, 0.2)\}$.*

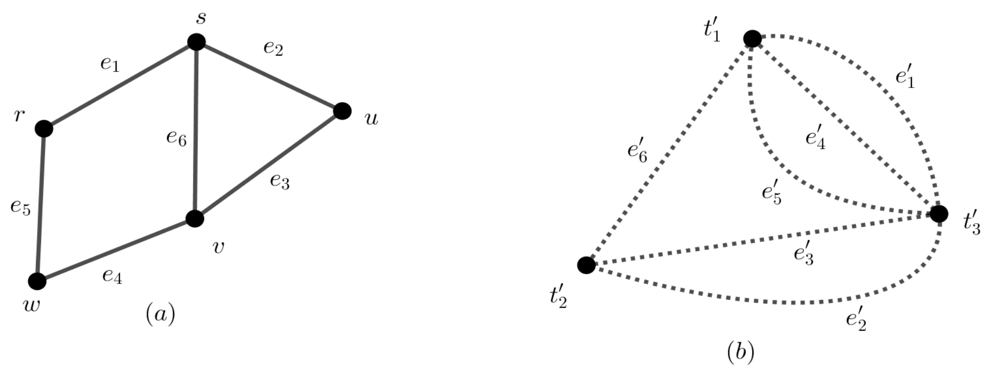

Figure 7: (a) A IFPG $\tilde{G}$ and (b) its IFCDG $\tilde{G}'_1$

Theorem 4.1. *Every 0.67-IFPG has a IFCDG.*

Proof. Let $\tilde{G}$ be 0.67-IFPG and $\tilde{G}'$ be the IFCDG. Then between the edges of $\tilde{G}$ and $\tilde{G}'_1$ there is a one-to-one correspondence such that the DMS and DNS of the edges of $\tilde{G}'_1$ are known. Let $\tilde{C}$ be a cycle of $\tilde{G}$ and it divides $\tilde{G}$ into two regions. Then we isolate the vertices of $\tilde{G}'_1$ into two non-empty subsets $\tilde{A}'$ and $\tilde{B}'$ (say), both are determined by the boundary of the cycle inside and outside $\tilde{C}$, respectively in $\tilde{G}$.
Let corresponding to the edges of $\tilde{C}$, we have a set of edges $\tilde{Ç}$ in $\tilde{G}'_1$ and removal of $\tilde{Ç}$ two subsets $\tilde{A}'$ and $\tilde{B}'$ becomes disjoint and $\tilde{G}'_1$ is disconnected. Thus $\tilde{Ç}$ is a cut set of $\tilde{G}'_1$.
Hence, each cycle of $\tilde{G}$ forms a cut set in $\tilde{G}'_1$. This proves the theorem. □

Example 4.7. *Consider a 0.67-IFPG $\tilde{G} = (\tilde{V}, \tilde{A}, \tilde{B})$, where $\tilde{V} = \{r, s, u, v, w\}$, $\tilde{A} = \{(r, 0.6, 0.1), (s, 0.5, 0.4), (u, 0.4, 0.3), (v, 0.3, 0.4), (w, 0.7, 0.2)\}$ and $\tilde{B} = \{(e_1, 0.5, 0.4), (e_2, 0.4, 0.3), (e_3, 0.3, 0.4), (e_4, 0.4, 0.4), (e_5, 0.3, 0.4), (e_6, 0.5, 0.4), (e_7, 0.3, 0.4), (e_8, 0.3, 0.4)\}$ and its IFCDG is $\tilde{G}'_1 = (\tilde{V}'_1, \tilde{A}'_1, \tilde{B}'_1)$, where $\tilde{V}'_1 = \{t'_1, t'_2, t'_3, t'_4, t'_5\}$, $\tilde{A}'_1 = \{(t'_1, 0.5, 0.3), (t'_2, 0.5, 0.4), (t'_3, 0.4, 0.4), (t'_4, 0.5, 0.3), (t'_5, 0.5, 0.4)\}$ and $\tilde{B}'_1 = \{(e'_1, 0.5, 0.4), (e'_2, 0.4, 0.3), (e'_3, 0.3, 0.4), (e'_4, 0.4, 0.4), (e'_5, 0.3, 0.4), (e'_6, 0.5, 0.4), (e'_7, 0.3, 0.4), (e'_8, 0.3, 0.4)\}$ (see Fig.8). Let $\tilde{C} = \{e_1, e_2, e_3, e_4\}$ be any cycle of $\tilde{G}$ such that $\tilde{A}' = \{t'_1, t'_3\}$ and and $\tilde{B}' = \{t'_2, t'_4, t'_5\}$ in $\tilde{G}'$. If we remove the corresponding edges of $\tilde{C}$, then $\tilde{G}'_1$ becomes disconnected. Hence, cycles of $\tilde{G}$ forms the cut sets in $\tilde{G}'_1$.*

Theorem 4.2. *Every IFCDG of a IFG has a 0.67-IFPG.*

Proof. Let K_5 or $K_{3,3}$ has a IFCDG. Both graphs has finite number of edges and one intersecting point can not be avoid for any representation of them.
Case-I: Let K_5 or $K_{3,3}$ has at least one weak edge in $\tilde{G}$ and this edge is not considered in IFG $\tilde{G}$. Then $\tilde{G}$ has no intersecting point between its edges and has a IFCDG $\tilde{G}'_1$. Thus K_5 or $K_{3,3}$ is a 0.67-IFPG.

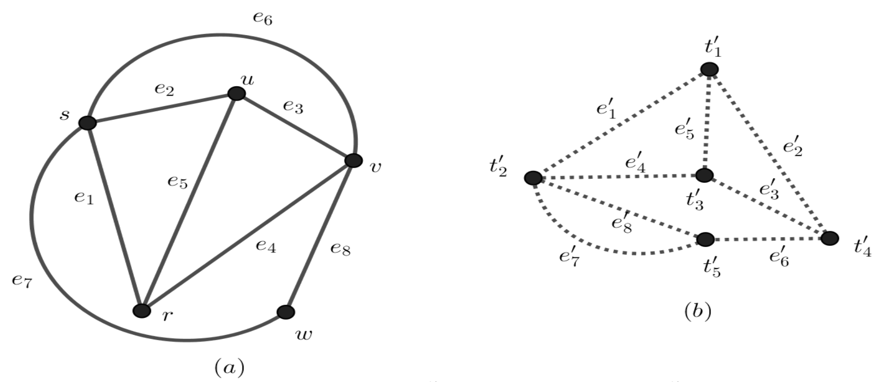

Figure 8: (a) A IFPG $\tilde{G}$ and (b) its IFCDG $\tilde{G}'_1$

Case-II: Let in K_5 or $K_{3,3}$ all edges are strong and there is only one intersecting point between strong edges. Then the DP is $f = (f_t, f_f)$, where $f_t < 0.67, f_f < 0.67$. Thus no dual graph can be drawn. Therefore, K_5 or $K_{3,3}$ does not have any 0.67-IFPG and IFCDG. □

Theorem 4.3. *A 0.67-IFPG is planar iff it has a IFCDG.*

Proof. Combining theorem 7.3 and theorem 7.5, we conclude it. □

5 Conclusion

This study relates the IFPGs and discussed its important consequences known as IFDGs and IFCDGs both are closely associated. For the 0.67-IFPG we define IFDG. But, when DP of IFPG is less than 0.67, then some modifications are needed to define it. IFMG, DP of a IFPG and IFF have also been introduced here and some corresponding results have been studied. This work can be viewed as the generalization of the study on fuzzy combinatorial dual graph.

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