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On generalized α regular-interior and generalized α regular-closure in Topological Spaces

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Abstract

In this paper, the authors introduce a new class of generalized α regular-interior and generalized α regular-closure in topological spaces. Some characterizations and several properties concerning generalized α regular-interior and generalized α regular-closure are obtained.

Keywords: $g\alpha r$-closed sets, $g\alpha r$-closed map, $g\alpha r$-continuous map, contra $g\alpha r$-continuity.

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1 Introduction

Levine introduced generalized closed sets in topology as a generalization of closed sets. This concept was found to be useful and many results in general topology were improved. Many researchers like Arya et al[5], Balachandran et al[6], Bhattarcharya et al[7], Arockiarani et al[4], Gnanambal [8], Nagaveni[14] and Palaniappan et al[15] have worked on generalized closed sets. Andrjivic[3] gave a new type of generalized closed set in topological space called b closed sets. A.A.Omari and M.S.M. Noorani [2] made an analytical study and gave the concepts of generalized b closed sets in topological spaces.

Sekar and Mariappa [18] gave rgb-interior and rgb-closure in topological spaces. In this paper, the notion of $g\alpha r$-interior is defined and some of its basic properties are investigated. Also we introduce the idea of $g\alpha r$-closure in topological spaces using the notions of $g\alpha r$-closed sets and obtain some related results. Through out this paper (X,τ) and (Y,σ) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. Let $A \subseteq X$, the closure of A and interior of A will be denoted by $cl(A)$ and $int(A)$ respectively, union of all $g\alpha r$-open sets X contained in A is called $g\alpha r$-interior of A and it is denoted by $g\alpha rint(A)$, the intersection of all $g\alpha r$-closed sets of X containing A is called $g\alpha r$-closure of A and it is denoted by $g\alpha rcl(A)$ [17].

2 Preliminaries

Definition 2.1. *Let a subset A of a topological space (X,τ), is called*

1) a α-open set [13] if $A \subseteq int(cl(int(A)))$.

2) a generalised-closed set (briefly g-closed) [10] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

3) a weakly-closed set(briefly w-closed) [16] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi open.

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4) *a generalized ∗-closed set (briefly g∗-closed) [20] if* $cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is g-open in* X.

5) *a generalized α-closed set (briefly gα-closed)[12] if* $\alpha cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is* α *open in* X.

6) *an α generalized-closed set (briefly αg-closed)[11] if* $\alpha cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is open in* X.

7) *a generalized b- closed set (briefly gb- closed) [1] if* $bcl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is open in* X.

8) *a semi generalized b-closed set (briefly sgb- closed) [9] if* $bcl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is semi open in* X.

9) *a generalized αb- closed set (briefly gαb- closed) [19] if* $bcl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is* α *open in* X.

10) *a regular generalized b- closed set (briefly rgb- closed) [13] if* $bcl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is regular open in* X.

11) *a generalized pre regular-closed set (briefly gpr-closed) [8] if* $pcl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is regular open in* X.

12) *a generalized α regular-closed set (briefly gαr-closed) [17] if* $\alpha cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and* U *is regular open in* X.

3 Generalized α regular - interior in Topological space

Definition 3.2. *Let* A *be a subset of* X. *A point* $x \in A$ *is said to be* $g\alpha r$ *- interior point of* A *is* A *is a* $g\alpha r$ *- neighbourhood of* x. *The set of all* $g\alpha r$ *- interior points of* A *is called the* $g\alpha r$ *- interior of* A *and is denoted by* $g\alpha r - int(A)$.

Theorem 3.1. *If* A *be a subset of* X. *Then* $g\alpha r - int(A) = \{\cup G : G \text{ is a } g\alpha r \text{ - open}, G \subset A\}$.

Proof. Let A be a subset of X.

$$\begin{aligned} x \in g\alpha r - int(A) &\Leftrightarrow x \text{ is a } g\alpha r - \text{ interior point of } A \\ &\Leftrightarrow A \text{ is a } g\alpha r \text{ - nbhd of point } x \\ &\Leftrightarrow \text{there exists } g\alpha r \text{ - open set } G \text{ such that } x \in G \subset A \\ &\Leftrightarrow x \in \{\cup G : G \text{ is a } g\alpha r\text{-open }, G \subset A\} \\ \text{Hence } g\alpha r - int(A) &= \{\cup G : G \text{ is a } g\alpha r\text{-open }, G \subset A\} \end{aligned}$$

□

Theorem 3.2. *Let* A *and* B *be subsets of* X. *Then*

(i) $g\alpha r - int(X) = X$ *and* $g\alpha r - int(\varphi) = \varphi$.

(ii) $g\alpha r - int(A) \subset A$.

(iii) *If* B *is any* $g\alpha r$ *- open set contained in* A, *then* $B \subset g\alpha r - int(A)$.

(iv) *If* $A \subset B$, *then* $g\alpha r - int(A) \subset g\alpha r - int(B)$.

(v) $g\alpha r - int\big(g\alpha r - int(A)\big) = g\alpha r - int(A)$.

Proof. (i) Since X and φ are $g\alpha r$ open sets, by Theorem 3.2

$$\begin{aligned} g\alpha r - int(X) &= \{\cup G : G \text{ is a } g\alpha r \text{ - open}, G \subset X\} \\ &= X \cup \{ \text{ all } g\alpha r \text{ open sets } \} \\ &= X \end{aligned}$$

(i.e.,) $g\alpha r\ int(X) = X$. Since φ is the only $g\alpha r$ - open set contained in $\varphi, g\alpha r - int(\varphi) = \varphi$.

(ii) Let $x \in g\alpha r - int(A)$

$$\begin{aligned} x \in g\alpha r - int(A) &\Rightarrow \text{ } x \text{ is a interior point of } A. \\ &\Rightarrow \text{ } A \text{ is a nbhd of } x. \\ &\Rightarrow x \in A \\ \text{Thus, } x \in g\alpha r - int(A) &\Rightarrow x \in A \\ \text{Hence } g\alpha r - int(A) &\subset A. \end{aligned}$$

(iii) Let B be any $g\alpha r$ - open sets such that $B \subset A$. Let $x \in B$. Since B is a $g\alpha r$ - open set contained in A. x is a $g\alpha r$ - interior point of A.
(i.e.,) $x \in g\alpha r - int(A)$. Hence $B \subset g\alpha r - int(A)$.

(iv) Let A and B be subsets of X such that $A \subset B$. Let $x \in g\alpha r - int(A)$. Then x is a $g\alpha r$ - interior point of A and so A is a $g\alpha r$ - nbhd of x. Since $B \supset A$, B is also $g\alpha r$ - nbhd of $x \Rightarrow x \in g\alpha r - int(B)$. Thus we have shown that $x \in g\alpha r - int(A) \Rightarrow x \in g\alpha r - int(B)$.

(v) Proof is obvious.

□

Theorem 3.3. *If a subset A of space X is $g\alpha r$ - open, then $g\alpha r - int(A) = A$.*

Proof. Let A be $g\alpha r$ - open subset of X. We know that $g\alpha r - int(A) \subset A$. Also, A is $g\alpha r$ - open set contained in A. From Theorem 3.3 (iii) $A \subset g\alpha r - int(A)$. Hence $g\alpha r - int(A) = A$. □

The converse of the above theorem need not be true, as seen from the following example.

Example 3.1. *Let $X = \{a, b, c\}$ with $\tau = \{X, \varphi, \{b\}, \{c\}, \{b, c\}\}$. Then $g\alpha r - O(X) = \{X, \varphi, \{a\}, \{b\}, \{c\}, \{b, c\}\}$. $g\alpha r - int(\{a, c\} = \{a\} \cup \{c\} \cup \{\varphi\} = \{a, c\}$. But $\{a, c\}$ is not $g\alpha r$ - open set in X.*

Theorem 3.4. *If A and B are subsets of X, then*
$g\alpha r - int(A) \cup g\alpha r - int(B) \subset g\alpha r - int(A \cup B)$.

Proof. We know that $A \subset A \cup B$ and $B \subset A \cup B$. We have Theorem 3.3 (iv) $g\alpha r - int(A) \subset g\alpha r - int(A \cup B)$, $g\alpha r - int(B) \subset g\alpha r - int(A \cup B)$. This implies that $g\alpha r - int(A) \cup g\alpha r - int(B) \subset g\alpha r - int(A \cup B)$. □

Theorem 3.5. *If A and B are subsets of X, then*
$g\alpha r - int(A \cap B) = g\alpha r - int(A) \cap g\alpha r - int(B)$.

Proof. We know that $A \cap B \subset A$ and $A \cap B \subset B$. We have $g\alpha r - int(A \cap B) \subset g\alpha r - int(A)$ and $g\alpha r - int(A \cap B) \subset g\alpha r - int(B)$.
This implies that

$$g\alpha r - int(A \cap B) \subset g\alpha r - int(A) \cap g\alpha r - int(B). \tag{3.1}$$

Again let $x \in g\alpha r - int(A) \cap g\alpha r - int(B)$. Then $x \in g\alpha r - int(A)$ and $x \in g\alpha r - int(B)$. Hence x is a $g\alpha r$ - int point of each of sets A and B. It follows that A and B is $g\alpha r$ - nbhds of x, so that their intersection $A \cap B$ is also a $g\alpha r$ - nbhds of x. Hence $x \in g\alpha r - int(A \cap B)$. Thus $x \in g\alpha r - int(A) \cap g\alpha r - int(A)$ implies that $x \in g\alpha r - int(A \cap B)$. Therefore

$$g\alpha r - int(A) \cap g\alpha r - int(B) \subset g\alpha r - int(A \cap B) \tag{3.2}$$

From (3.1) and (3.2),
We get $g\alpha r - int(A \cap B) = g\alpha r - int(A) \cap g\alpha r - int(B)$. □

Theorem 3.6. *If A is a subset of X, then $int(A) \subset g\alpha r - int(A)$.*

Proof. Let A be a subset of X.

$$\begin{aligned}
\text{Let } x \in int(A) &\Rightarrow x \in \{\cup G : G \text{ is open}, G \subset A\} \\
&\Rightarrow \text{there exists an open set } G \\
&\quad\ \text{such that } x \in G \subset A \\
&\Rightarrow \text{there exist a } g\alpha r \text{ - open set } G \\
&\quad\ \text{such that } x \in G \subset A, \text{ as every open set is} \\
&\quad\ \text{a } g\alpha r \text{ - open set in } X \\
&\Rightarrow x \in \{\cup G : G \text{ is } g\alpha r \text{ - open}, G \subset A\} \\
&\Rightarrow x \in g\alpha r - int(A) \\
\text{Thus } x \in int(A) &\Rightarrow x \in g\alpha r - int(A) \\
\text{Hence } int(A) &\subset g\alpha r - int(A).
\end{aligned}$$

This completes the proof. □

Remark 3.1. *Containment relation in the above theorem may be proper as seen from the following example.*

Example 3.2. *Let $X = \{a,b,c\}$ with $\tau = \{X, \varphi, \{b\}, \{c\}, \{b, c\}\}$. Then $g\alpha r - O(X) = \{X, \varphi, \{a\}, \{b\}, \{c\}, \{b,c\}\}$. Let $A = \{b,c\}$. Now $g\alpha r - int(A) = \{b,c\}$ and $int(A) = \{b\}$. It follows that $int(A) \subset g\alpha r - int(A)$ and $int(A) \neq g\alpha r - int(A)$.*

Theorem 3.7. *If A is a subset of X, then $g - int(A) \subset g\alpha r - int(A)$, where $g - int(A)$ is given by $g - int(A) = \cup\{G : G \text{ is } g \text{ - open}, G \subset A\}$.*

Proof. Let A be a subset of X.

$$\begin{aligned}
\text{Let } x \in int(A) &\Rightarrow x \in \{\cup G : G \text{ is } g \text{ - open}, G \subset A\} \\
&\Rightarrow \text{there exists an } g \text{ - open set } G \\
&\quad\ \text{such that } x \in G \subset A \\
&\Rightarrow \text{there exist a } g\alpha r \text{ - open set } G \\
&\quad\ \text{such that } x \in G \subset A, \text{ as every } g \text{ open set} \\
&\quad\ \text{is a } g\alpha r \text{ - open set in } X \\
&\Rightarrow x \in \{\cup G : G \text{ is } g\alpha r \text{ - open}, G \subset A\} \\
&\Rightarrow x \in g\alpha r - int(A) \\
\text{Hence } g - int(A) &\subset g\alpha r - int(A).
\end{aligned}$$

This completes the proof. □

Remark 3.2. *Containment relation in the above theorem may be proper as seen from the following example.*

Example 3.3. *Let $X = \{a,b,c\}$ with $\tau = \{X, \varphi, \{a\}, \{b,c\}\}$. Then $g\alpha r - O(X) = \{X, \varphi, \{a\}, \{b,c\}\}$. and g - open $(X) = \{X, \varphi, \{a\}, \{b\}, \{a,b\}\}$. Let $A = \{b,c\}$, $g\alpha r - int(A) = \{b,c\}$ and $g - int(A) = \{b\}$. It follows that $g - int(A) \subset g\alpha r - int(A)$ and $g - int(A) \neq g\alpha r - int(A)$.*

4 Generalized α regular - closure in Topological space

Definition 4.3. *Let A be a subset of a space X. We define the $g\alpha r$ - closure of A to be the intersection of all $g\alpha r$ - closed sets containing A.*
In symbols, $g\alpha r - cl(A) = \{\cap F : A \subset F \in g\alpha rc(X)\}$.

Theorem 4.8. *If A and B are subsets of a space X. Then*

(i) $g\alpha r - cl(X) = X$ *and* $g\alpha r - cl(\varphi) = \varphi$

(ii) $A \subset g\alpha r - cl(A)$

(iii) *If B is any $g\alpha r$ - closed set containing A, then $g\alpha r - cl(A) \subset B$*

(iv) *If $A \subset B$ then $g\alpha r - cl(A) \subset g\alpha r - cl(B)$*

Proof. (i) By the definition of $g\alpha r$ - closure, X is the only $g\alpha r$ - closed set containing X. Therefore $g\alpha r - cl(X) =$ Intersection of all the $g\alpha r$ - closed sets containing $X = \cap\{X\} = X$. That is $g\alpha r - cl(X) = X$. By the definition of $g\alpha r$ - closure, $g\alpha r - cl(\varphi) =$ Intersection of all the $g\alpha r$ - closed sets containing $\varphi = \{\varphi\} = \varphi$. That is $g\alpha r - cl(\varphi) = \varphi$.

(ii) By the definition of $g\alpha r$ - closure of A, it is obvious that $A \subset g\alpha r - cl(A)$.

(iii) Let B be any $g\alpha r$ - closed set containing A. Since $g\alpha r - cl(A)$ is the intersection of all $g\alpha r$ - closed sets containing A, $g\alpha r - cl(A)$ is contained in every $g\alpha r$ - closed set containing A. Hence in particular $g\alpha r - cl(A) \subset B$.

(iv) Let A and B be subsets of X such that $A \subset B$. By the definition
$g\alpha r - cl(B) = \{\cap F : B \subset F \in g\alpha r - c(X)\}$. If $B \subset F \in g\alpha r - c(X)$, then $g\alpha r - cl(B) \subset F$. Since $A \subset B, A \subset B \subset F \in g\alpha r - c(X)$,
we have $g\alpha r - cl(A) \subset F$. Therefore $g\alpha r - cl(A) \subset \{\cap F : B \subset F g\alpha r - c(X)\} = g\alpha r - cl(B)$. (i.e.,) $g\alpha r - cl(A) \subset g\alpha r - cl(B)$.

□

Theorem 4.9. *If $A \subset X$ is $g\alpha r$ - closed, then $g\alpha r - cl(A) = A$.*

Proof. Let A be $g\alpha r$ - closed subset of X. We know that $A \subset g\alpha r - cl(A)$. Also $A \subset A$ and A is $g\alpha r$ - closed. By Theorem 4.2 (iii) $g\alpha r - cl(A) \subset A$. Hence $g\alpha r - cl(A) = A$. □

Remark 4.3. *The converse of the above theorem need not be true as seen from the following example.*

Example 4.4. *Let $X = \{a, b, c\}$ with topology $\tau = \{X, \varphi, \{b\}, \{b, c\}\}$. Then $g\alpha r - C(X) = \{X, \varphi, \{a\}, \{b, c\}\}$. $g\alpha r - cl(\{c\}) = \{b, c\}$. But $\{c\}$ is not $g\alpha r$ - closed set in X.*

Theorem 4.10. *If A and B are subsets of a space X, then*
$g\alpha r - cl(A \cap B) \subset g\alpha r - cl(A) \cap g\alpha r - cl(B)$.

Proof. Let A and B be subsets of X. Clearly $A \cap B \subset A$ and $A \cap B \subset B$. By Theorem $g\alpha r - cl(A \cap B) \subset g\alpha r - cl(A)$ and $g\alpha r - cl(A \cap B) \subset g\alpha r - cl(B)$. Hence $g\alpha r - cl(A \cap B) \subset g\alpha r - cl(A) \cap g\alpha r - cl(B)$. □

Theorem 4.11. *If A and B are subsets of a space X then*
$g\alpha r - cl(A \cup B) = g\alpha r - cl(A) \cup g\alpha r - cl(B)$.

Proof. Let A and B be subsets of X. Clearly $A \subset A \cup B$ and $B \subset A \cup B$. We have

$$g\alpha r - cl(A) \cup g\alpha r - cl(B) \subset g\alpha r - cl(A \cup B) \tag{4.3}$$

Now to prove $g\alpha r - cl(A \cup B) \subset g\alpha r - cl(A) \cup g\alpha r - cl(B)$.
Let $x \in g\alpha r - cl(A \cup B)$ and suppose $x \notin g\alpha r - cl(A) \cup g\alpha r - cl(B)$. Then there exists $g\alpha r$ - closed sets A_1 and B_1 with $A \subset A_1, B \subset B_1$ and $x \notin A_1 \cup B_1$. We have $A \cup B \subset A_1 \cup B_1$ and $A_1 \cup B_1$ is $g\alpha r$ - closed set by Theorem such that $x \notin A_1 \cup B_1$. Thus $x \notin g\alpha r - cl(A \cup B)$ which is a contradiction to $x \in g\alpha r - cl(A \cup B)$. Hence

$$g\alpha r - cl(A \cup B) \subset g\alpha r - cl(A) \cup g\alpha r - cl(B) \tag{4.4}$$

From (4.3) and (4.4), we have $g\alpha r - cl(A \cup B) = g\alpha r - cl(A) \cup g\alpha r - cl(B)$. □

Theorem 4.12. *For an $x \in X$, $x \in g\alpha r - cl(A)$ if and only if $V \cap A \neq \varphi$ for every $g\alpha r$ - open sets V containing x.*

Proof. Let $x \in X$ and $x \in g\alpha r - cl(A)$. To prove $V \cap A \neq \varphi$ for every $g\alpha r$ - open set V containing x.
Prove the result by contradiction. Suppose there exists a $g\alpha r$ - open set V containing x such that $V \cap A = \varphi$. Then $A \subset X - V$ and $X - V$ is $g\alpha r$-closed. We have $g\alpha r - cl(A) \subset X - V$. This shows that $x \notin g\alpha r - cl(A)$, which is a contradiction. Hence $V \cap A \neq \varphi$ for every $g\alpha r$ - open set V containing x.
Conversely, let $V \cap A = \varphi$ for every $g\alpha r$ - open set V containing x. To prove $x \in g\alpha r - cl(A)$. We prove the result by contradiction. Suppose $x \notin g\alpha r - cl(A)$. Then $x \in X - F$ and $S - F$ is $g\alpha r$ - open. Also $(X - F) \cap A = \varphi$, which is a contradiction. Hence $x \in g\alpha r - cl(A)$. □

Theorem 4.13. *If A is a subset of a space X, then $g\alpha r - cl(A) \subset cl(A)$.*

Proof. Let A be a subset of a space S. By the definition of closure, $cl(A) = \{\cap F : A \subset F \in C(X)\}$. If $A \subset F \in C(X)$, Then $A \subset F \in g\alpha r - C(X)$, because every closed set is $g\alpha r$ - closed. That is $g\alpha r - cl(A) \subset F$. Therefore $g\alpha r - cl(A) \subset \{\cap F \subset X : F \in C(X)\} = cl(A)$. Hence $g\alpha r - cl(A) \subset cl(A)$. □

Theorem 4.14. *If A is a subset of X, then $g\alpha r - cl(A) \subset g - cl(A)$, where $g - cl(A)$ is given by $g - cl(A) = \{\cap F \subset X : A \subset F$ and f is a g - closed set in $X\}$.*

Proof. Let A be a subset of X. By definition of $g - cl(A) = \{\cap F \subset X : A \subset F$ and f is a g - closed set in $X\}$. If $A \subset F$ and F is g - closed subset of x, then $A \subset F \in g\alpha r - cl(X)$, because every g closed is $g\alpha r$ - closed subset in X. That is $g\alpha r - cl(A) \subset F$. Therefore $g\alpha r - cl(A) \subset \{\cap F \subset X : A \subset F$ and f is a g - closed set in $X\} = g - cl(A)$. Hence $g\alpha r - cl(A) \subset g - cl(A)$. □

Corollary 4.1. *Let A be any subset of X. Then*

(i) $\left(g\alpha r - int(A)\right)^c = g\alpha r - cl(A^c)$

(ii) $g\alpha r - int(A) = \left(g\alpha r - cl(A^c)\right)^c$

(iii) $g\alpha r - cl(A) = \left(g\alpha r - int(A^c)\right)^c$

Proof. (i) Let $x \in \left(g\alpha r - int(A)\right)^c$. Then $x \notin g\alpha r - int(A)$. That is every $g\alpha r$ - open set U containing x is such that U not subset of A. That is every $g\alpha r$ - open set U containing x is such that $U \cap A^c \neq \varphi$. By Theorem $x \in \left(g\alpha r - cl(A^c)\right)$ and therefore $\left(g\alpha r - int(A)\right)^c \subset g\alpha r - cl(A^c)$.

Conversely, let $x \in g\alpha r - cl(A^c)$. Then by theorem, every $g\alpha r$ - open set U containing x is such that $U \cap A^c \neq \varphi$. That is every $g\alpha r$ - open set U containing x is such that U not subset of A. This implies by definition of $g\alpha r$ - interior of A, $x \notin g\alpha r - int(A)$. That is $x \in \left(g\alpha r - int(A)\right)^c$ and $g\alpha r - cl(A^c) \subset \left(g\alpha r - int(A)\right)^c$. Thus $\left(g\alpha r - int(A)\right)^c = g\alpha r - cl(A^c)$.

(ii) Follows by taking complements in (i).

(iii) Follows by replacing A by A^c in (i).

□

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